An Improved Deterministic Truncation of Monte Carlo Solutions for Nuclear Reactor Analysis

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1. Introduction

An improved deterministic truncation of Monte Carlo (iDTMC) solutions method has been preliminarily established by introducing a new calculational strategy applied with a coupled coarse-mesh finite difference (CMFD) and a decoupled fine-mesh finite difference (FMFD) methods in a Monte Carlo (MC) simulation [1]. The iDTMC method has been newly devised to overcome the numerical instability and eliminate potential bias of the solutions in big size nuclear reactor problems. The preliminary study showed that the iDTMC method can eliminate the numerical stability and enable early accurate truncation of the MC solution within only a few active cycles [1].

In the meantime, due to the strong correlation between the cycle-wise FMFD parameters, the variance of the deterministic solutions could be highly underestimated and slowly decrease along the simulation even though the variance is already much lower than that of conventional methods from the beginning of cycle. In this work, two different numerical schemes for the iDTMC method are considered and tested to lessen the correlation of the parameters and take further reduction of the stochastic errors. The concept of the iDTMC method will be presented, and numerical performance in terms of the solution estimation and variance reduction will be evaluated in a big size PWR problem.

2. Methods and Results

2.1 iDTMC Method

The DTMC method has been developed for acceleration and variance reduction of the laborious MC neutronic calculation [2-4]. The deterministic solutions can accelerate the convergence of the fission source distribution (FSD) and provide reliable solutions. However, in applications of the DTMC method in large scale reactor problems, it was found that numerical instability may happen in the FMFD calculations and the solutions can be biased unless the number of neutron histories is big enough. Therefore, the DTMC method is improved to eliminate the numerical instability and potential bias, and guarantee stable and consistent DTMC solution.

In the preliminary iDTMC study [1], the FMFD parameters are accumulated all the way from the initial cycle as shown in Fig. 1. However, the variance of the deterministic solutions with the long cumulative parameters can be highly underestimated and slowly decreased due to the strong cycle-wise correlation. Therefore, a new scheme has been considered to weaken the correlation of the parameters and further decrease the variance. Once the simulation enters the active cycle, one oldest data is removed when a new active cycle data is added to generate the one-group FMFD parameters as shown in Fig. 2.

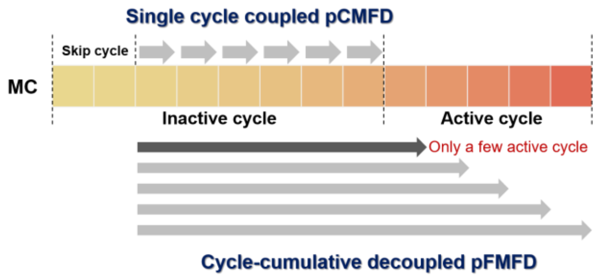

Fig. 1 Improved DTMC scheme (1) – iDTMC1

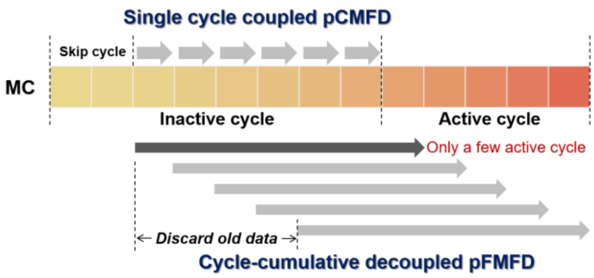

Fig. 2 Improved DTMC scheme (2) – iDTMC2

2.2 Numerical Results

The quarter core of the first-cycle APR 1400 model [5] is considered for the numerical analysis. The configuration of the core is described in Figs. 3 and 4. To evaluate the performance of the iDTMC method, the reference solution is first calculated with 1,200 active cycles, and 10,000,000 histories per cycle. The multiplication factor is estimated to be 1.20392 and its apparent standard deviation (SD) is 0.82 pcm. The normalized pin power distribution is also obtained, and its maximum 2D pin uncertainty is 0.627%.

The multiplication factor is estimated by the two different iDTMC schemes, and the results are compared with the reference solution and the conventional CMFD method [6-7]. For the CMFD and iDTMC methods, the first 5 inactive cycles are skipped, and more than 20 inactive cycles, 10 active cycles and 50 million histories per cycle are considered which have been determined by the m-PRUP method [8]. To quantify the statistical errors for each method, 35 batches were independently simulated with the different random seeds.

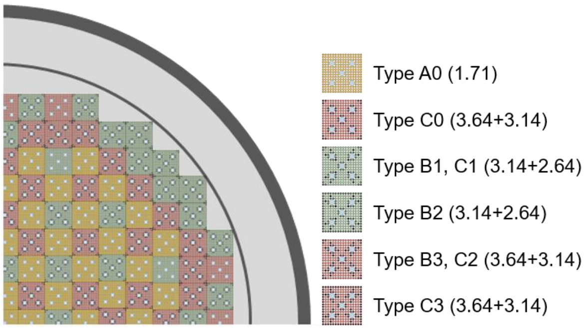

Fig. 3. Radial configuration of APR 1400 quarter core

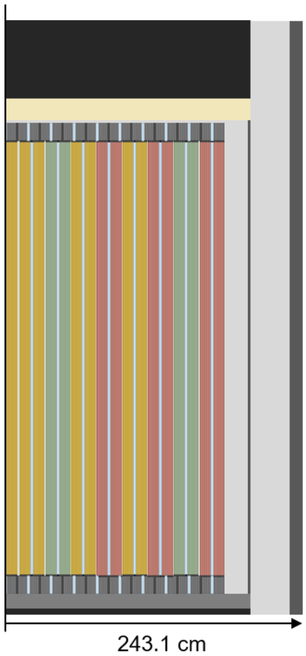

Fig. 4. Axial configuration of APR 1400

Figure 5 depicts the convergence behavior of the FSD in terms of the Shannon entropy, which was evaluated using a 10 X 10 X 20 cm mesh configuration of the 3D problem. The entropy seems to converge at around cycle 20.

Figures 6 and 7 are the cycle-averaged FMFD parameters at two specific pin positions. One position, (3,1,19), is near the center of the core, and the other one, (100,100,19), is in the peripheral region. Node average total cross section and FMFD correction factors at a surface of the pin node are plotted with the MC cycles. Both parameters at the center region quickly converge, while their convergence is relatively slow in the peripheral position.

Table I compares the 1st batch multiplication factor, and Table II lists the associated apparent (σ_a) and real (σ_r) standard deviations (SDs). The real SDs are also compared in Fig. 8. Both iDTMC and convectional MC (MC-CMFD) methods provide very accurate multiplication factor from the first active cycle, but the iDTMC method is a lot more reliable from the beginning of the active cycles. Compared to the MC-CMFD method, the iDTMC method has 2~4 times lower stochastic uncertainties for the k-eff values in the early active cycles. However, two different iDTMC schemes showed just similar real SDs each other.

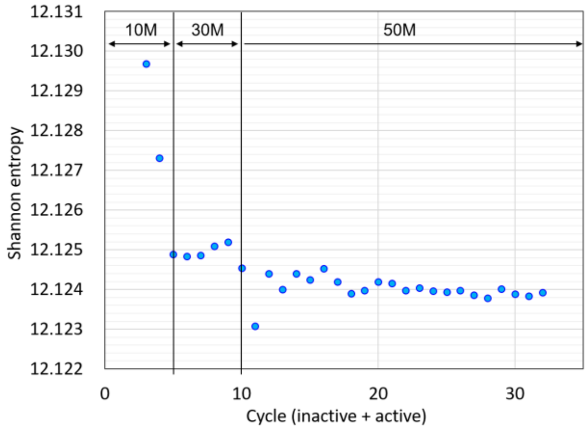

Fig. 5 Convergence behavior of FSD

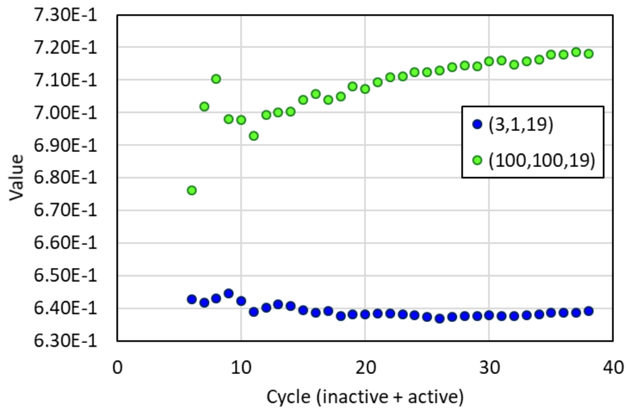

Fig. 6 Cycle cumulative total cross section

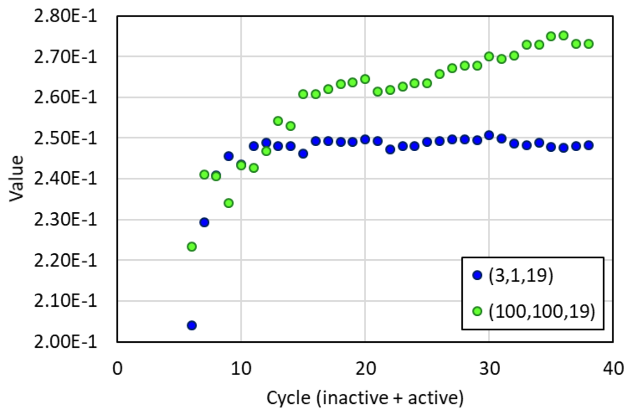

Fig. 7 Cycle cumulative correction factors

It is clear that the apparent SD of the iDTMC schemes in Table II is extremely underestimated.

Currently, it is hard to estimate the acceptable σ_a of the iDTMC solution in the middle of active cycles by the conventional generation-based calculation because the variables are strongly correlated. Therefore, it is necessary to devise a way to quantify the SD by a single MC calculation for the iDTMC method.

Table I. Comparison of the multiplication factor

Cycle	MC-CMFD	iDTMC1	iDTMC2
1	1.20389	1.20389	1.20392
2	1.20390	1.20390	1.20392
3	1.20389	1.20389	1.20392
4	1.20390	1.20390	1.20390
5	1.20391	1.20391	1.20389
6	1.20390	1.20390	1.20389
7	1.20391	1.20391	1.20391
8	1.20391	1.20391	1.20391
9	1.20390	1.20390	1.20392
10	1.20390	1.20390	1.20391

Table II. Standard deviations of the k-eff (pcm)

Cycle	MC-CMFD		iDTMC1		iDTMC2	
	σ_a	σ_r	σ_a	σ_r	σ_a	σ_r
1	-	15.5	-	3.1	-	2.9
2	4.5	10.9	0.24	3.1	0.31	3.1
3	5.7	8.4	0.23	3.0	0.32	2.9
4	5.6	7.9	0.23	2.9	0.31	3.0
5	5.5	7.1	0.22	2.8	0.30	2.9
6	4.6	5.9	0.20	2.7	0.30	2.9
7	4.8	5.9	0.18	2.7	0.30	2.4
8	4.7	6.0	0.18	2.6	0.32	2.5
9	4.2	5.5	0.17	2.6	0.33	2.8
10	4.1	5.2	0.17	2.6	0.34	2.5

Table III presents the maximum and average errors of the axially averaged 2D pin power distribution in the 1st batch results. The iDTMC method has less than 0.4% average error and no more than 3.5% maximum pin error throughout the active cycles. The error of the iDTMC methods is almost half that of the MC-CMFD method in the 1st and 2nd cycles, and the discrepancy in the power errors gradually dwindles with active cycles. Also, no big difference is observed between the two different iDTMC schemes in the pin power distribution, except very early active cycles.

Figures 9 and 10 display the 2D distribution of the relative real standard deviation for each method. The relative real SD can be calculated by Eq. (1). It is clear that the conventional MC solution has much higher uncertainties for the pin power distribution, while the iDTMC method has no more than 2% real SD in the whole core.

$$\sigma_{rr}^{i,j} = \frac{\sigma_r^{i,j}}{p_{i,j}}, \qquad (1)$$

where $\sigma_r^{i,j}$ is the real standard deviation and $p_{i,j}$ is the normalized pin power at node (i,j). The average of the relative real SDs is estimated to be 0.95% in the MC-CMFD and 0.33% in the iDTMC. In short, the iDTMC method provided the detailed pin power profile with 3 times lower stochastic errors compared to the conventional CMFD method.

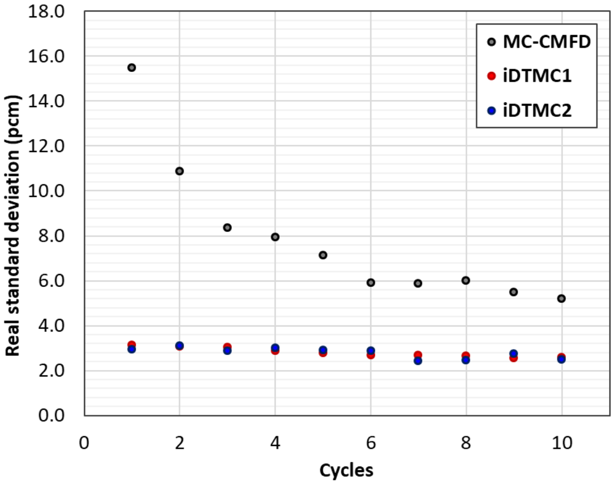

Fig. 8 Real standard deivation of k_{eff}

Table III. 2D pin power distribution (%)

Cycle	MC-CMFD		iDTMC1		iDTMC2	
	Avg. error	Max. error	Avg. error	Max. error	Avg. error	Max. error
1	1.51	14.1	-	-	0.34	3.0
2	0.60	6.5	0.34	3.3	-	-
3	0.53	5.5	0.34	3.2	0.33	3.3
5	0.44	4.3	0.33	3.2	0.33	3.1
10	0.38	3.3	0.32	3.1	0.34	2.9

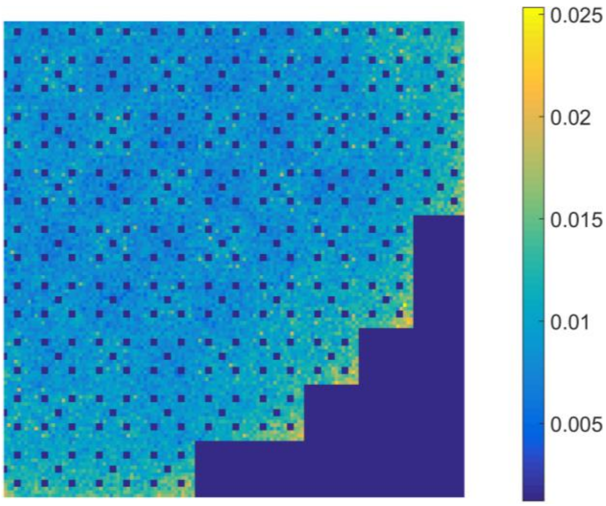

Fig. 9 Real SD distribution at cycle 10 (MC-CMFD)

Last, the computing time and figure-of-merit (FOM) are evaluated and compared. Since the deterministic calculations are so cheap compared to the MC calculation for the big PWR core and the FMFD calculation is well accelerated by the 1-CMFD scheme

[9], the computing time of the CMFD and iDTMC method turns out to be almost identical within the uncertainty. Table IV shows the average computing time of 35 independent batches.

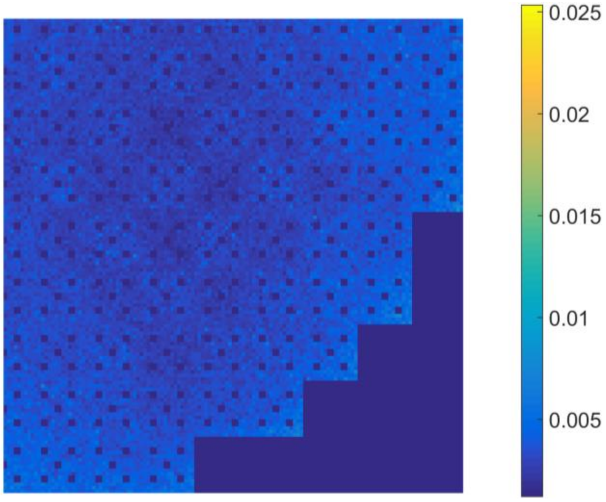

Fig. 10 Real SD distribution at cycle 10 (iDTMC2)

Table IV. Computing time of CMFD and iDTMC

	CMFD	iDTMC
Inactive cycle (hr.)	6.0	6.0
Active cycle (hr.)	3.7	3.7
Total time (hr.)	9.7	9.7

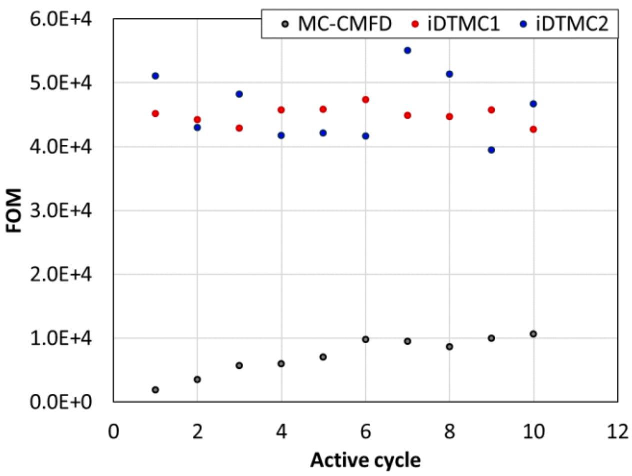

Fig. 11 FOM for the multiplication factor

The FOM for the multiplication factor is estimated as shown in Fig. 11. Because the iDTMC method can provide more reliable solution than the CMFD method while requiring almost identical computing time, the FOM is almost 5 times higher with the iDTMC method than the conventional CMFD approach during the whole active cycles. Because two iDTMC schemes attained similar SDs, the FOM for the multiplication factor are also comparable each other.

3. Conclusions

An iDTMC method with a new numerical strategy has been successfully implemented to analyze the large scale APR1400 core problem. Two different numerical schemes for the iDTMC method have been considered to weaken the correlation of the parameters and hence further reduce the stochastic errors. From the numerical analysis, it is shown that no big difference is observed between two numerical schemes in terms of the stochastic errors. In the meantime, the iDTMC method provides much more reliable and accurate solution from the very early active cycles compared to the conventional CMFD method. In addition, the computing cost of the iDTMC method is very similar to the conventional CMFD approach. Therefore, the FOM is much greater in the iDTMC method, which attests to the high potential of the iDTMC scheme for a quick and accurate truncation of the high-fidelity MC solutions.

ACKNOWLEDGEMENT

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