Study on APEC-corrected Macroscopic Depletion in 2-D Nodal Analysis

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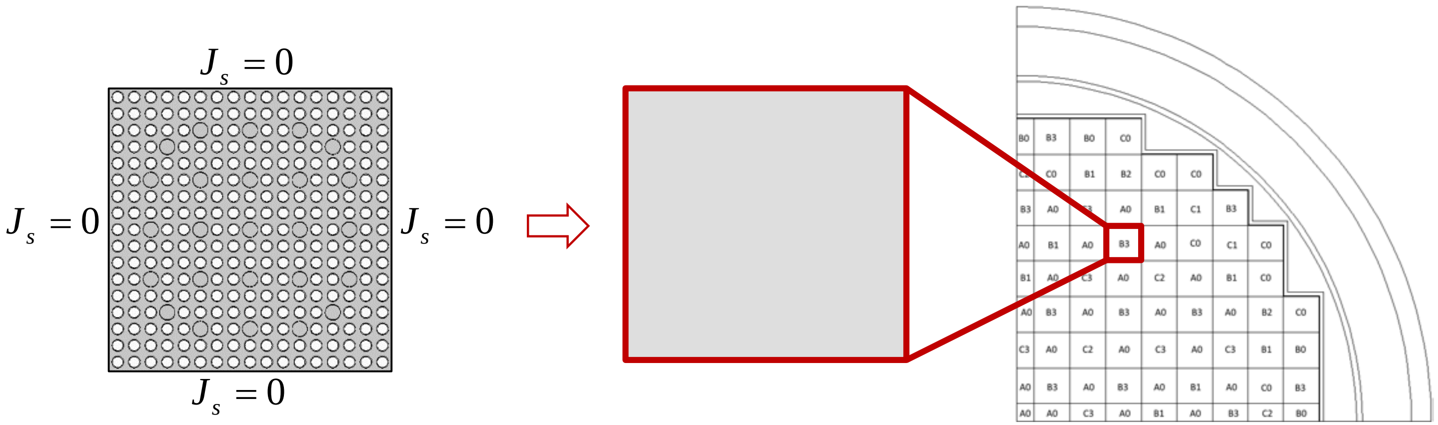
Introduction (1/4)

❖ **Generalized equivalence theory (GET) for 2-step procedures**

➢ Assembly homogenization using all reflective BC (infinite lattice)

- Computing cost for whole-core transport calculation is too large.
- Very successful for the analysis of conventional LWR.

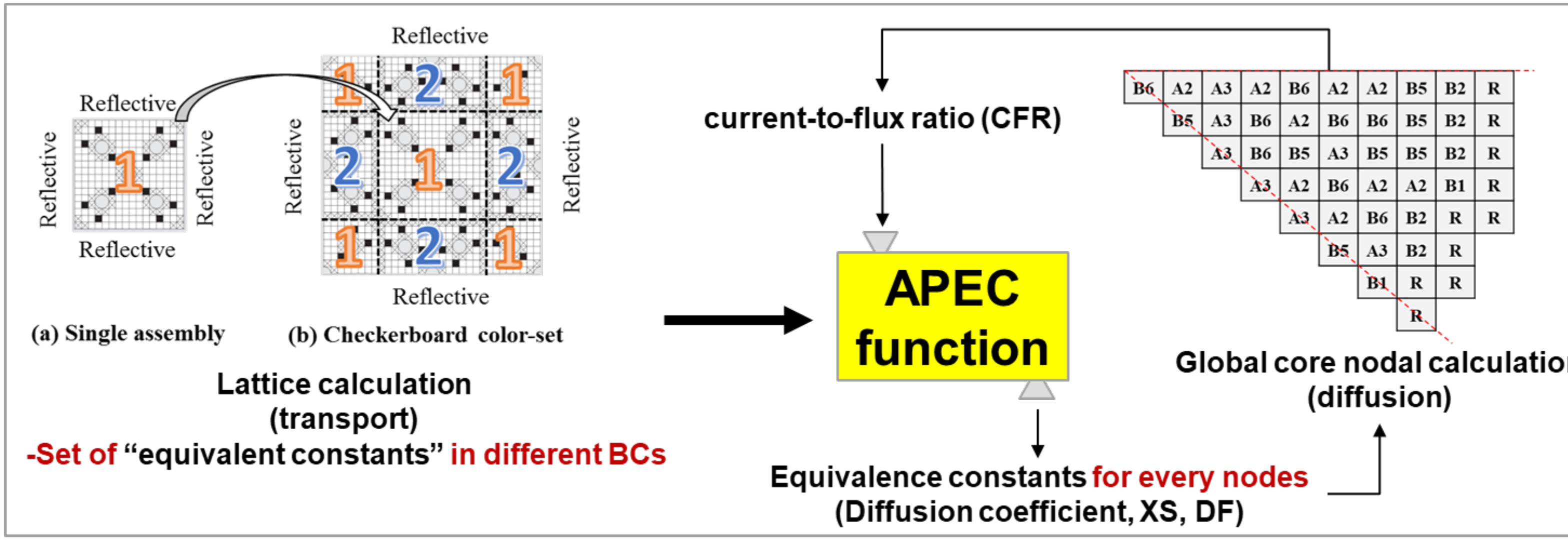


Lattice calculation (transport) | **Homogenization and condensation** | **Global core calculation (diffusion)**

➢ Limitation of conventional 2-step procedures

- Limited accuracy when the neighborhood effect is rather strong.
- The ad-hoc critical spectrum correction was used for the leakage correction.

❖ **Albedo-corrected parameterized equivalence constants (APEC) method**



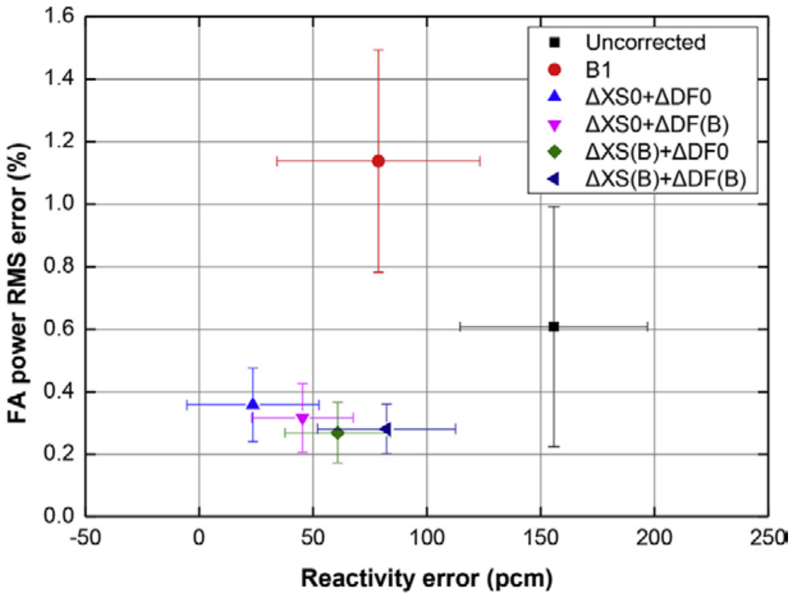
- ➢ Conventional 2-step-consistent approach for computational efficiency
  - Equivalence constants are updated during nodal calculation using actual leakage information.
  - Once determined APEC functions can be used for thousands of nodal calculations.
- ➢ Almost reproduced the transport solution of UOX loaded PWR initial core analyses:
  ~1 % of Max. assembly power error.
- ➢ Application of APEC leakage correction to commercial DeCART2D/MASTER code
  - Corrected group constants in the middle of a microscopic core depletion.
  - Approximated burnup-dependent APEC functions:
    FA of interest to be burned and that the surrounding FAs to be fresh.
- ➢ Improved APEC DF modeling and APEC leakage correction for the BR region

Introduction (3/4)

❖ APEC leakage-corrected microscopic depletion analysis

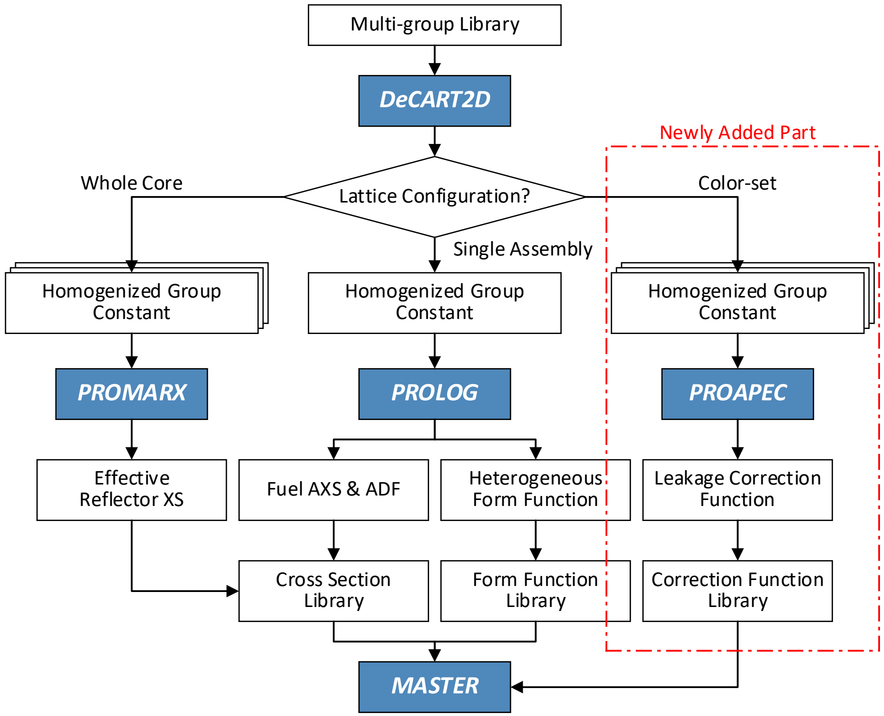
➢ DeCART2D/MASTER code system

- Burnup-independent and -dependent APEC leakage correction were implemented to commercial DeCART2D/MASTER code system
- Approximated burnup-dependent APEC functions were applied.
- [†]*"The results showed that the APEC correction can enhance the nodal equivalence significantly in terms of the neutron multiplication factor and nodal power distribution with inexpensive additional calculations."*



[†]Fig. 24. Distribution of errors in terms of reactivity and FA power.

Benchmark problems: SMR that consists of 17x17 UOX-loaded FAs



[†]Fig. 1. Flow chart of the DeCART2D/MASTER code system.

[†] K. Lee, W. Kim, Y. Kim, Improved nodal equivalence with leakage-corrected cross sections and discontinuity factors for PWR depletion analysis, Nucl. Eng. Technol. 51 (2019) 1195–1208.

Introduction (4/4)

❖ Objective of this paper

- Application of the APEC leakage correction for macroscopic depletion analysis.

❖ Codes and benchmark problem

- DeCART2D code
  - MOC based 2D transport lattice code.
  - Whole-core transport calculations (reference solution).
  - Lattice and color-set calculations.
- In-house nodal code
  - Implementation of the NEM within the p-CMFD formulation.
  - APEC leakage correction for FAs.
  - Macroscopic depletion analysis.
- UOX-loaded and Partially MOX-loaded SMR benchmark problem

Options
▹ 47-group library.

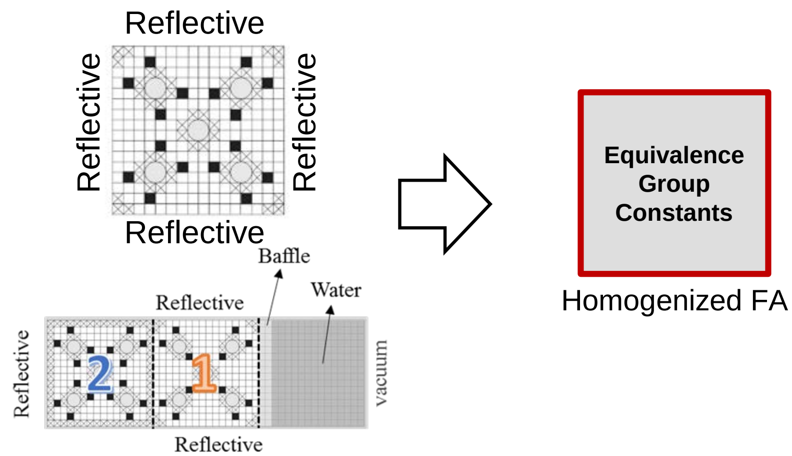
▹ Transport corrected P0 XS for anisotropic scattering treatment.

▹ Default ray tracing
- two polar angles for 90°
- eight azimuthal angles for 90°
- ray-spacing of 0.02 cm

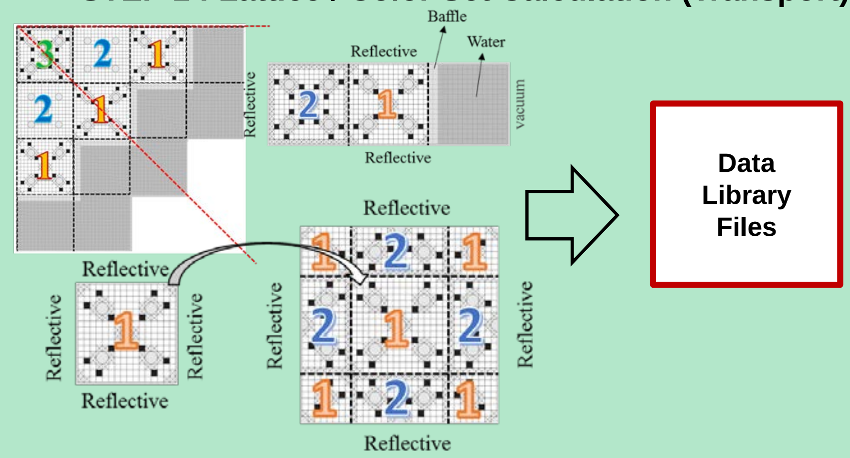
APEC-corrected Macroscopic Depletion Method (1/7)

❖ Summary of APEC method compared to conventional 2-step method

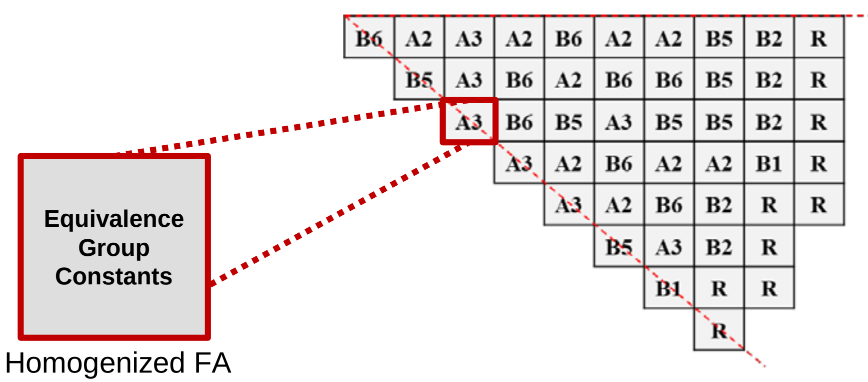
STEP 1 : Lattice / Color Set Calculation (Transport)



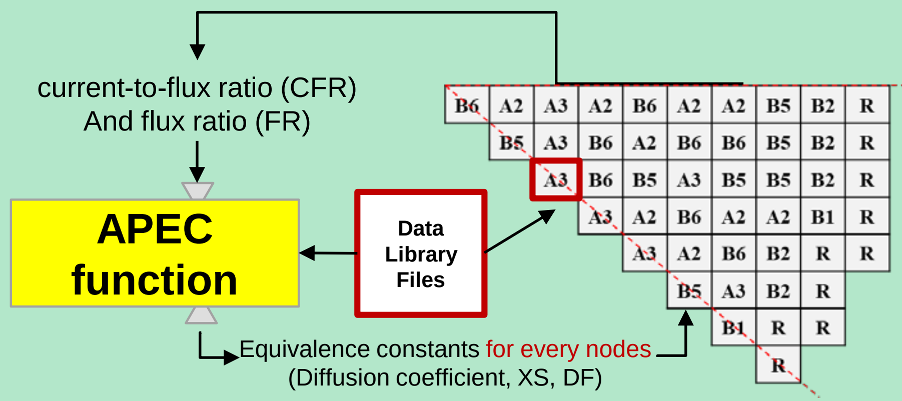
STEP 1 : Lattice / Color Set Calculation (Transport)



STEP 2 : Whole core nodal calculation (Diffusion)



STEP 2 : Whole core nodal calculation (Diffusion)



<Conventional 2-step Method> <APEC Method >

APEC-corrected Macroscopic Depletion Method (2/7)

❖ **APEC leakage-corrected macroscopic depletion method in NEM analysis**

- Xe/Sm calculation
  - Transient Xe/Sm number densities are obtained by applying forward-differencing on the time derivative terms.

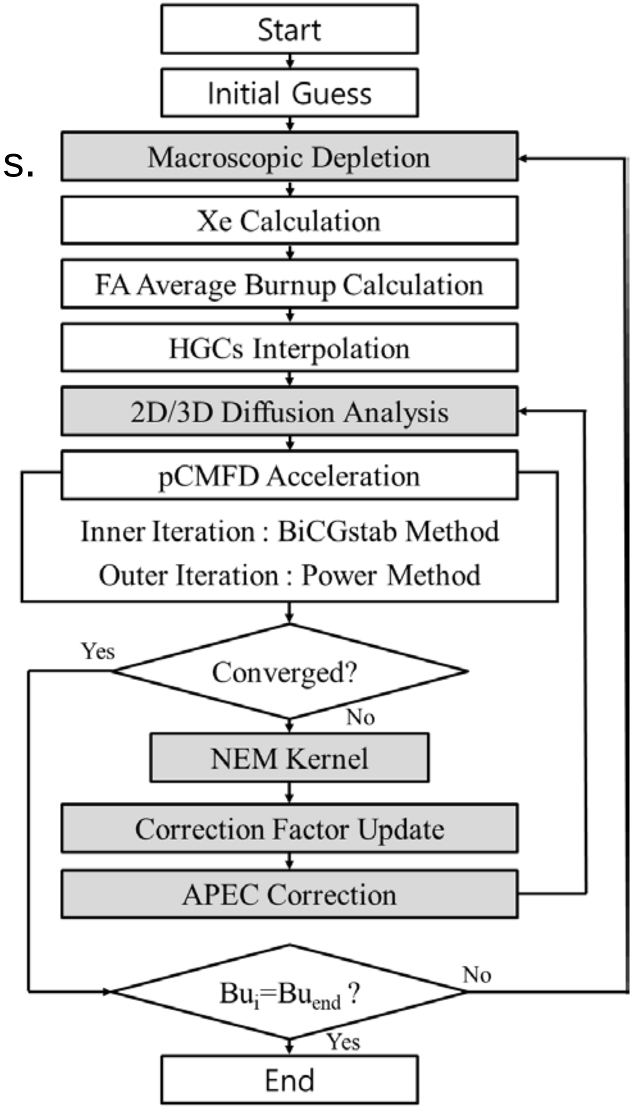
$$N_I(t_n+\Delta t) = N_I(t_n) + \Delta t\left(\gamma_I \sum_{g=1}^{2} \Sigma_{f,g}(t_n)\phi_g(t_n) - \lambda_I N_I(t_n)\right) \quad (1)$$

$$N_{Xe}(t_n+\Delta t) = N_{Xe}(t_n) + \Delta t(\lambda_I N_I(t_n) + \gamma_{Xe}\sum_{g=1}^{2}\Sigma_{f,g}(t_n)\phi_g(t_n) - \lambda_{Xe}N_{Xe}(t_n) - \sum_{g=1}^{2}\sigma_{Xe,a,g}(t_n)N_{Xe}(t_n)\phi_g(t_n)) \quad (2)$$

- Macroscopic depletion
  - The depletion at each fuel assembly is approximated as proportional to power per heavy metal.

$$\Delta B_i = \Delta B_c \frac{P_i}{G_i} / \frac{P_c}{G_c} \quad (3)$$

- APEC leakage correction
  - Burnup-independent and –dependent APEC functions are implemented.



APEC-corrected Macroscopic Depletion Method (3/7)

- **Burnup-independent APEC functions**
  - APEC XS and DF Functions for FAs
    - APEC XS Functions

$$\Sigma_{x,g}^{FA} = \Sigma_{s,g}^{SA} + \Delta\Sigma_{x,g}^{FA}, \quad (4)$$

$$\Delta\Sigma_{x,F}^{FA} = a_{x,F} CFR_F^N + b_{x,F} CFR_T^N + c_{x,F}, \quad (4.1)$$

$$\Delta\Sigma_{x,T}^{FA} = a_{x,T} CFR_T^N + b_{x,T} \left(CFR_T^N \right)^2 + c_{x,T}. \quad (4.2)$$

    - APEC DF Functions

$$DF_{g,s}^{FA} = ADF_{g,s}^{SA} + \Delta DF_{g,s}^{FA}, \quad (5)$$

$$\Delta DF_{g,s}^{FA} = a_{g,1} FR_g^S + a_{g,2} CFR_g^S + a_{g,3} CFR_g^N + c_g. \quad (5.1)$$

where,

Σ^{SA} : single assembly HGC,

F : fast group, T : thermal group,

$J_{g,s}$: surface current, $\phi_{g,s}$: surface flux, $\overline{\phi}_g^{Avg}$: average flux,

$$c_{x,G} \begin{cases} = 0 & \text{for interior FA} \\ \neq 0 & \text{for peripheral FA} \end{cases}.$$

Assembly-averaged CFR

$$CFR_g^N \equiv \frac{\sum_s J_{g,s}}{\sum_s \phi_{g,s}}$$

Assembly-surface CFR

$$CFR_g^S \equiv \frac{J_{g,s}}{\phi_{g,s}}$$

Assembly-surface FR

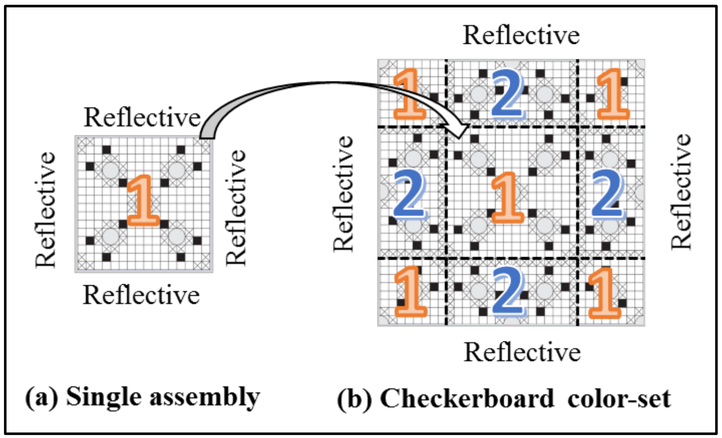
$$FR_g^S \equiv \frac{\overline{\phi}_g^{Avg}}{\phi_{g,s}}$$

APEC-corrected Macroscopic Depletion Method (4/7)

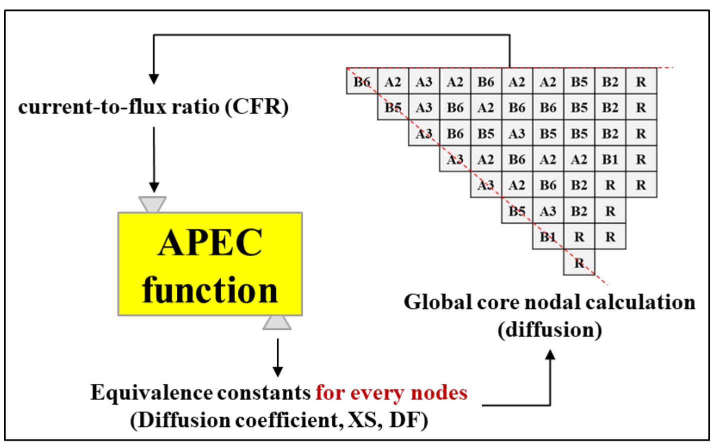
❖ Burnup-independent APEC functions

➢ APEC XS Functions

- The procedure of determining the APEC XS functions.



Lattice calculation (transport analysis)



Nodal analysis (diffusion analysis)

① Arrangement of data obtained by color-set calculation

$$a_{x,F} CFR_F + b_{x,F} CFR_T + c_{x,F} = \Sigma_{x,F} - \Sigma_{s,F}^{SA} = \Delta\Sigma_{x,F}$$

$$a_{x,T} CFR_T + b_{x,T} \left(CFR_T \right)^2 + c_{x,T} = \Sigma_{x,T} - \Sigma_{s,T}^{SA} = \Delta\Sigma_{x,T}$$

② Determining coefficients by multiple linear regression

$$a_{x,F}, b_{x,F}, c_{x,F} \qquad a_{x,T}, b_{x,T}, c_{x,T}$$

③ Updating APEC XS by corresponding CFRs

$$\Delta\Sigma_{x,F} = a_{x,F} CFR_F + b_{x,F} CFR_T + c_{x,F},$$

$$\Delta\Sigma_{x,T} = a_{x,T} CFR_T + b_{x,T} \left(CFR_T \right)^2 + c_{x,T}.$$

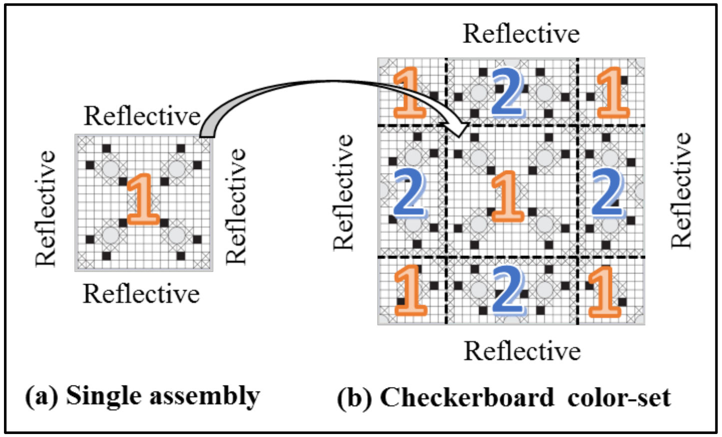
$$\Sigma_{x,g} = \Sigma_{s,g}^{SA} + \Delta\Sigma_{x,g}$$

APEC-corrected Macroscopic Depletion Method (5/7)

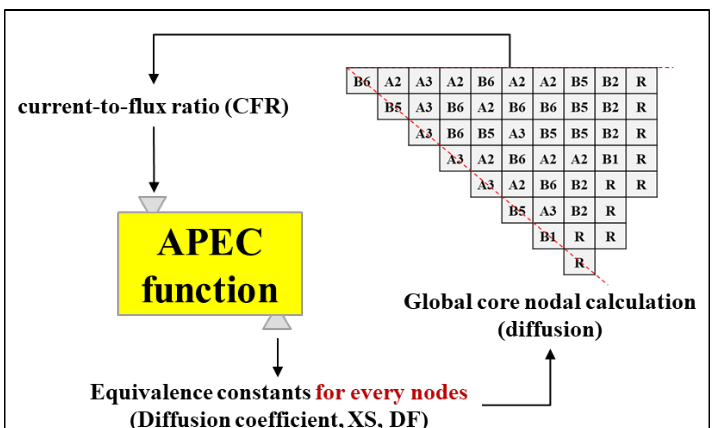
❖ Burnup-independent APEC functions

➢ APEC DF Functions

- The procedure of determining the APEC DF functions.



Lattice calculation (transport analysis)



Nodal analysis (diffusion analysis)

① **Arrangement of data obtained by color-set calculation**

$$a_{g,1}FR_{g,s} + a_{g,2}CFR_{g,s} + a_{g,3}CFR_{g} + c_{g} = DF_{g,s} - ADF_{g,s}^{SA} = \Delta DF_{g,s}$$

② **Determining coefficients by multiple linear regression**

$$a_{g1}, a_{g2}, a_{g3}, c_{g}$$

③ **Updating APEC XS by corresponding CFRs**

$$\Delta DF_{g,s} = a_{g,1}FR_{g,s} + a_{g,2}CFR_{g,s} + a_{g,3}CFR_{g} + c_{g}$$

$$DF_{g,s} = ADF_{g,s}^{SA} + \Delta DF_{g,s}$$

APEC-corrected Macroscopic Depletion Method (6/7)

❖ **Predetermined burnup-dependent APEC functions**

➢ Burnup-dependent APEC XS and DF Functions

- APEC XS Functions

$$\Sigma^{FA}_{x,g,Bu} = \Sigma^{SA}_{s,g,Bu} + \Delta\Sigma^{FA}_{x,g,Bu}, \quad (6)$$

$$\Delta\Sigma^{FA}_{x,F,Bu} = a_{x,F,Bu} CFR^{N}_{F,Bu} + b_{x,F,Bu} CFR^{N}_{T,Bu} + c_{x,F,Bu}, \quad (6.1)$$

$$\Delta\Sigma^{FA}_{x,T,Bu} = a_{x,T,Bu} CFR^{N}_{T,Bu} + b_{x,T,Bu} \left(CFR^{N}_{T,Bu}\right)^2 + c_{x,T,Bu}. \quad (6.2)$$

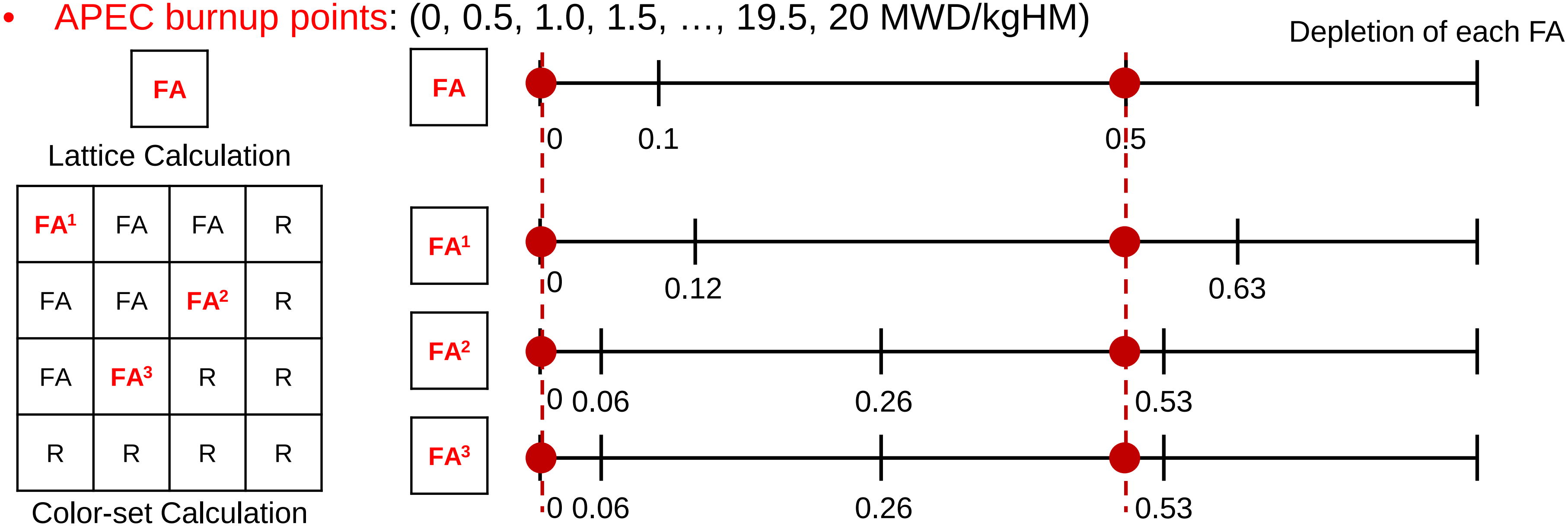
- APEC DF Functions

$$DF^{FA}_{g,s,Bu} = ADF^{SA}_{g,s,Bu} + \Delta DF^{FA}_{g,s,Bu}, \quad (7)$$

$$\Delta DF^{FA}_{g,s} = a_{g,1} FR^{S}_{g} + a_{g,2} CFR^{S}_{g} + a_{g,3} CFR^{N}_{g} + c_{g}. \quad (7.1)$$

➢ Approximation for burnup-dependent APEC functions

- Average core burnup points for lattice and color-set calculation: (0, 0.1, 0.5, 1.0, 1.5, …, 24.5, 25 MWD/kgHM)
- APEC burnup points: (0, 0.5, 1.0, 1.5, …, 19.5, 20 MWD/kgHM)



APEC-corrected Macroscopic Depletion Method (7/7)

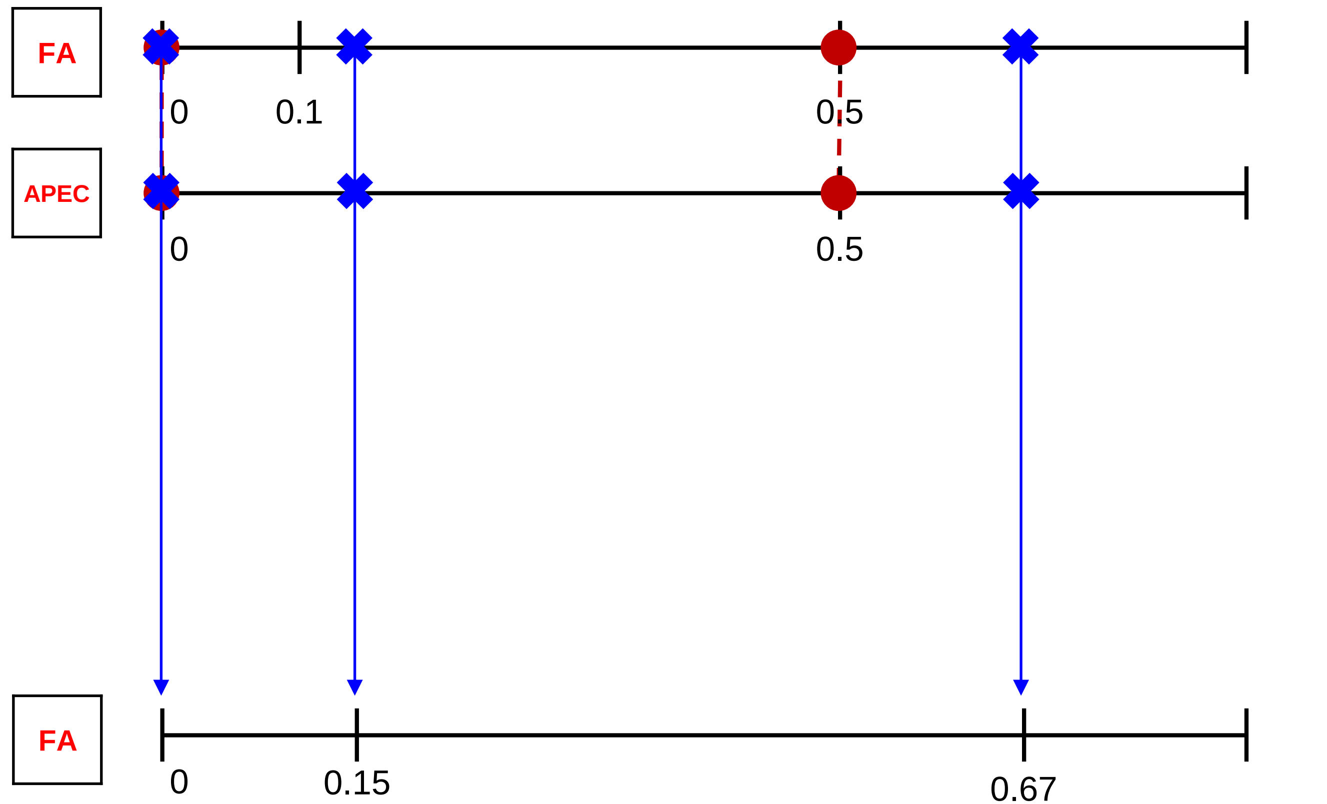
❖ **Application burnup-dependent APEC functions in nodal analysis**

➢ Approximation for burnup-dependent APEC functions

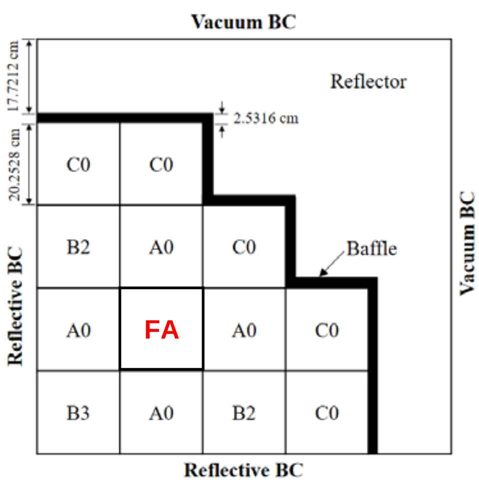
- Average core burnup points for nodal analysis:
  (0, 0.1, 0.5, 1.0, 1.5, …, 14.5, 15.0 MWD/kgHM)
- APEC burnup points: (0, 0.5, 1.0, 1.5, …, 19.5, 20 MWD/kgHM)

FA

HGCs and ADFs
<Lattice Calculation>



Depletion of each FA

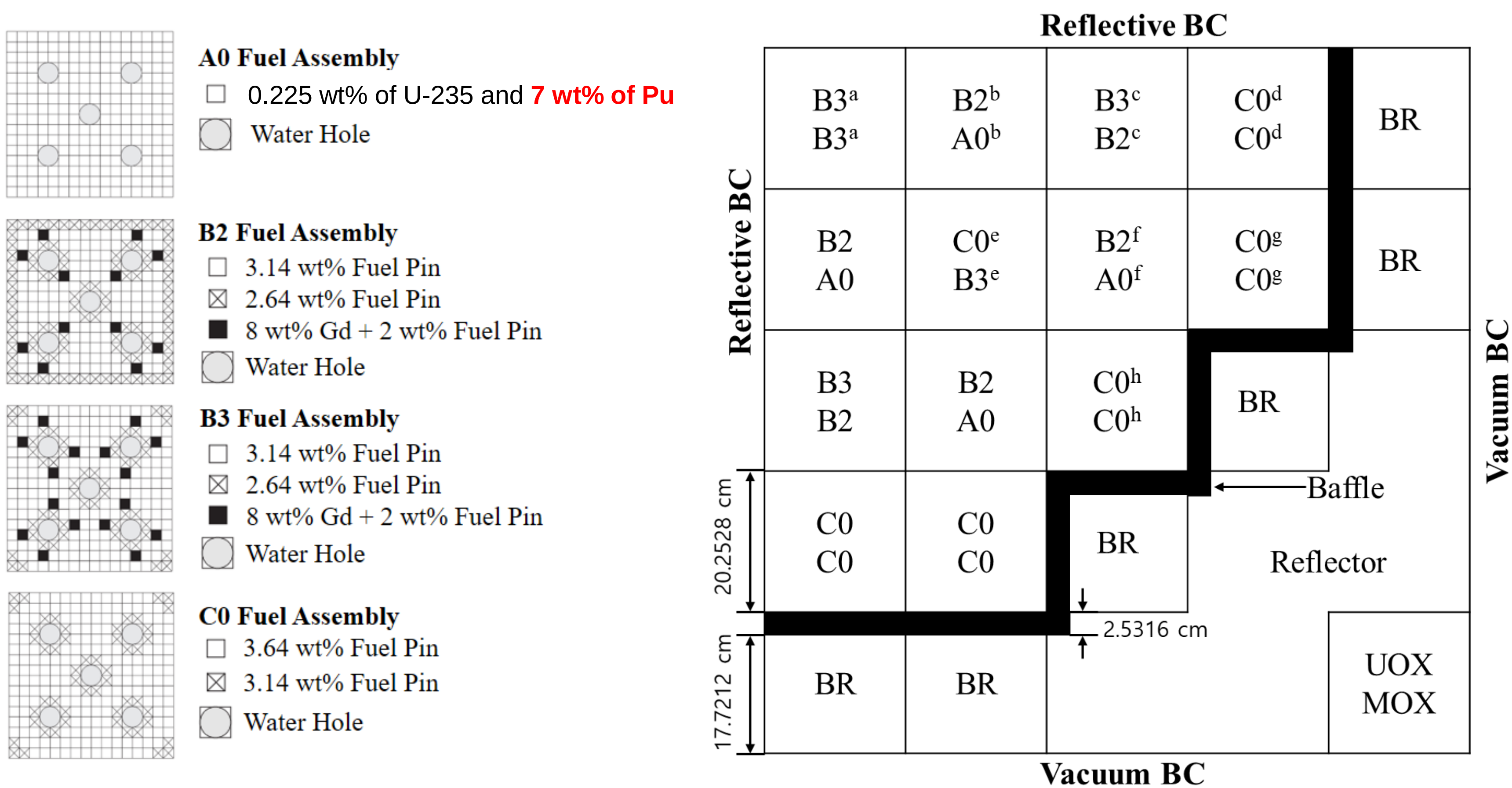


Color-set Calculation

❖ Benchmark Problem

➢ UOX-loaded and partially MOX-loaded SMR problem

- Soluble boron free condition
- Hot zero power (HZP) condition → No TH feedback consideration



❖ Benchmark Problem

➢ UOX-loaded and partially MOX-loaded SMR problem

- Soluble boron free condition
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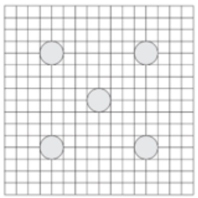
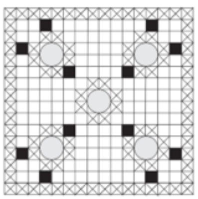


A0 Fuel Assembly

□ 0.225 wt% of U-235 and **7 wt% of Pu**

○ Water Hole



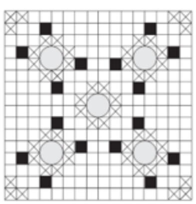
B2 Fuel Assembly

□ 3.14 wt% Fuel Pin

⊠ 2.64 wt% Fuel Pin

■ 8 wt% Gd + 2 wt% Fuel Pin

○ Water Hole



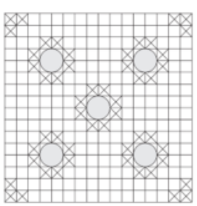
B3 Fuel Assembly

□ 3.14 wt% Fuel Pin

⊠ 2.64 wt% Fuel Pin

■ 8 wt% Gd + 2 wt% Fuel Pin

○ Water Hole

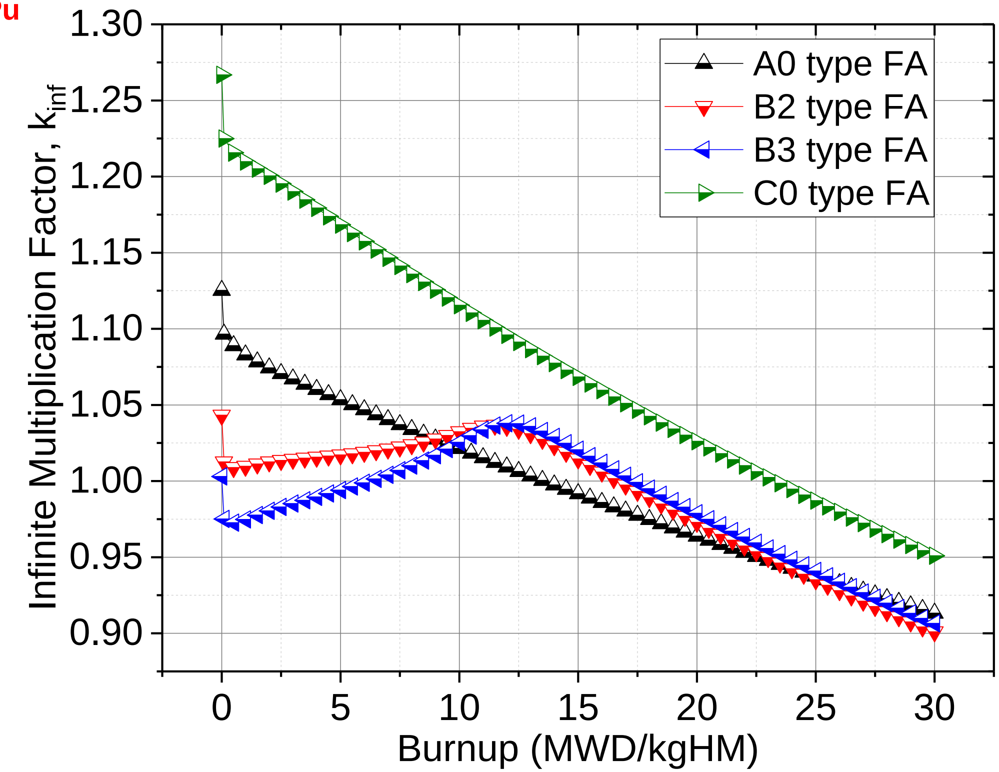


C0 Fuel Assembly

□ 3.64 wt% Fuel Pin

⊠ 3.14 wt% Fuel Pin

○ Water Hole



Evolution of infinite multiplication factor of each FA

❖ List of Color-set Calculation

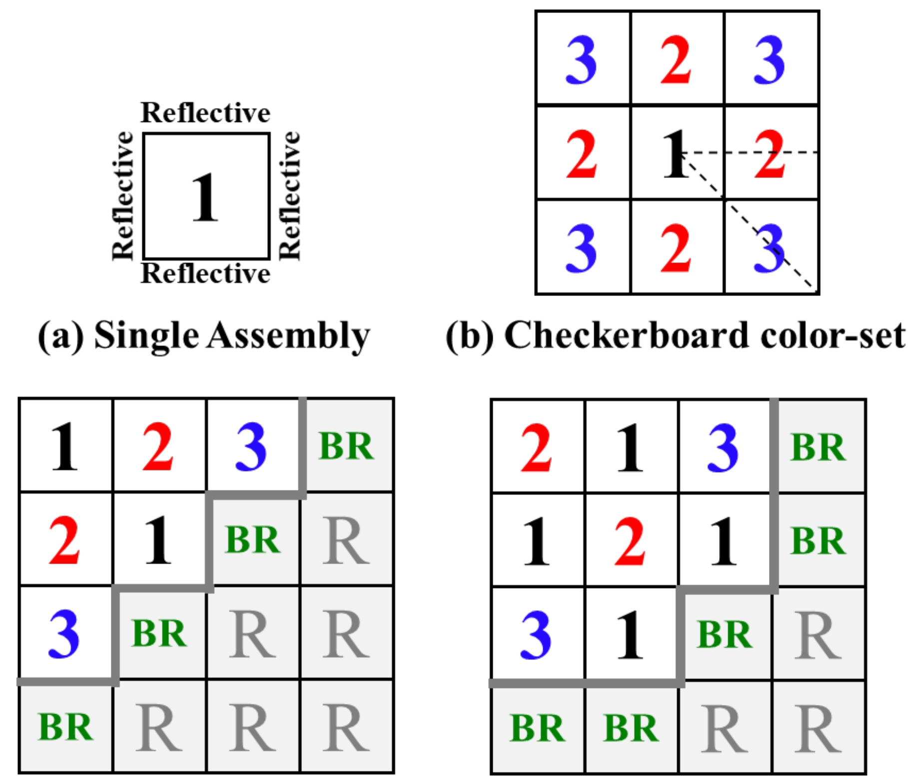
➢ Average core burnup points for color-set calculation:
(0, 0.1, 0.5, 1.0, 1.5, …, 24.5, 25 MWD/kgHM)

Color-set Model	Combination of FAs
Checkerboard	(B2,B3,C0), (B3,C0,B2), (C0,B3,B2), (A0,B2,B3), (B3,C0,A0), (C0,A0,B2)
L-Shape Type1	(B2,B3,C0), (B2,C0,B3), (B3,C0,B2), (B3,B2,C0), (C0,B2,B3), (C0,B3,B2), (A0,B2,C0), (A0,C0,B3), (B2,A0,C0), (B2, B3, A0)
L-Shape Type2	(B2,B3,C0), (B2,C0,B3), (B3,C0,B2), (B3,B2,C0), (C0,B2,B3), (C0,B3,B2), (B2,B3,B2), (C0,B2,C0), (B3,C0,B3), (C0,B2,A0), (A0,B3,C0), (B2,A0,B3), (B3,B2,A0), (A0,C0,A0), (C0,A0,C0), (B3,A0,B3)

Options
▹ 47-group library.

▹ Transport corrected P0 XS for anisotropic scattering treatment.

▹ Default ray tracing
- two polar angles for 90°
- eight azimuthal angles for 90°
- ray-spacing of 0.02 cm



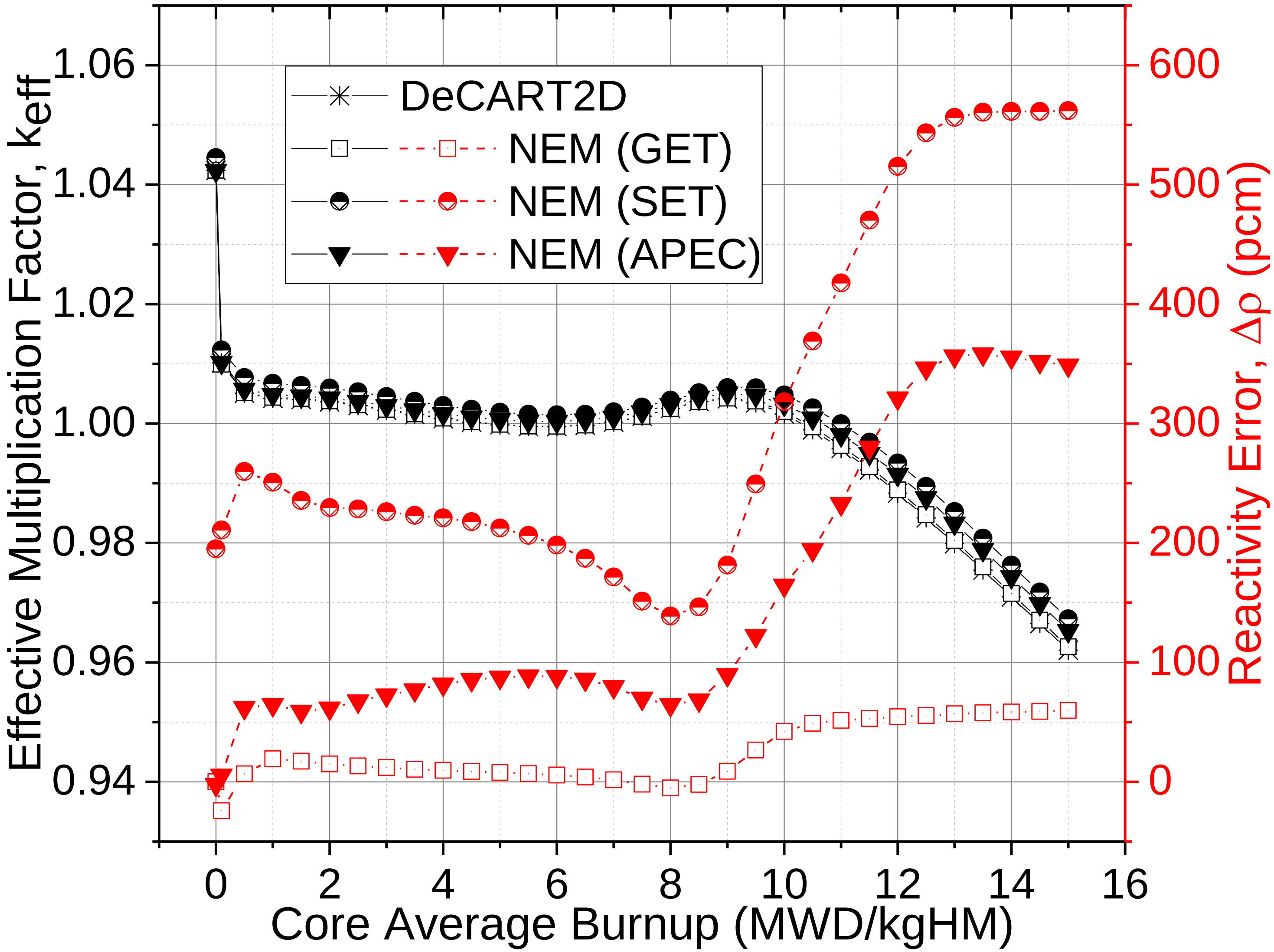
(a) Single Assembly (b) Checkerboard color-set

(c) L-shape color-set type 1 (d) L-shape color-set type 2

Numerical Results (4/11)

❖ **UOX-loaded SMR core analysis (burnup-independent APEC functions)**

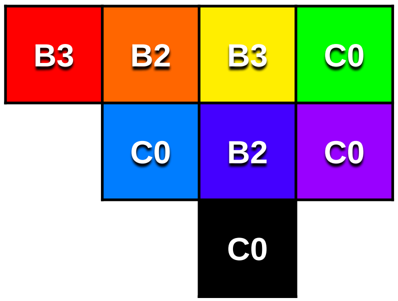
➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)

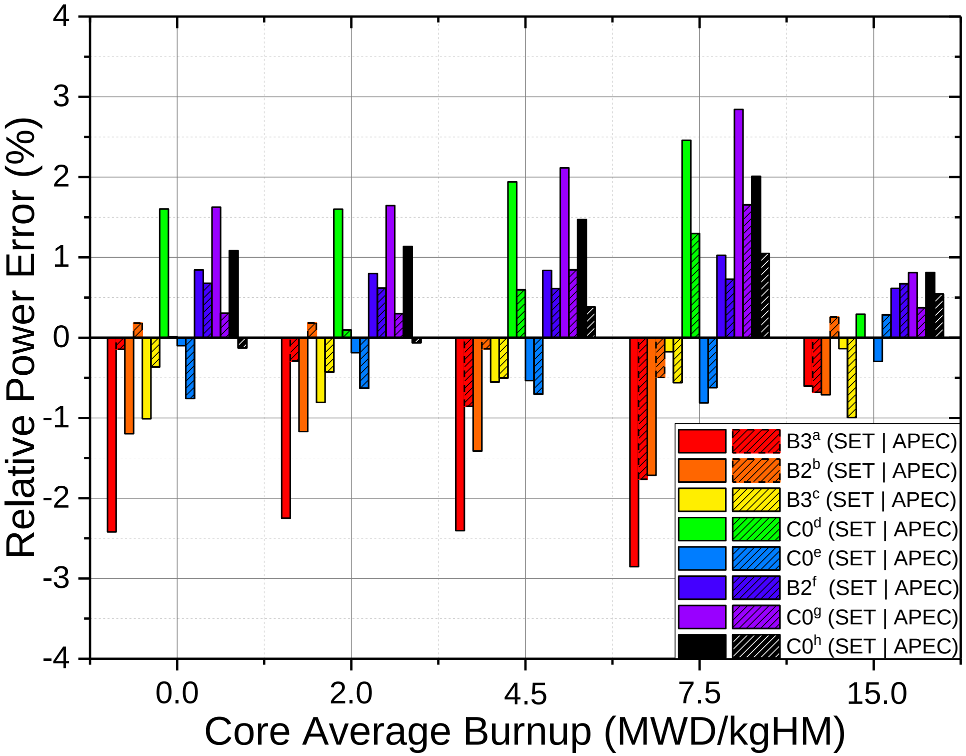


Numerical Results (5/11)

❖ **UOX-loaded SMR core analysis (burnup-independent APEC functions)**

➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)



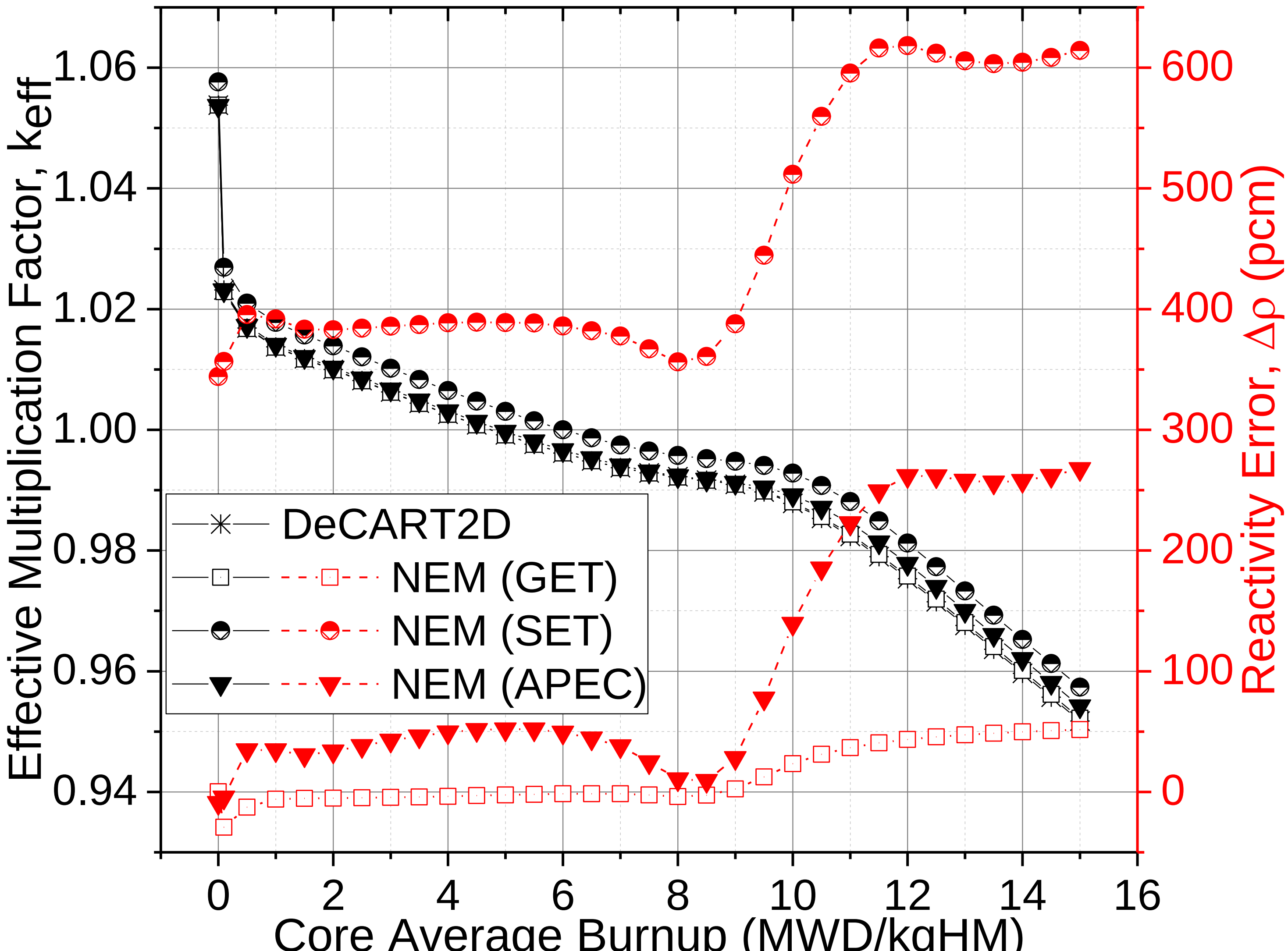


HGCs	k_{eff}	$\Delta\rho$(pcm)	RRMS[a] (%)	Min.[b] (%)	Max.[c] (%)
Ref.	*1.042409*	*0.00 MWD/kgHM*			
GET[d]	1.042409	-0.02	0.003	-0.006	0.002
SET[e]	1.044532	195.01	1.353	-2.419	1.625
APEC[f]	1.042393	-1.52	0.396	-0.759	0.676
Ref.	*1.003631*	*2.00 MWD/kgHM*			
GET	1.003663	3.14	0.120	-0.123	0.224
SET	1.005950	229.68	1.308	-2.249	1.645
APEC	1.004274	63.78	0.380	-0.630	0.617
Ref.	*1.000213*	*4.50 MWD/kgHM*			
GET	1.000224	1.10	0.192	-0.198	0.353
SET	1.002396	217.75	1.537	-2.405	2.115
APEC	1.001100	88.60	0.608	-0.855	0.845
Ref.	*1.001227*	*7.50 MWD/kgHM*			
GET	1.001064	-16.23	0.305	-0.397	0.546
SET	1.002745	151.22	1.944	-2.853	2.843
APEC	1.001955	72.56	1.096	-1.764	1.654
Ref.	*0.962057*	*15.00 MWD/kgHM*			
GET	0.962393	36.28	0.226	-0.286	0.456
SET	0.967287	561.98	0.582	-0.712	0.812
APEC	0.965341	353.58	0.564	-0.995	0.673

Numerical Results (6/11)

❖ **Partially MOX-loaded SMR core analysis (burnup-independent APEC functions)**

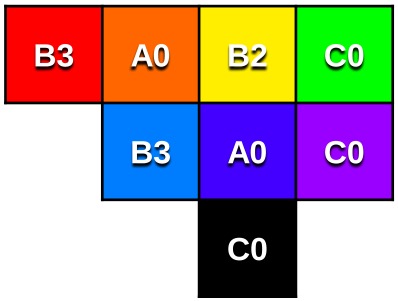
➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)

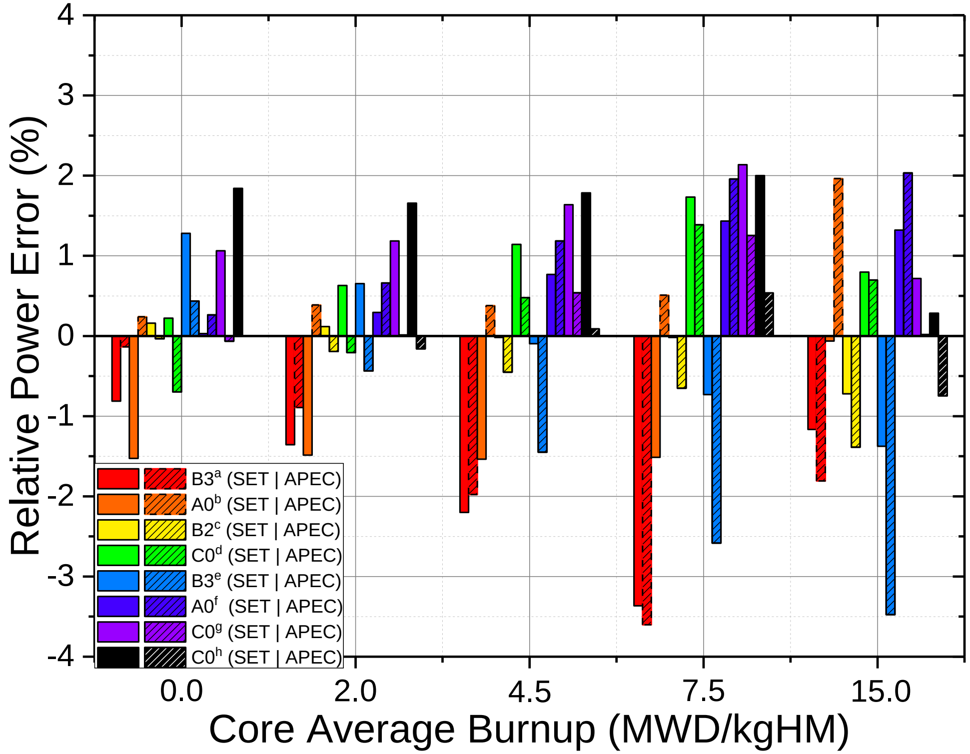


Numerical Results (7/11)

❖ **Partially MOX-loaded SMR core analysis (burnup-independent APEC functions)**

➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)



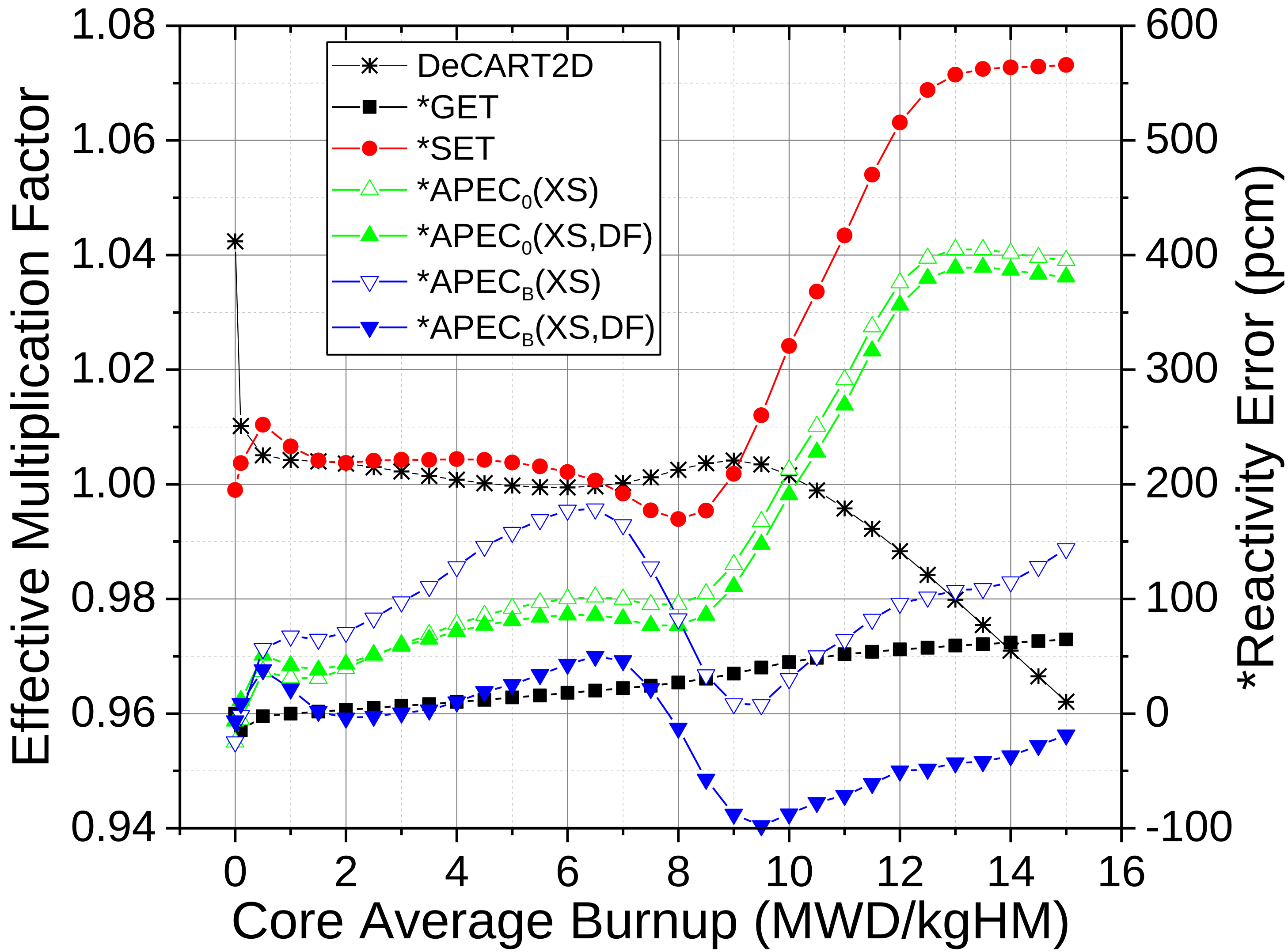


HGCs	k_{eff}	Δρ(pcm)	RRMS[a] (%)	Min.[b] (%)	Max.[c] (%)
Ref.	*1.053803*	*0.00 MWD/kgHM*			
GET[d]	1.053804	0.09	0.004	-0.006	0.005
SET[e]	1.057639	344.19	0.991	-1.527	1.841
APEC[f]	1.053702	-9.06	0.333	-0.697	0.435
Ref.	*1.009993*	*2.00 MWD/kgHM*			
GET	1.009940	-5.17	0.148	-0.160	0.236
SET	1.013913	382.82	1.008	-1.485	1.657
APEC	1.010337	33.68	0.425	-0.893	0.660
Ref.	*1.000835*	*4.50 MWD/kgHM*			
GET	1.000805	-3.02	0.178	-0.176	0.310
SET	1.004749	389.23	1.298	-2.200	1.784
APEC	1.001351	51.50	0.901	-1.978	1.184
Ref.	*0.992845*	*7.50 MWD/kgHM*			
GET	0.992821	-2.43	0.232	-0.310	0.397
SET	0.996476	367.04	1.747	-3.364	2.136
APEC	0.993089	24.77	1.663	-3.601	1.958
Ref.	*0.951762*	*15.00 MWD/kgHM*			
GET	0.952230	51.62	0.180	-0.113	0.358
SET	0.957359	614.27	0.884	-1.375	1.320
APEC	0.954193	267.63	1.680	-3.476	2.033

Numerical Results (8/11)

❖ **UOX-loaded SMR core analysis (burnup-dependent APEC functions)**

➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)

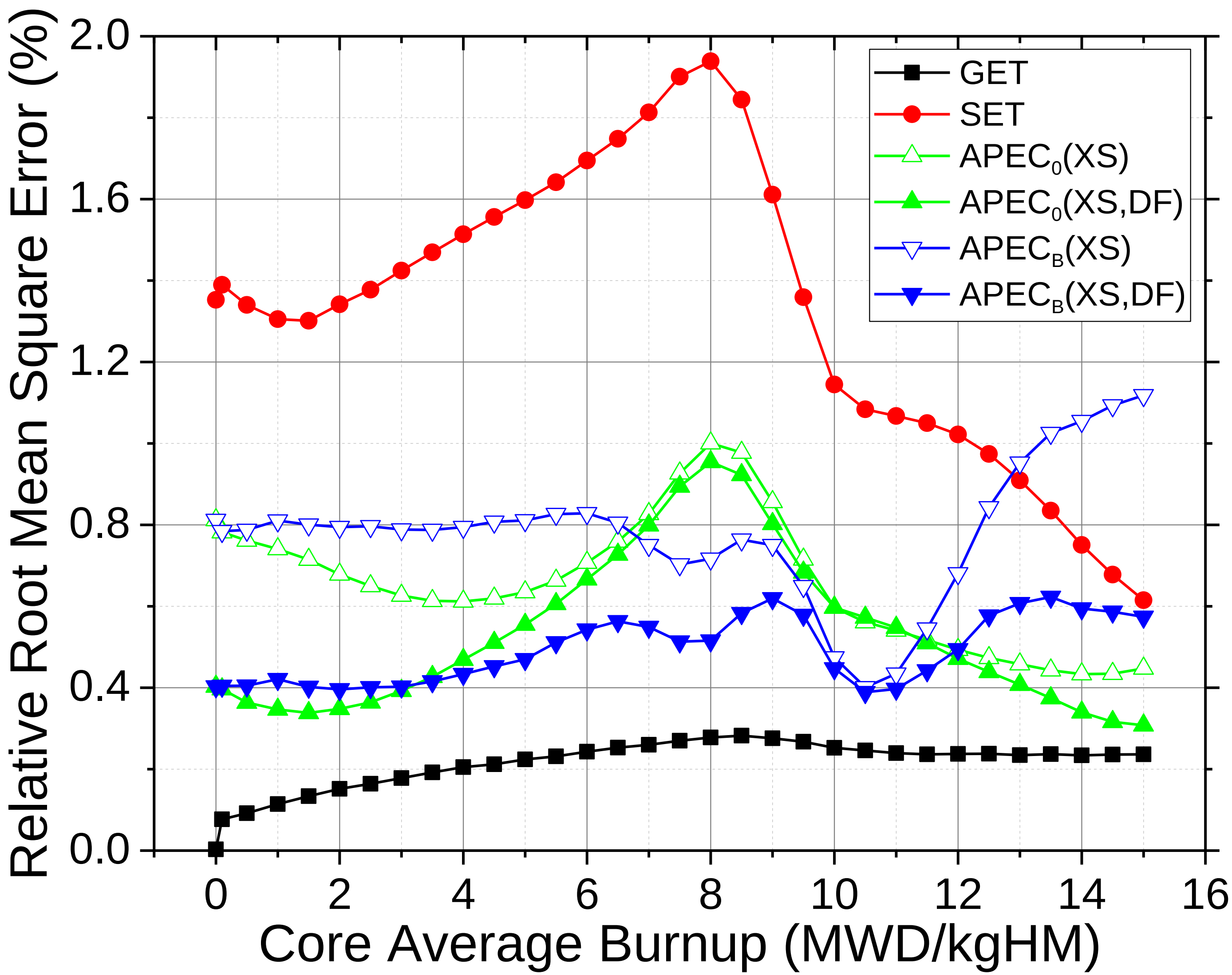


APEC burnup points: (0, 1, 2, …, 19, 20 MWD/kgHM)

Numerical Results (9/11)

❖ **UOX-loaded SMR core analysis (burnup-dependent APEC functions)**

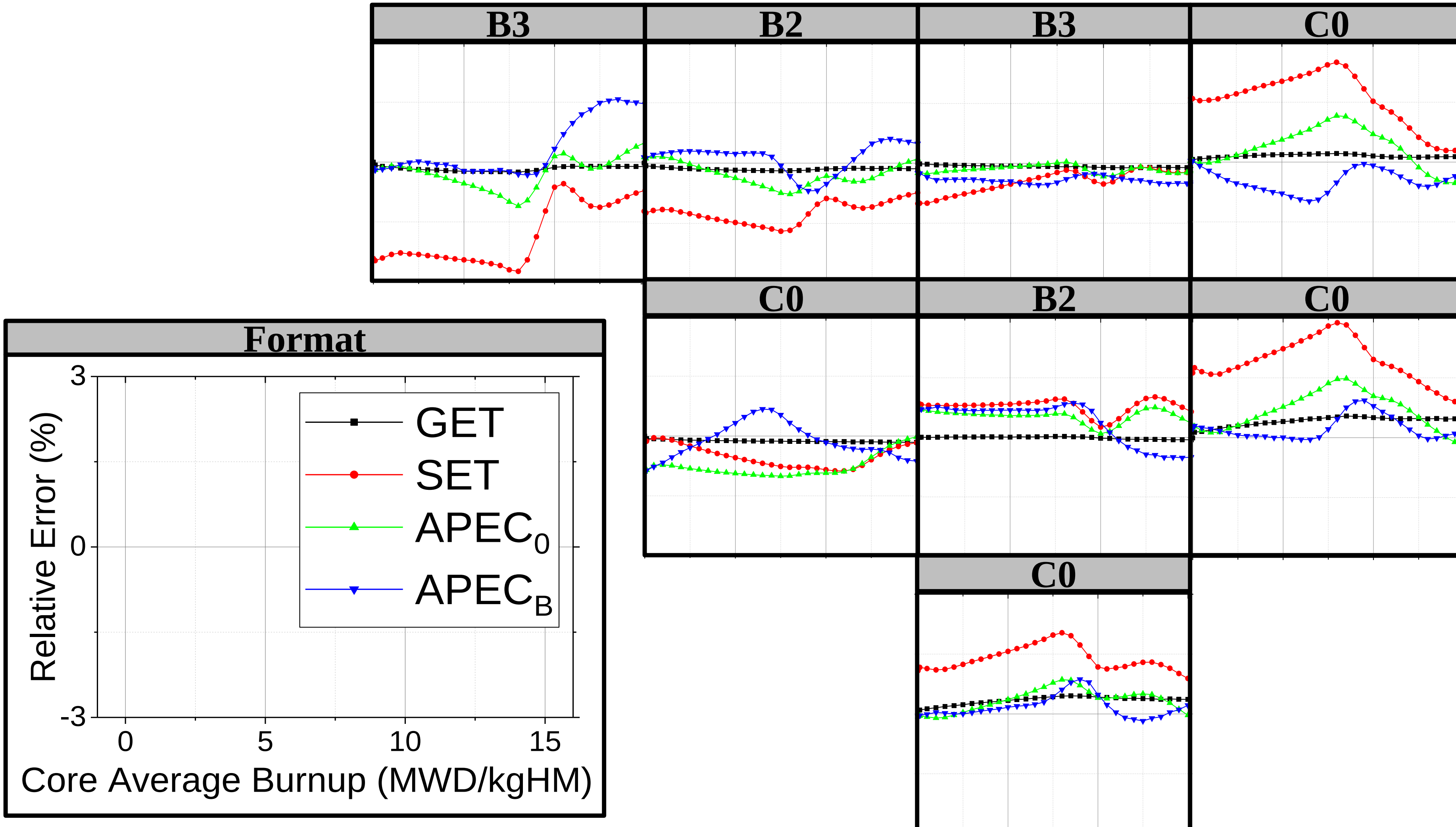
➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)



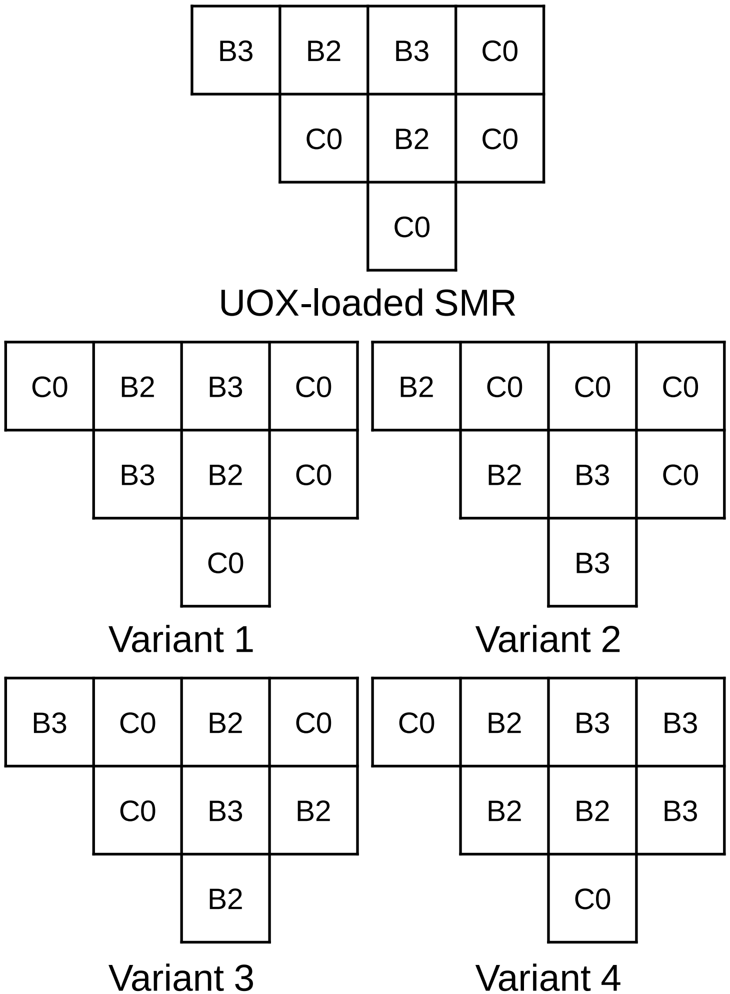
Numerical Results (10/11)

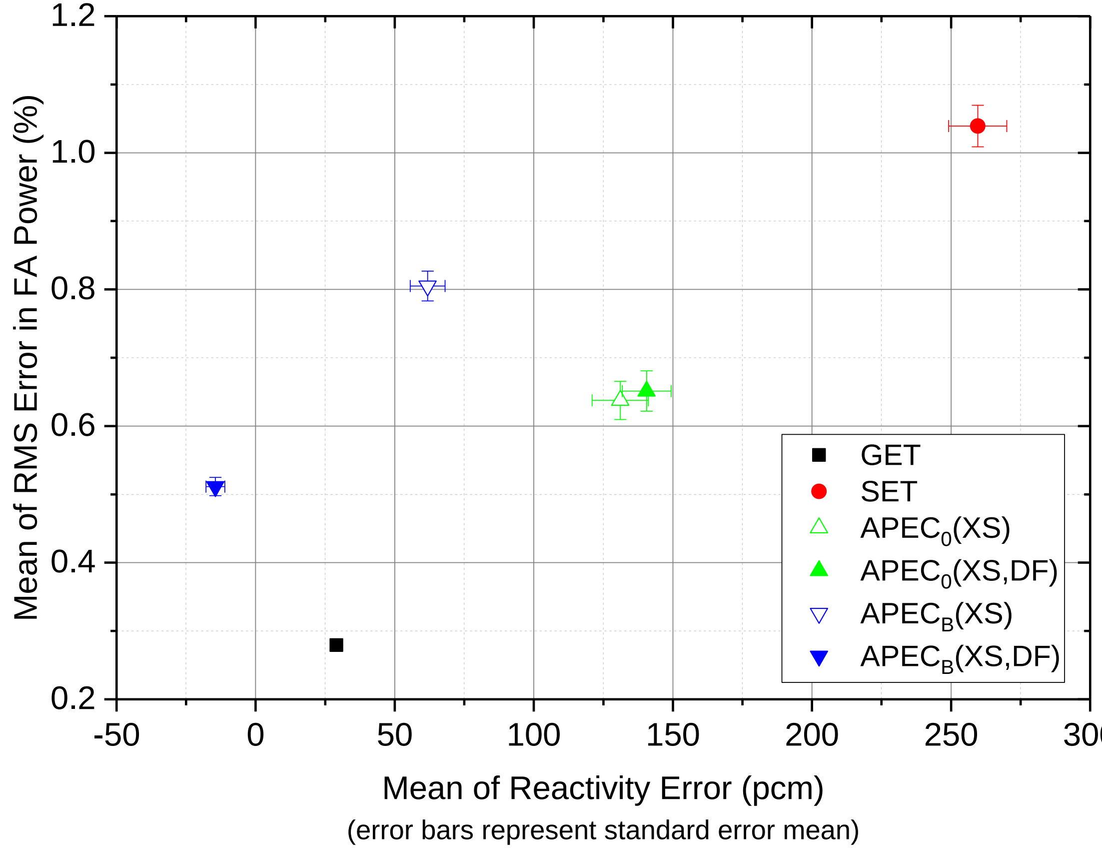
❖ **UOX-loaded SMR core analysis (burnup-dependent APEC functions)**

➢ Evolution of effective multiplication factor and corresponding reactivity error (pcm)



- **UOX-loaded SMR core analysis (burnup-dependent APEC functions)**
  - Evolution of effective multiplication factor and corresponding reactivity error (pcm)





Summary

❖ **Summary**

➢ APEC leakage correction for the macroscopic depletion in nodal analysis

- DeCART2D code was used to whole-core, lattice, and color-set depletion calculations.
- In-house nodal code was used to perform the APEC leakage correction in macroscopic depletion analysis.
- The burnup-independent and -dependent APEC functions were proposed.

➢ UOX-loaded and Partially MOX-loaded SMR benchmark problem

- UOX-loaded and Partially MOX-loaded SMR problems were introduced for performing burnup-independent APEC leakage correction for the macroscopic depletion.
- UOX-loaded SMR variant core problems were additionally analyzed for the burnup-dependent APEC leakage correction in macroscopic depletion analysis.

Conclusions and Future Works

❖ Conclusions

- The burnup-independent and -dependent APEC leakage correction have been implemented to macroscopic depletion in NEM-based nodal analysis.

- It was demonstrated that both APEC leakage correction can enhance the nodal equivalence in terms of reactivity error (pcm) and relative error (%) in assembly power.

- It was proven that the burnup-dependent APEC leakage correction can substantially improve the accuracy of nodal solution compared to those of burnup-independent APEC leakage correction.

- It is concluded that the APEC correction can lead to a very accurate and reliable multiplication factor and power distribution in the macroscopic depletion analysis.

❖ Future Works

- Optimization of the color-set depletion calculation for predetermining burnup-dependent APEC functions.

Thank you for your attention!