An AFEN Response Matrix Method in the Two-dimensional Trigonal Geometry

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1. Introduction

The trigonal node based Analytic Function Expansion Nodal (AFEN) method has been developed to deal with the asymmetric inhomogeneity inside the hexagonal fuel assembly. Often, this inhomogeneity caused by burnable absorbers or control rods loaded asymmetrically within the assembly is handled by a hexagonal node based nodal method after being homogenized throughout the hexagonal assembly. However, such handling inevitably causes the homogenization error which is sometimes unacceptable in the reactor design.

The AFEN method which was originally developed for rectangular geometry [1] and then expanded to hexagonal geometry [2-4] is attempted for the first time in this paper for triangular geometry. Therefore, this paper will have a special meaning of completing the final puzzle to encompass all AFEN methods for all the important geometries of reactor physics. Given that the number of nodes increases by six times compared to the hexagonal method, the trigonal method can be the original AFEN method [1-4] rather than the refined AFEN method [5]. The former uses only the neutron flux and the latter uses both the neutron flux and the flux moment as the nodal unknown at an interface. For a reactor core consisting of N hexagonal assemblies, there are N nodes and 3N interfaces in the hexagonal nodal method, whereas 6N nodes and 9N interfaces in the trigonal nodal method. For this core, the hexagonal refined AFEN method will have 6N interface unknowns, whereas the trigonal original AFEN method will have 9N interface unknowns. Therefore, we expect that the trigonal original AFEN method without interface flux moments provides better accuracy than the hexagonal refined AFEN method with interface flux moments.

The trigonal AFEN method developed in this paper is the one based on the response matrix method which is known to be numerically efficient and parallel computation friendly [6]. The response matrix AFEN method uses the interface partial currents as nodal unknowns instead of the interface fluxes. A triangular nodal response matrix formulation based on the polynomial flux expansion can be found in Reference [7].

This paper presents the results of a two-dimensional reactor core analysis using the proposed method. The main numerical characteristics will remain almost the same even if it is expanded to three dimensions.

2. Methodology

Deriving the AFEN Response Matrix which expresses the outgoing interface partial currents into the incoming interface partial currents has two steps for convenience. In the first step, the AFEN single node is solved to obtain the relationship between the interface fluxes and the interface currents. In the second step, the response matrix is derived by replacing the interface fluxes and interface currents in this relationship with the incoming and outgoing interface partial currents.

Since the two step procedure is similar to that described in Reference [6], we follow it with a lots of descriptions overlapping with the reference.

2.1 AFEN Solution of Single-Node Problem

2.1.1 Intranodal Flux Expansion

Solving the single trigonal node problem with the boundary condition of three interface currents by the AFEN method starts from expending the intranodal flux distribution into analytic expansion basis functions. We can easily obtain the intranodal flux expansion symmetrical to the 120 degrees apart three-side directions of a trigonal node shown in Fig. 1 by taking the flux expansion of the form below as introduced in the hexagonal AFEN method [2,6].

$$\boldsymbol{\phi}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{\varphi}(\boldsymbol{x},\boldsymbol{y}) + \boldsymbol{\varphi}(\boldsymbol{u},\boldsymbol{v}) + \boldsymbol{\varphi}(\boldsymbol{p},\boldsymbol{q}) \qquad (1)$$

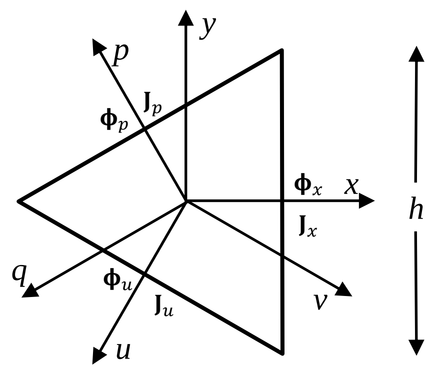

Fig. 1. Coordinate systems and nodal unknowns

Determining the analytic expansion basis function $\boldsymbol{\varphi}(x,y)$ is not as straightforward as in the rectangular or hexagonal AFEN method. Since we have three interface currents in which the flux expansion is expressed, we run into the dilemma of having to choose one between even and odd functions as the basis function $\boldsymbol{\varphi}(x,y)$ in one of three directions. This is quite awkward and even unphysical considering that the basis function harmoniously consists of both even and odd functions in the rectangular or hexagonal AFEN method. To look at the physical meaning of this dilemma and to seek its solution, we continue to expand the intranodal flux distribution with choice of e.g., an odd function as the basis function.

$$\boldsymbol{\varphi}(x,y) = \mathbf{A}_x \sinh(\sqrt{\boldsymbol{\Lambda}}x) \quad (2)$$

where

$$\boldsymbol{\Lambda} = \mathbf{D}^{-1}\boldsymbol{\Sigma} \quad (3)$$

and $\mathbf{D}$ and $\boldsymbol{\Sigma}$ are the diffusion coefficient and crosssection matrixes, respectively and $\mathbf{A}_x$ is the expansion coefficient.

Note that the flux expansion Eq. (1) has three terms with one coefficient each and all of them completely satisfy the diffusion equation for the node. Of course, both the basis functions and the coefficients of this expansion are square matrices with the number of energy groups as its order. However, thanks to the matrix function theory, they can be treated like scalar as long as they are functions of single matrix $\boldsymbol{\Lambda}$.[8,9]

The average flux of the node is defined from this flux expansion as follows.

$$\bar{\boldsymbol{\phi}} = \frac{4\sqrt{3}}{3h^2}\int_{-\frac{\sqrt{3}}{3}h}^{\frac{\sqrt{3}}{6}h}\int_{-\frac{\sqrt{3}}{3}x-\frac{1}{3}h}^{\frac{\sqrt{3}}{3}x+\frac{1}{3}h} \boldsymbol{\phi}(x,y)\,dydx \quad (4)$$

The interface flux e.g., at the x interface is defined by

$$\boldsymbol{\phi}_x = \frac{1}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \boldsymbol{\phi}\left(\frac{\sqrt{3}}{6}h, y\right)dy \quad (5)$$

Strictly speaking, when applying equivalence theory[10,11], the interface flux in Eq. (5) is homogeneous one. This is multiplied by the discontinuity factor to yield the heterogeneous one. However, for simplicity of derivation, we ignore the discontinuity factor at this moment. In implementing, of course, the discontinuity factor is involved.

Further, the interface current coming into the node through the example interface is consistently defined by

$$\mathbf{J}_x = \frac{\mathbf{D}}{h}\int_{-\frac{h}{2}}^{\frac{h}{2}} \frac{\partial}{\partial x}\boldsymbol{\phi}(x,y)\,dy\Bigg|_{x=\frac{\sqrt{3}}{6}h} \quad (6)$$

2.1.2 AFEN Solution of Single-Node Problem

Solving the single node problem in Fig. 1 to obtain the intranodal flux distribution means expressing three coefficients of the flux expansion Eq. (1) in terms of three interface currents. Please remember that we introduced the two decoupling transformations in the reference [6] to simplify this procedure: parity transformation and direction transformation. Since there is only one interface in each direction, only the direction transformation among them can be applicable in the trigonal AFEN method for both the expansion coefficients and the nodal unknowns as follow:

$$\mathbf{A}_\theta = \frac{\mathbf{A}_x+\mathbf{A}_u+\mathbf{A}_p}{3}, \mathbf{A}_\varepsilon = \frac{2\mathbf{A}_x-\mathbf{A}_u-\mathbf{A}_p}{3}, \mathbf{A}_\chi = \frac{\mathbf{A}_u-\mathbf{A}_p}{3} \quad (7)$$

$$\boldsymbol{\phi}_\theta = \frac{\boldsymbol{\phi}_x+\boldsymbol{\phi}_u+\boldsymbol{\phi}_p}{3} - \bar{\boldsymbol{\phi}}, \boldsymbol{\phi}_\varepsilon = \frac{2\boldsymbol{\phi}_x-\boldsymbol{\phi}_u-\boldsymbol{\phi}_p}{3}, \boldsymbol{\phi}_\chi = \frac{\boldsymbol{\phi}_u-\boldsymbol{\phi}_p}{3} \quad (8)$$

$$\mathbf{J}_\theta = \frac{\mathbf{J}_x+\mathbf{J}_u+\mathbf{J}_p}{3},\ \mathbf{J}_\varepsilon = \frac{2\mathbf{J}_x-\mathbf{J}_u-\mathbf{J}_p}{3},\ \mathbf{J}_\chi = \frac{\mathbf{J}_u-\mathbf{J}_p}{3} \quad (9)$$

Then, the flux expansion (1) with the original coefficients becomes the following expansion decoupled into three components corresponding to three transformed coefficients.

$$\boldsymbol{\phi}(x,y) = \boldsymbol{\varphi}_\theta^{sn}(x,y) + \boldsymbol{\varphi}_\varepsilon^{sn}(x,y) + \boldsymbol{\varphi}_\chi^{sn}(x,y) \quad (10)$$

where three transformed flux distribution components are given by

$$\boldsymbol{\varphi}_\theta^{sn}(x,y) = \left\{-2\sinh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\cosh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right) + \sinh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\theta \quad (11)$$

$$\boldsymbol{\varphi}_\varepsilon^{sn}(x,y) = \left\{\sinh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\cosh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right) + \sinh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\varepsilon \quad (12)$$

$$\boldsymbol{\varphi}_\chi^{sn}(x,y) = -3\cosh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\sinh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right)\mathbf{A}_\varepsilon \quad (13)$$

Thanks to the direction transformation, each transformed component in Eq. (10) is related to only the corresponding transformed unknown. Therefore, we can say that the flux distribution induced by each transformed unknown is the corresponding flux distribution component in equations (11) to (13). For example, the flux distribution induced by $\boldsymbol{\phi}_\varepsilon$ is $\boldsymbol{\varphi}_\varepsilon^{sn}(x,y)$.

$\boldsymbol{\phi}_\varepsilon$ can be said to be odd in the x direction because it is related to the difference between $2\boldsymbol{\phi}_x$ (located right in Fig. 1) and $\boldsymbol{\phi}_u+\boldsymbol{\phi}_p$ (located left in Fig. 1). This quantity becomes even in the y direction because there is no upward or downward bias in locations of $2\boldsymbol{\phi}_x$ and $\boldsymbol{\phi}_u+\boldsymbol{\phi}_p$. Therefore, $\boldsymbol{\varphi}_\varepsilon^{sn}(x,y)$ induced by $\boldsymbol{\phi}_\varepsilon$ should also be odd in the x direction and even in the y direction. This corresponds to reality as shown in Eq. (12). Similarly, the distribution $\boldsymbol{\varphi}_\chi^{sn}(x,y)$ in Eq. (13) is also reasonable from this point of view.

However, $\boldsymbol{\varphi}_\theta^{sn}(x,y)$ is quite unreasonable because it is odd in the x direction and even in the y direction but $\boldsymbol{\phi}_\theta$ is even in the both directions. This unphysical phenomena of $\boldsymbol{\varphi}_\theta^{sn}(x,y)$ can also be shown mathematically. By eliminating the coefficient $\mathbf{A}_\theta$ from the two equations with the coefficient $\mathbf{A}_\theta$ (one for the transformed flux $\boldsymbol{\phi}_\theta$ and the other for the current $\mathbf{J}_\theta$), we can get the relationship between the transformed flux and current as follows:

$$\boldsymbol{\phi}_\theta = \mathbf{T}_\theta^{sn}\mathbf{J}_\theta \quad (14)$$

We expand the relationship matrix $\mathbf{T}_\theta^{sn}$ **(**which is a matrix function of $\boldsymbol{\Lambda}$) in Taylor series of $\boldsymbol{\Lambda}$.

$$\mathbf{T}_\theta^{sn} = \frac{\sqrt{3}}{8\boldsymbol{\Lambda}} + \frac{\sqrt{3}}{21} - \frac{\sqrt{3}\boldsymbol{\Lambda}}{294} + O(\boldsymbol{\Lambda}^2) \quad (15)$$

It is obvious that the relationship matrix $\mathbf{T}_\theta^{sn}$ is singular at $\boldsymbol{\Lambda} = \mathbf{0}$ and this is unphysical.

Partial deficiency in the intranodal flux expansion due to the selection of the odd basis function as in Eq. (2) is compensated by the selection of the even basis function.

$$\boldsymbol{\varphi}(x,y) = \mathbf{A}_x \cosh(\sqrt{\boldsymbol{\Lambda}}x) \quad (16)$$

Repeating the process described for the case of the odd basis function so far, we can get the flux expansion decoupled into components by the direction transformation for the case of this even basis function.

$$\boldsymbol{\phi}(x,y) = \boldsymbol{\varphi}_\theta^{cs}(x,y) + \boldsymbol{\varphi}_\varepsilon^{cs}(x,y) + \boldsymbol{\varphi}_\chi^{cs}(x,y) \quad (17)$$

where

$$\boldsymbol{\varphi}_\theta^{cs}(x,y) = \left\{2\cosh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\cosh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right) + \cosh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\theta \quad (18)$$

$$\boldsymbol{\varphi}_\varepsilon^{cs}(x,y) = \left\{-\cosh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\cosh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right) + \cosh(\sqrt{\boldsymbol{\Lambda}}x)\right\}\mathbf{A}_\varepsilon \quad (19)$$

$$\boldsymbol{\varphi}_\chi^{cs}(x,y) = 3\sinh\left(\frac{\sqrt{\boldsymbol{\Lambda}}}{2}x\right)\sinh\left(\frac{\sqrt{3\boldsymbol{\Lambda}}}{2}y\right)\mathbf{A}_\varepsilon \quad (20)$$

The even-odd test done in each of x- and y-directions for these three expansion components showed that only the first component Eq. (18) is suitable for the flux expansion.

The dilemma caused by selecting only one of even and odd basis functions is solved by constructing the expansion function with only suitable ones among all the expansion components derived from the even or odd basis function.

$$\boldsymbol{\phi}(x,y) = \boldsymbol{\varphi}_\theta^{cs}(x,y) + \boldsymbol{\varphi}_\varepsilon^{SN}(x,y) + \boldsymbol{\varphi}_\chi^{SN}(x,y) \quad (21)$$

In this way, the flux expansion function is harmoniously composed of even and odd basis functions. Please note that this procedure can also be applied to find a polynomial which is symmetrical and harmonious in the triangular geometry.

Expressing the transformed unknowns in terms of the transformed expansion coefficients, the original 3x3 matrix equation is decoupled into three scalar equations due to the decoupling transformation.

$$\boldsymbol{\phi}_\theta = \mathbf{P}_\theta^{cs}\mathbf{A}_\theta, \quad \boldsymbol{\phi}_\varepsilon = \mathbf{P}_\varepsilon^{sn}\mathbf{A}_\varepsilon, \quad \boldsymbol{\phi}_\chi = \mathbf{P}_\chi^{sn}\mathbf{A}_\chi \quad (22)$$

and

$$\mathbf{J}_\theta = \mathbf{D}\mathbf{Q}_\theta^{cs}\mathbf{A}_\theta, \quad \mathbf{J}_\varepsilon = \mathbf{D}\mathbf{Q}_\varepsilon^{sn}\mathbf{A}_\varepsilon, \quad \mathbf{J}_\chi = \mathbf{D}\mathbf{Q}_\chi^{sn}\mathbf{A}_\chi \quad (23)$$

By eliminating the coefficient vector, we can get the relationship between the transformed interface flux and current:

$$\boldsymbol{\phi}_\alpha = \mathbf{T}_\alpha\mathbf{A}_\alpha \quad (24)$$

where α is θ, ε or χ and

$$\mathbf{T}_\theta = \mathbf{P}_\theta^{cs}{\mathbf{Q}_\theta^{cs}}^{-1}\mathbf{D}^{-1} \quad (25)$$

$$\mathbf{T}_\beta = \mathbf{P}_\beta^{sn}{\mathbf{Q}_\beta^{sn}}^{-1}\mathbf{D}^{-1}, \ \beta = \ \varepsilon \ or \ \chi \quad (26)$$

Fortunately, we have only two relationship matrices because we realized that $\mathbf{T}_\varepsilon = \mathbf{T}_\chi$.

2.2 AFEN Response Matrix

The response matrix that computes the output, which are the outgoing interface partial currents out of a node, from the input, which are the incoming interface partial currents into the node, is derived by noting that the interface partial current at the interface *s* is expressed in terms of the interface flux Eq. (5) and current Eq. (6).

$$\mathbf{j}_s^f = \frac{\mathbf{J}_s^f}{2} + \frac{\boldsymbol{\phi}_s}{4} \quad (27)$$

where flow direction index *f* is *in* or *out*, interface index *s* is *x*, *u*, or *p*. Then, the interface flux and current are equivalently given by

$$\mathbf{J}_s^{in} = \mathbf{j}_s^{in} - \mathbf{j}_s^{out}, \quad \boldsymbol{\phi}_s = 2(\mathbf{j}_s^{in} + \mathbf{j}_s^{out}) \quad (28)$$

Since the relationship (27) is linear and the direction transformation explained in the previous section is also linear, the partial current shall have its transformed form with respect to the transformation and this form shall have the relationship corresponding to that of Eq. (28) as follows,

$$\mathbf{J}_\alpha^{in} = \mathbf{j}_\alpha^{in} - \mathbf{j}_\alpha^{out}, \ \ \boldsymbol{\phi}_\alpha = 2\left(\mathbf{j}_\alpha^{in} + \mathbf{j}_\alpha^{out}\right), \ \boldsymbol{\alpha = \theta, \varepsilon, or\ \chi} \quad (29)$$

Substituting these relationship into Eq. (24) and solving for the transformed outgoing partial current, we finally obtain the response matrix in the transformed system as follows,

$$\mathbf{j}_\alpha^{out} = \mathbf{R}_\alpha\mathbf{j}_\alpha^{in}, \ \boldsymbol{\alpha = \theta, \varepsilon, or\ \chi} \quad (30)$$

where $\mathbf{R}_\alpha = -(2\mathbf{I} + \mathbf{T}_\alpha)^{-1}(2\mathbf{I} - \mathbf{T}_\alpha)$. Again, $\mathbf{R}_\varepsilon = \mathbf{R}_\chi$ because $\mathbf{T}_\varepsilon = \mathbf{T}_\chi$.

Note that the interface partial currents can be easily transformed into their linearly transformed partners and vice versa. Once the interface incoming partial currents are given for a node, the interface outgoing partial currents can be calculated by the response matrix Eq. (30). Then, these outgoing partial currents become the partial currents incoming into its neighboring nodes. This provides an iterative process to solve the global core eigenvalue problem through the well-known inner-outer iteration. Generally, the number of inner iterations per outer iteration is an issue in this type of iteration. As in many other nodal methods, we used one for this value throughout this paper.

As mentioned in Ref. [5], the inverse of any matrix function of $\boldsymbol{\Lambda}$ is singular when one of the eigenvalues of the crosssection matrix is very small. This singularity is a numerical singularity which is different from a mathematical singularity like the one involved in T_θ^{sn} in Eq. 14. The numerical singularity can be easily removed in the manner described in the reference.

2.3 RGB-BW Sweeping Scheme

The response matrix calculations are performed only within a single node regardless of neighboring nodes. Therefore, these calculations are carried out by sequentially moving from one node to another. In this case, it is advantageous to sweep the nodes by grouping them in two steps as shown in Fig. 2. In the first step, the core is divided into red (R), green (G), and blue (B) hexagonal assemblies like in the reference (6) and in the second step, a hexagonal assembly is divided into black and white triangular nodes.

This kind of iteration schemes is known to be good in convergence and stability due to geometrical balance. It further enhances the advantage in parallel-computing that the response matrix method has already.

In addition, the memory required is saved by storing inputs of the response calculation, i.e., incoming partial currents and outputs, i.e., outgoing partial currents in the same storage. This is because the outgoing partial currents resulting from previous two other color types of node calculations automatically become incoming partial currents for the third kind of node calculations.

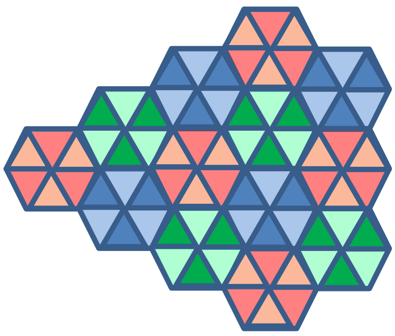

Fig. 2. RGB sweeping scheme

3. Numerical Results and Discussion

The accuracy of the AFEN response method in the trigonal geometry is demonstrated by the MHTGR-350 problem. The MHTGR-350 problem represents a 350MWth hexagonal prismatic block type HTGR core with graphite moderator and helium coolant. It has an active core of 66 fuel blocks in the fourth, fifth and sixth rings of the core, surrounded by graphite reflectors with about three rings thick inward and outward. The assembly homogenized cross-sections directly come from the Reference [6].

In Fig. 3, the assembly-wise relative powers of the trigonal AFEN method were compared with those of the hexagonal AFEN method. Since both the AFEN methods are based on the response matrix method, the zero incoming partial current boundary condition was applied. And, the hexagonal AFEN method against which the trigonal method is benchmarked is the refined AFEN method which replaces the corner-point fluxes with the flux moments. The step weighting function is used in the definition of the flux moment. Accuracy of this method was well shown in the reference [5]. This accuracy is also demonstrated by showing that, when compared with the CAPP solution of the MHTGR-350 problem with the cubic finite element option, a 22 pcm error occurs in the effective multiplication factor and up to 0.2% error in the block-wise power distribution.

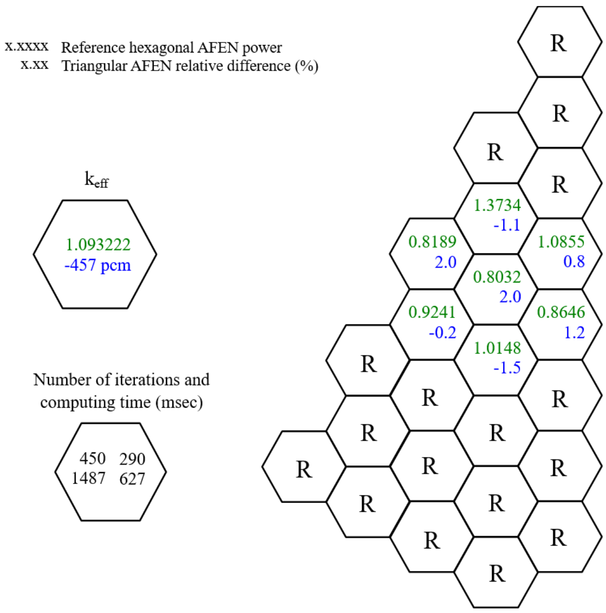

Fig. 3. Power distribution of the MHTGR-350 benchmark problem (1/12 core).

This figure shows that, contrary to our expectations, the triangular AFEN method results in quite large errors in the multiplication factor and in the block-wise power distribution even though it takes more than twice the computation time. This may be due to the looser flux continuity constraint at the interface between two adjacent hexagonal blocks in the triangular AFEN method than in the hexagonal AFEN method. While the hexagonal AFEN method divides a block interface in half and applies the continuity condition to each half, the triangular method applied only one continuity condition for the entire block interface. Of course, it additionally applies the continuity conditions to the six interfaces divided by six triangular nodes inside a hexagonal block. However, please note that in the hexagonal method, the flux is intrinsically continuous across these interfaces. Identification of another possible causes of the large error and how to overcome them will be pursued in the future.

4. Conclusions

The triangular AFEN response matrix method was developed with the primary purpose of dealing with hexagonal assemblies with asymmetrical internals rather than increasing accuracy. The version of AFEN methods implemented here is the original one having only the interface fluxes as nodal unknown, considering that the number of nodes increases six times compared the hexagonal AFEN method.

Eventually, with this paper, we have a complete set of AFEN methods for the key geometries that are important in core neutronic analysis, including rectangular, hexagonal and triangular geometries. In addition, this paper presents a methodology for how to construct the flux expansion function that is not only geometrically symmetric but also even-odd harmonious within a triangular node.

This AFEN method was tested against the MHTGR-350 benchmark problems. Unfortunately, the results show that this method cannot be used for actual core neutronic analysis due to its poor accuracy. It remains to be done in the future to identify the obvious cause of this poor accuracy and to find the way to overcome it.

Acknowledgements

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