Simulation on Pinch Plasma using SPH with resistive MHD models

SPH *: Smoothed Particle Hydrodynamics*
MHD *: Magneto-hydrodynamics*

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1. Introduction

Motivation & Objective of Study

◆ Motivation

- ✓ Pinch is the phenomenon that appears in a plasma when it is compressed by magnetic forces.
- ✓ In recent years, the pinch plasma has received large attention as an efficient source of radiation and a way to explore high-density plasma physics [1-3].
- ✓ Magnetohydrodynamic (MHD) simulation is one of the powerful tools for understanding the pinch phenomenon, and it is appropriate to use the resistive MHD model since the pinch plasmas have varying local resistivity with temperature and pressure.
- ✓ SPH has many advantages, particularly in pinch plasma simulations, as it can handle the problem of complex and deformable boundaries relatively easily.

◆ Objective of Research

1. Investigation of the physical models required for the pinch plasma analysis
2. Implementation and verification of resistive MHD-based SPH model for the pinch plasma analysis

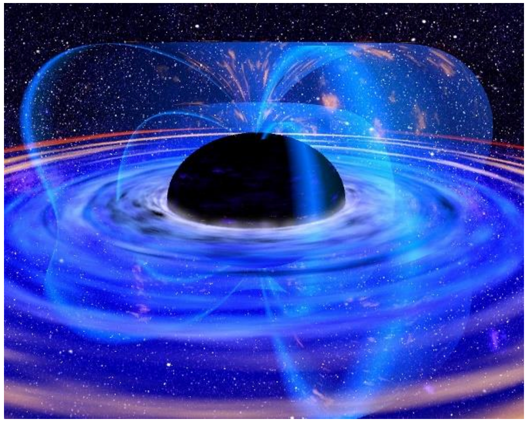
2. MHD Governing Equations

- What is MHD?
- Ideal MHD Governing Equations
- Resistive MHD Governing Equations

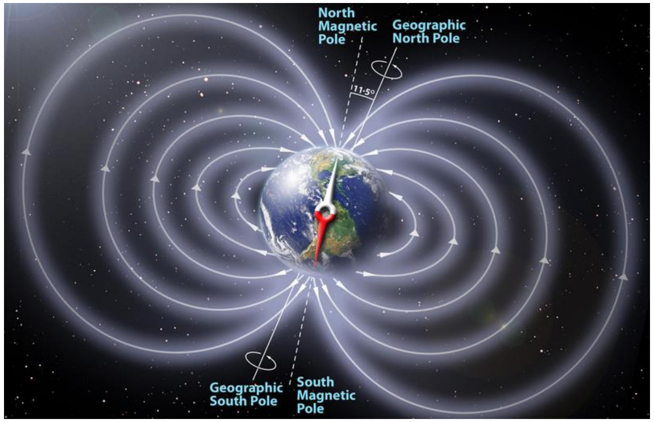
What is MHD?

❖ Magnetohydrodynamics (MHD)

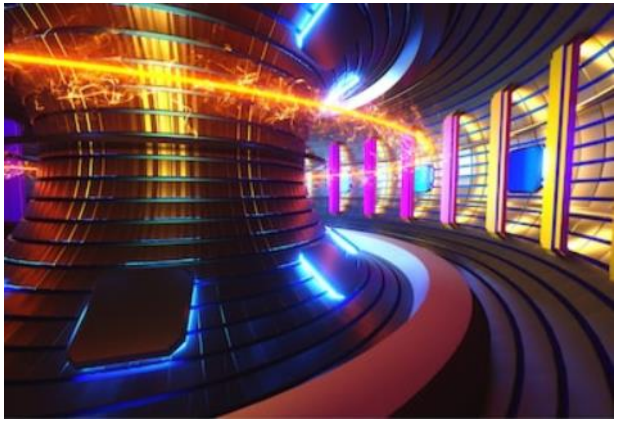
- ✓ Magnetohydrodynamics (MHD) is the study for the magnetic properties and behavior of **electrically conducting fluids** such as plasma.
- ✓ Magnetic forces act on charged particles and change their momentum and energy.
- ✓ In return, particles alter the strength and direction of the magnetic field.
- ✓ MHD plays a crucial role in various applications such as astrophysics, planetary magnetism, and controlled nuclear fusion etc.



Astrophysics



Planetary Magnetism



Nuclear Fusion

What is MHD?

❖ MHD governing equations

- ✓ The set of MHD equations can be summarized as the **combination** of **Navier–Stokes equations** of fluid dynamics and **Maxwell's equations** of electromagnetism.
- ✓ Various MHD model can be derived depending on the type of plasma and applied assumption.

◆ MHD governing equations

▪ Fluid dynamic equations

Mass conservation: $\frac{D\rho}{Dt} + \rho(\nabla \cdot \boldsymbol{v}) = 0$

Navier-Stokes equation: $\frac{D\boldsymbol{v}}{Dt} = -\frac{1}{\rho}\nabla p + \frac{\boldsymbol{f}}{\rho}$

Energy conservation: $\frac{Du}{Dt} = -\frac{P}{\rho}\nabla \cdot \boldsymbol{v}$

Equation of State: $P = (\gamma - 1)\rho u$

+

▪ Maxwell's equations

Gauss's law: $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$

Gauss's law for magnetism: $\nabla \cdot \vec{B} = 0$

Ampere's law: $\nabla \times \vec{B} = \mu_0 \vec{J}$

Faraday's law: $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Ohm's law (for resistive MHD): $\vec{E} + \vec{v} \times \vec{B} = \eta \vec{J}$

Ideal MHD Governing Equations

❖ Ideal MHD model

- ✓ The **simplest MHD models**, Ideal MHD, assumes that the fluid has so little resistivity that it can be treated as a perfect conductor.
- ✓ In ideal-MHD, various physical quantities such as displacement current, electrical resistivity, viscosity, and thermal conduction are neglected.
- ✓ The ideal MHD equations consist of the continuity equation, the Cauchy momentum equation, the induction equation, and the energy conservation equation.

- **Ideal-MHD governing equations**

Mass conservation:	$\frac{D\rho}{Dt} + \rho(\nabla \cdot \boldsymbol{v}) = 0$
Mtm conservation:	$\frac{D\boldsymbol{v}}{Dt} = \frac{1}{\rho}\nabla \cdot \left(\frac{\boldsymbol{BB}}{\mu_0} - \left(\frac{1}{2\mu_0}\boldsymbol{B}^2 + P\right)\overleftrightarrow{I}\right)$
Induction equation:	$\frac{D\boldsymbol{B}}{Dt} = -\boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v}$
Energy conservation:	$\frac{Du}{Dt} = -\frac{P}{\rho}\nabla \cdot \boldsymbol{v}$
Equation of State:	$P = (\gamma - 1)\rho u$

Neglecting

① Displacement current
② Electrical resistivity
③ Viscosity
④ Thermal conduction

Resistive MHD Governing Equations

❖ Resistive MHD model

- ✓ When the **fluid cannot be considered as completely conductive**, but the other conditions for ideal MHD are satisfied, it is possible to use an extended model called resistive MHD.
- ✓ In this model, some resistive term are added to the induction equation and energy equation of the ideal MHD model, and additional calculations are performed to obtain the current density.
- ✓ It is appropriate to use the resistive MHD model since the pinch plasmas have varying local resistivity with temperature and pressure.

- **Resistive-MHD governing equations**

Mass conservation: $$\frac{D\rho}{Dt} + \rho(\nabla \cdot \boldsymbol{v}) = 0$$

Mtm conservation: $$\frac{D\boldsymbol{v}}{Dt} = \frac{1}{\rho}\nabla \cdot \left(\frac{\boldsymbol{BB}}{\mu_0} - \left(\frac{1}{2\mu_0}\boldsymbol{B}^2 + P\right)\overleftrightarrow{I}\right)$$

Current density: $$\boldsymbol{J} = \frac{1}{\mu_0}(\nabla \times \boldsymbol{B})$$

Induction equation: $$\frac{D\boldsymbol{B}}{Dt} = -\boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - \eta\nabla \times \boldsymbol{J}$$

Energy conservation: $$\frac{Du}{Dt} = -\frac{P}{\rho}\nabla \cdot \boldsymbol{v} + \frac{1}{\rho}\eta\boldsymbol{J}^2$$

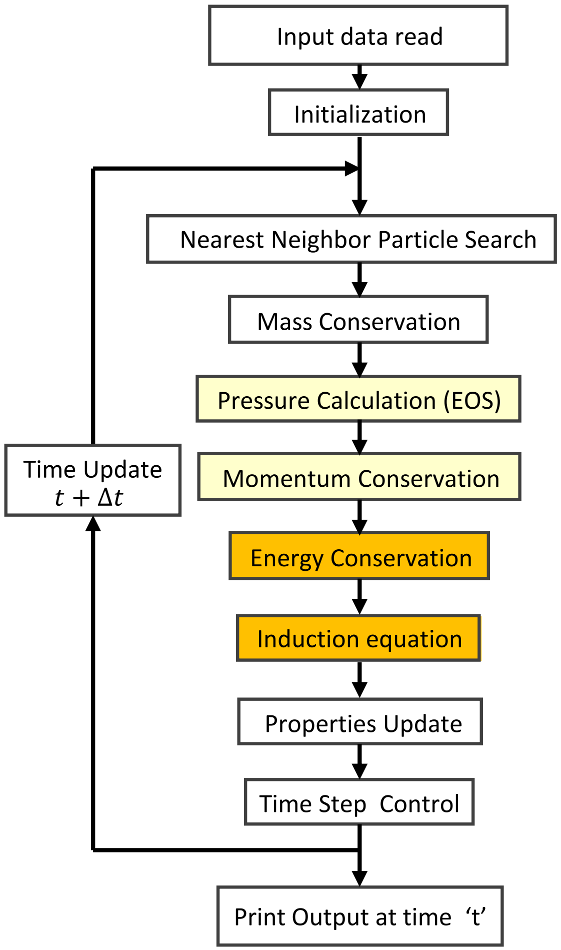
Equation of State: $$P = (\gamma - 1)\rho u$$

3. SPMHD code Development

- SPMHD code Structure
- Effect of $\nabla \cdot \boldsymbol{B}$ Correction Term
- Effect of Dissipation Term
- ASPH Methodology
- SPH Governing Equations

SPMHD code Structure

❖ Algorithm of SPMHD model



- Induction equation, energy equation are added to the existing SPH based CFD code (SOPHIA).
- Momentum equation and EOS are modified.
- Resistive terms is added in induction equation, and energy equation. (for resistive MHD)
- Several artificial dissipation terms that capture the shock and reduce numerical instability are incorporated.
- The divergence B correction term to maintain the divergence constraint of the plasma ($\nabla \cdot \boldsymbol{B} = 0$) is incorporated.

Momentum conservation

$$\frac{d\boldsymbol{v}_i}{dt} = \sum_j m_j \left(\frac{\overleftrightarrow{\boldsymbol{M}}_i}{\rho_i^2} + \frac{\overleftrightarrow{\boldsymbol{M}}_j}{\rho_j^2} + \Pi_{ij}\overleftrightarrow{I}\right) \cdot \nabla_i W_{ij} \underbrace{- \boldsymbol{B}_i \sum_j m_j \left(\frac{\boldsymbol{B}_i}{\rho_i^2} + \frac{\boldsymbol{B}_j}{\rho_j^2}\right) \cdot \nabla W_{ij}}_{\nabla \cdot B \text{ Correction}}$$

Induction equation

$$\frac{d\boldsymbol{B}_i}{dt} = \frac{1}{\rho_i} \sum_j m_j (\boldsymbol{B}_i \boldsymbol{v}_{ij} - \boldsymbol{v}_{ij} \boldsymbol{B}_i) \cdot \nabla_i W_{ij} + \left(\frac{d\boldsymbol{B}_i}{dt}\right)_{dissapation} + \underbrace{\left(\frac{d\boldsymbol{B}_a}{dt}\right)_{\eta}}_{\text{Resistive-MHD}}$$

Energy conservation

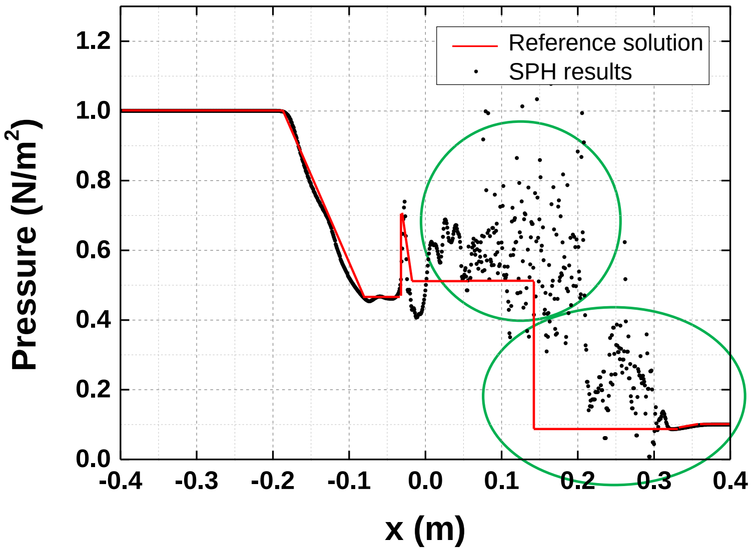
$$\frac{du_i}{dt} = \frac{1}{2} \sum_j m_j \left(\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2} + \Pi_{ij}\right) \boldsymbol{v}_{ij} \cdot \nabla_i W_{ij} + \underbrace{\left(\frac{du_a}{dt}\right)_{\eta}}_{\text{Resistive-MHD}}$$

SPMHD *: SPH-MHD*

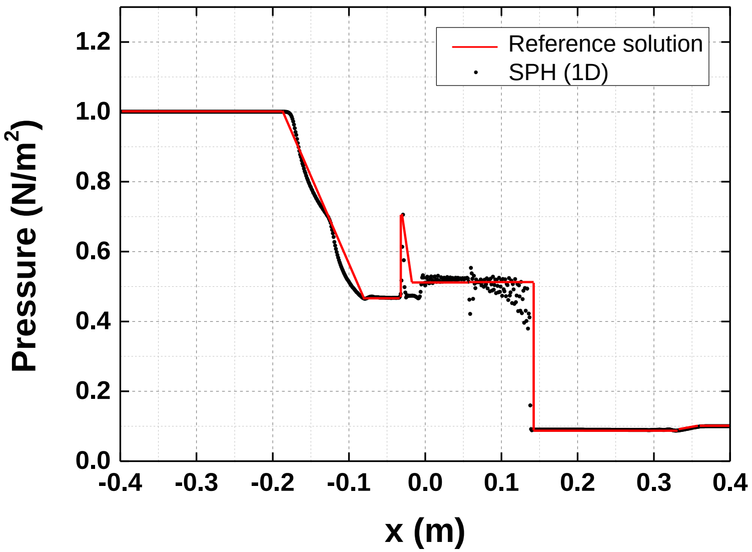
Effect of ∇·$\boldsymbol{B}$ Correction Term

- In simulations of magnetohydrodynamic (MHD) processes, the violation of the divergence constraint ($\nabla \cdot B = 0$) causes severe stability problems. (◯)
- In the MHD simulation, $\nabla \cdot B$ is not completely zero, and an additional term to correct it is applied.
- After applying the $\nabla \cdot B$ correction term, it is confirmed that the numerical instability can be significantly controlled.

$$\left(\frac{d\boldsymbol{v}_i}{dt}\right)_{correction} = -\boldsymbol{B}_i \sum_j m_j \left(\frac{\boldsymbol{B}_i}{\rho_i^2} + \frac{\boldsymbol{B}_j}{\rho_j^2}\right) \cdot \nabla W_{ij}$$



[Pressure profiles (t=0.1 s, $\nabla \cdot \boldsymbol{B}$ **correction X**)]



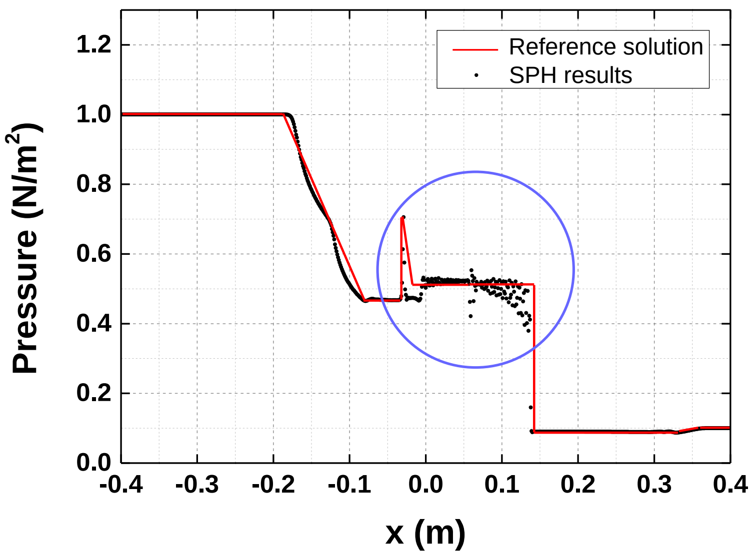
[Pressure profiles (t=0.1 s , $\nabla \cdot \boldsymbol{B}$ **correction O**)]

Effect of Dissipation Term

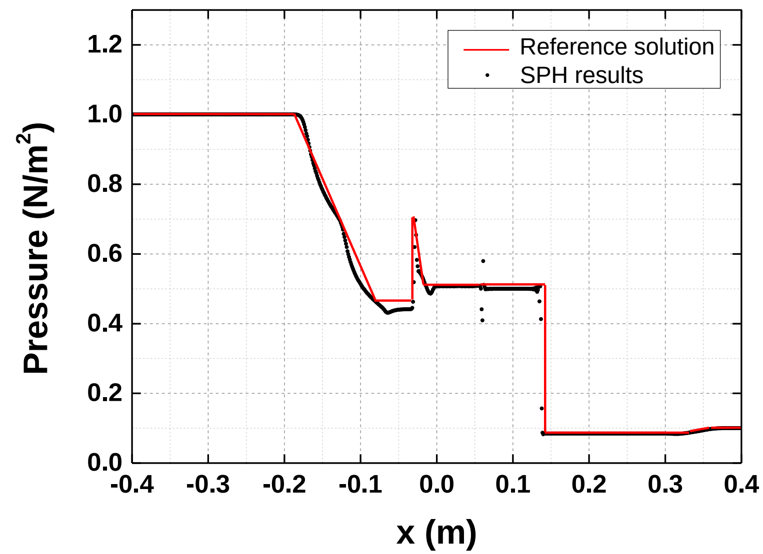
- When there is a sudden discontinuous interface due to shock, unphysical oscillations occur, and **artificial viscosity** has been widely used in reducing these numerical errors [12-14].
- In MHD simulations, the addition of an **artificial resistivity** term in the induction equation in order to deal with discontinuities in the magnetic field is the main requirement.

$$\left(\frac{d\boldsymbol{v}_i}{dt}\right)_{diss} = \sum_j m_j \frac{\alpha v_{sig}(\boldsymbol{v_i} - \boldsymbol{v}_j)}{\bar{\rho}_{ij}} \cdot \frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|} \nabla W_{ij}$$

$$\left(\frac{d\boldsymbol{B}_i}{dt}\right)_{diss} = \rho_i \sum_j m_j \frac{\alpha_B v_{sig}^B}{2\overline{\rho_{ij}}^2} (\boldsymbol{B}_i - \boldsymbol{B}_j) \hat{\boldsymbol{r}}_{\boldsymbol{ij}} \cdot \nabla W_{ij}$$



[Pressure profiles (t=0.1 s, **Artificial resistivity X**)]

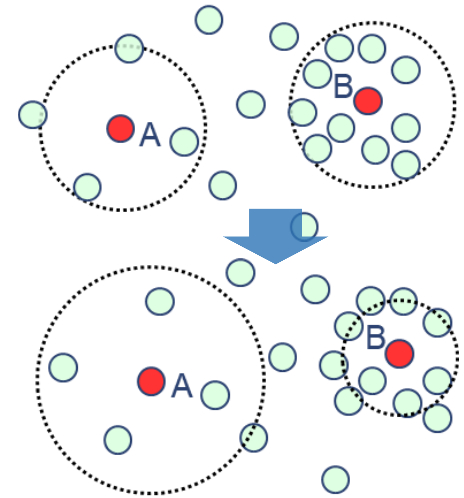


[Pressure profiles (t=0.1 s , **Artificial resistivity O**)]

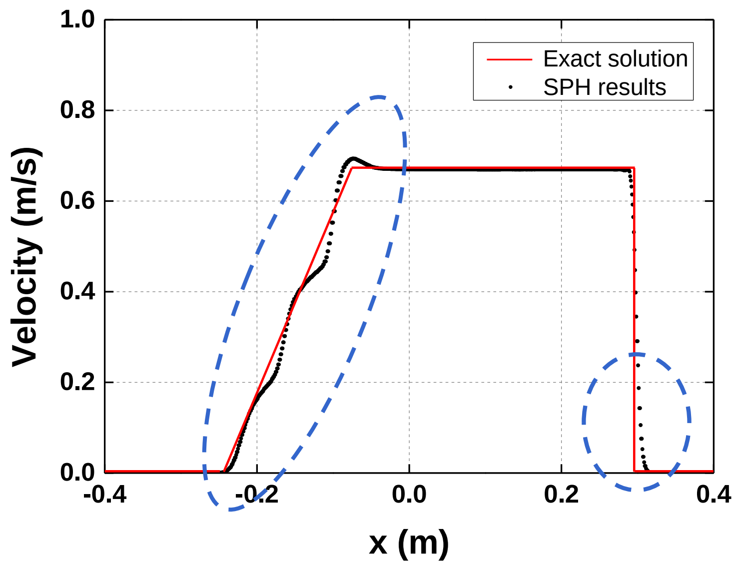
ASPH Methodology

❖ Application of ASPH

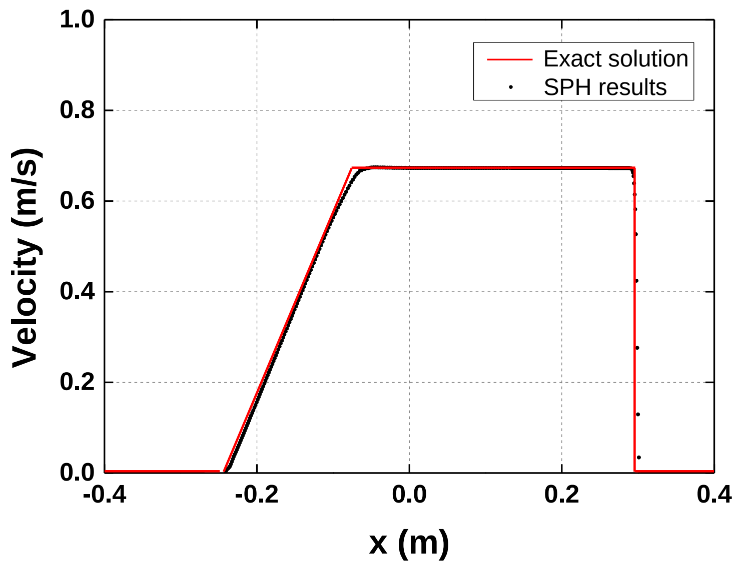
- The Adaptive SPH (ASPH) method is a technique that changes the smoothing length according to the particle number density.
- It replaces the isotropic smoothing algorithm of standard SPH.



$$\frac{du_i}{dt} = \frac{1}{2}\sum_j m_j \boldsymbol{v}_{ij}\left[\frac{P_i}{\rho_i^2} + \frac{P_j}{\rho_j^2}\right]\nabla_i W_{ij} \Rightarrow \frac{du_i}{dt} = \frac{1}{2}\sum_j m_j \boldsymbol{v}_{ij}\left[\frac{P_i}{\Omega_i\rho_i^2}\nabla_i W_{ij}(h_i) + \frac{P_j}{\Omega_j\rho_j^2}\nabla_i W_{ij}(h_j)\right]$$



[Velocity profiles (t=0.2 s, ***ASPH*** X)]



[Velocity profiles (t=0.2 s, ***ASPH*** O)]

SPH Governing Equations

❖ Resistive-MHD governing equations (SPH formulation)

Mass conservation

$$\frac{d\rho_a}{dt} = \frac{1}{\Omega_a}\sum_b \frac{m_b}{\rho_b}\boldsymbol{v}_{ab}\nabla_a W_{ab}(h_a)$$

Mtm conservation

$$\frac{d\boldsymbol{v}_a}{dt} = \sum_b m_b\left(\frac{\overleftrightarrow{\boldsymbol{M}_a}}{\Omega_a\rho_a^2}\cdot\nabla W_a + \frac{\overleftrightarrow{\boldsymbol{M}_b}}{\Omega_b\rho_b^2}\cdot\nabla W_b\right) + \underbrace{\sum_b m_b \frac{\alpha v_{sig}(\boldsymbol{v}_a - \boldsymbol{v}_b)}{\bar{\rho}_{ab}}\cdot\frac{\boldsymbol{r}_{ab}}{|\boldsymbol{r}_{ab}|}\nabla\bar{W}_{ab}}_{\text{Artificial viscosity}} \underbrace{- \boldsymbol{B}_a\sum_b m_b\left(\frac{\boldsymbol{B}_a}{\Omega_a\rho_a^2}\cdot\nabla_a W_{ab}(h_a) + \frac{\boldsymbol{B}_b}{\Omega_b\rho_b^2}\cdot\nabla_a W_{ab}(h_b)\right)}_{\nabla\cdot B \text{ Correction}},$$

Current density

$$\boldsymbol{J}_a = \frac{1}{\mu_0}\nabla\times\boldsymbol{B} = -\frac{\rho_a}{\mu_0}\sum_b m_j\left(\frac{\boldsymbol{B}_a}{\Omega_a\rho_a^2}\times\nabla_a W_{ab}(h_a) + \frac{\boldsymbol{B}_b}{\Omega_b\rho_b^2}\times\nabla_b W_{ab}(h_b)\right)$$

Induction equation

$$\frac{d\boldsymbol{B}_a}{dt} = -\frac{1}{\Omega_a\rho_a}\sum_b m_b[\boldsymbol{v}_{ab}(\boldsymbol{B}_a\cdot\nabla W_a) - \boldsymbol{B}_a(\boldsymbol{v}_{ab}\cdot\nabla W_a)] + \underbrace{\rho_a\sum_b m_b\frac{\alpha_B v_{sig}^B(\boldsymbol{B}_a - \boldsymbol{B}_b)}{\bar{\rho}_{ab}^2}\cdot\frac{\boldsymbol{r}_{ab}}{|\boldsymbol{r}_{ab}|}\nabla\bar{W}_{ab}}_{\text{Artificial resistivity}} \underbrace{- \rho_a\sum_b m_b\left(\frac{\eta_a\boldsymbol{J}_a}{\Omega_a\rho_a^2}\times\nabla_a W_{ab}(h_a) + \frac{\eta_b\boldsymbol{J}_b}{\Omega_b\rho_b^2}\times\nabla_b W_{ab}(h_b)\right)}_{\text{Resistive induction term}}$$

Energy conservation

$$\frac{du_a}{dt} = \frac{1}{2}\sum_b m_b\boldsymbol{v}_{ab}\left[\frac{p_a}{\Omega_a\rho_a^2}\nabla_a W_{ab}(h_a) + \frac{p_b}{\Omega_b\rho_b^2}\nabla_a W_{ab}(h_b) + \Pi_{ab}\cdot\overline{\nabla_a W_{ab}}\right] + \frac{1}{\rho_a}\eta_a\boldsymbol{J}_a^2$$

Equation of State

$$P_a = (\gamma - 1)\rho_a u_a$$

Dissipation switch

$$\alpha_{B,a} = \frac{h_a|\nabla\boldsymbol{B}_a|}{|\boldsymbol{B}_a|}$$

$$v_{sig}^B = \frac{1}{2}\sqrt{\frac{B_a^2}{\mu_0\rho_a} + \frac{B_b^2}{\mu_0\rho_b}},\qquad \boldsymbol{v}_{ab} = \boldsymbol{v}_a - \boldsymbol{v}_b,\qquad \Omega_a = 1 - \frac{\partial h_a}{\partial\rho_a}\sum_b m_b\frac{\partial W_{ab}(h_a)}{\partial h},$$

$$\overline{\nabla_a W_{ab}} = \frac{1}{2}\{\nabla_a W_{ab}(h_a) + \nabla_a W_{ab}(h_b)\},\qquad \overleftrightarrow{\boldsymbol{M}_a} = \boldsymbol{B}\boldsymbol{B}_a - \left(\frac{1}{2}\boldsymbol{B}_a^2 + P_a\right)\overleftrightarrow{I}$$

4. V&V and Test simulations

- V&V of Developed SPMHD model
  1. Hydrodynamic shock problem
  2. Ideal MHD problem
  3. Resistive MHD problem
  4. Pinch problem
- Preliminary simulation of X-pinch (on-going)

V&V of Developed SPMHD model

❖ V&V Simulation Cases

✓ The models required for the pinch plasma simulation have been sequentially incorporated.

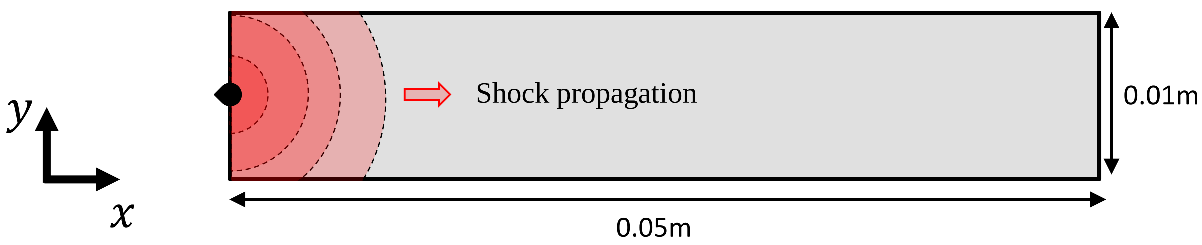
✓ The simulations using the implemented models are compared with some reference Eulerian MHD simulations and analytical solutions.

V&V Cases	Assessment objectives
➢ **Hydrodynamic shock problem** - *Slab detonation* - *SOD shock tube*	1. Capturing shock (artificial viscosity) 2. Applying ASPH methodology
➢ **Ideal MHD problem** - *Brio&Wu shock tube* - *Orszag-tang vortex*	1. Calculating magnetic field (induction equation) 2. Controlling numerical instability (artificial resistivity)
➢ **Resistive MHD problem** - *Resistive MHD shock tube (w/ constant resistivity)* - *Resistive MHD shock tube (w/ varying resistivity)*	1. Calculating current density (Ampere's law) 2. Calculating resistive terms 3. Incorporating the plasma resistivity model
➢ **Pinch plasma problem** - *Magnetized Noh Z-pinch problem*	1. Reviewing the comprehensive model

V&V of Developed SPMHD model

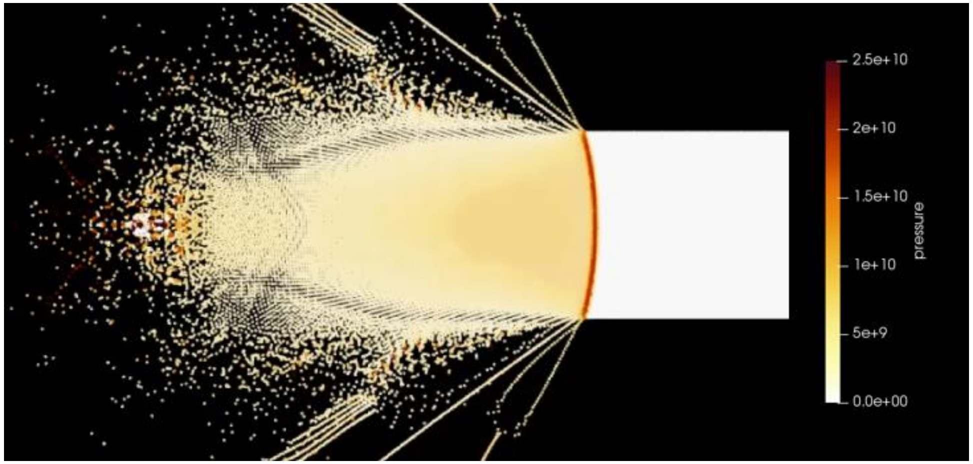
❖ Hydrodynamic shock problem (1/2)

- 2D Slab detonation simulation

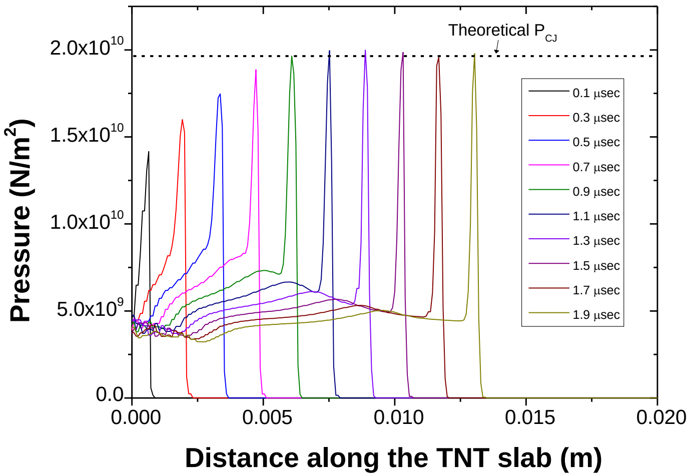


●	Detonation point
ρ_0	1630 (kg/m^3)
U_0	4.29×10^6(J/kg)
D_{CJ}	6930 (m/s)

[2D slab detonation simulation conditions]



[2D slab detonation simulation (Pressure)]

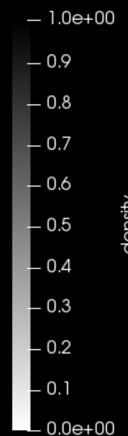


[Pressure distribution]

V&V of Developed SPMHD model

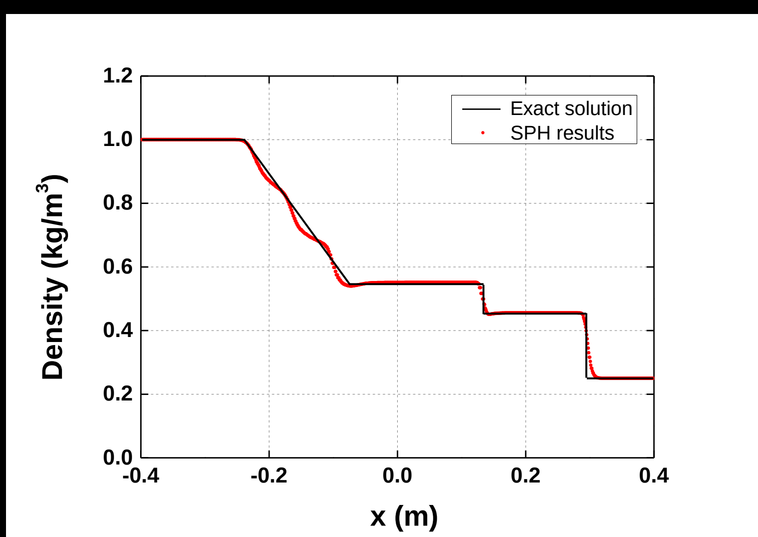
❖ Hydrodynamic shock problem (2/2)

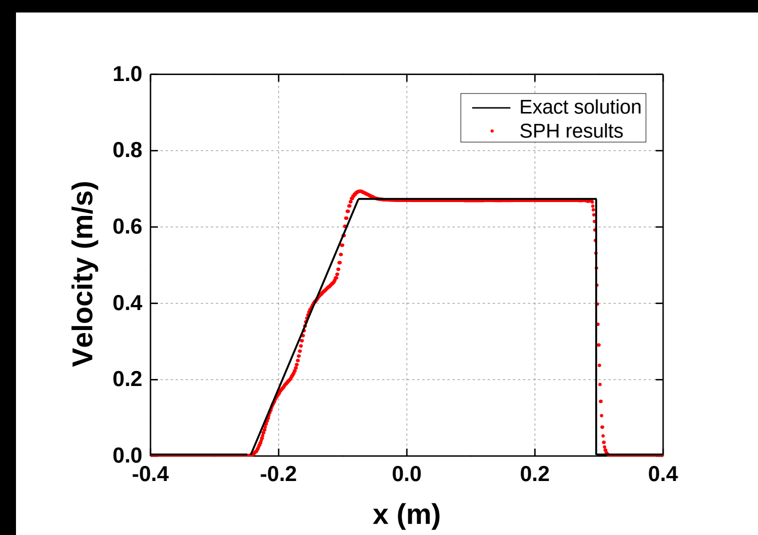
- 2D SOD shock tube



[2D sod shock tube simulation (Density)]

$$V_L = \begin{bmatrix} \rho_L \\ v_L \\ P_L \end{bmatrix} = \begin{bmatrix} 1\ kg/m^3 \\ 0\ m/s \\ 1\ N/m^2 \end{bmatrix} \qquad V_R = \begin{bmatrix} \rho_R \\ v_R \\ P_R \end{bmatrix} = \begin{bmatrix} 0.25\ kg/m^3 \\ 0\ m/s \\ 0.1795\ N/m^2 \end{bmatrix}$$

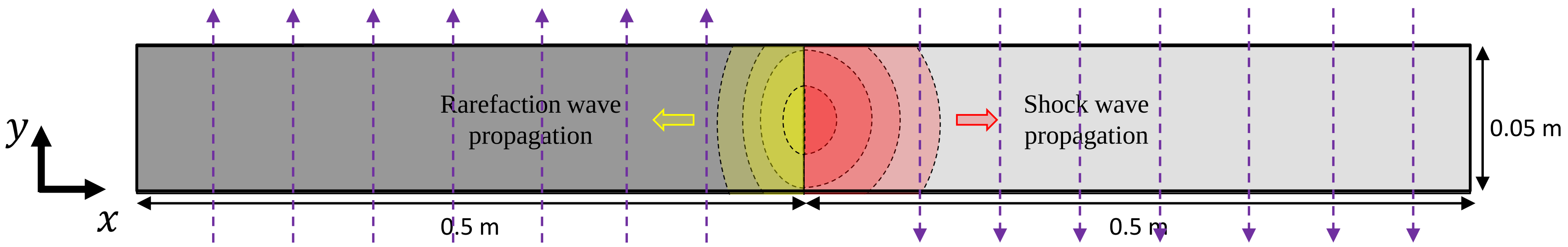




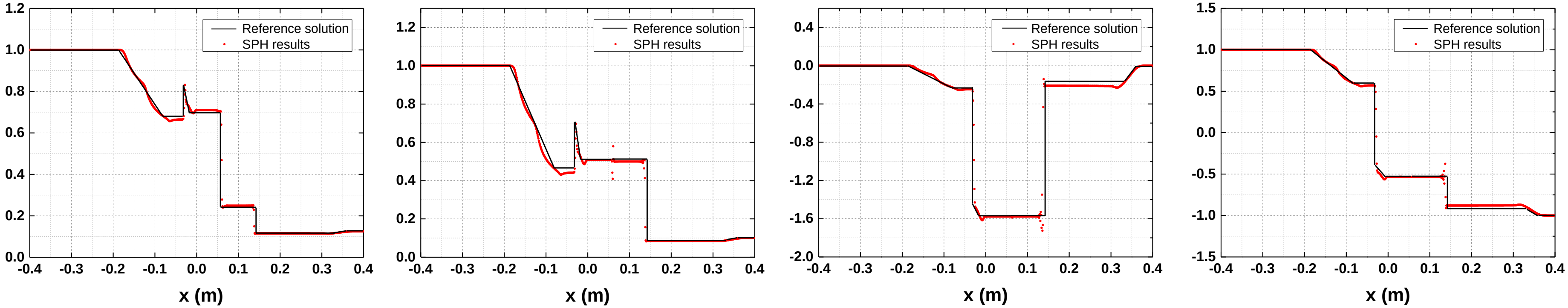
V&V of Developed SPMHD model

❖ Ideal MHD problem

- Brio & Wu shock tube



$$V_L = \begin{bmatrix} \rho \\ v_x \\ B_y \\ p \end{bmatrix} = \begin{bmatrix} 1\ kg/m^3 \\ 0\ m/s \\ 1 \\ 1\ N/m^2 \end{bmatrix} \qquad V_R = \begin{bmatrix} \rho \\ v_x \\ B_y \\ p \end{bmatrix} = \begin{bmatrix} 0.125\ kg/m^3 \\ 0\ m/s \\ -1 \\ 0.1\ N/m^2 \end{bmatrix}$$



[Density (kg/m³)] [Pressure (N/m²)] [Velocity in y direction (m/s)] [Magnetic field]

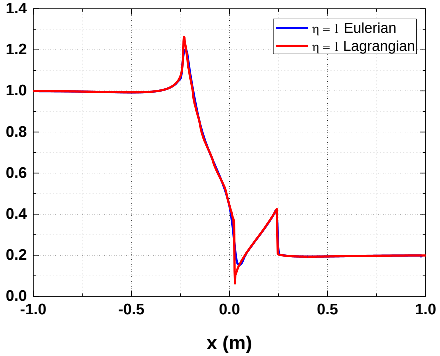
V&V of Developed SPMHD model

❖ Resistive MHD problem

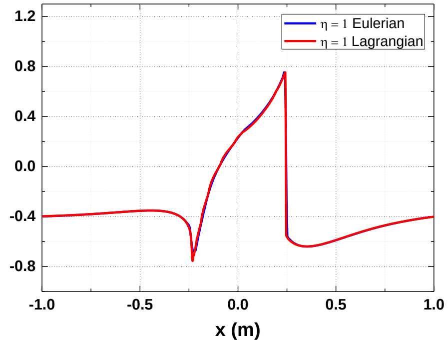
- Resistive MHD shock tube

$$(\rho^L, v^L, P^L) = (1.0, 0.4, 1.0), \quad (\rho^R, v^R, P^R) = (0.2, 0.4, 0.1)$$

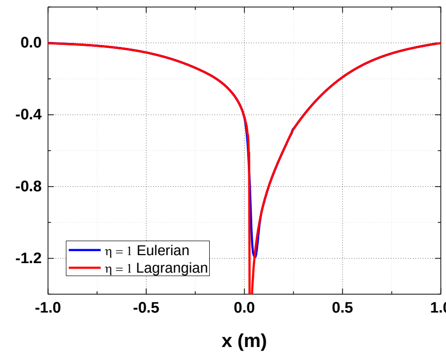
➢ Constant resistivity ($\eta = 1$)



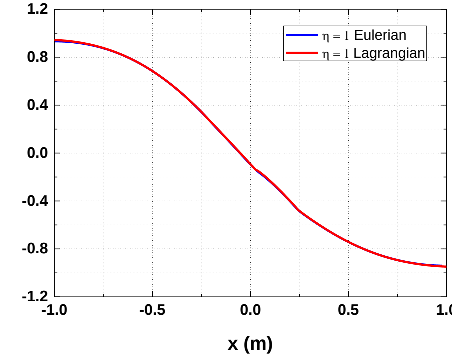
[Density (kg/m³)]



[Velocity in x direction (m/s)]

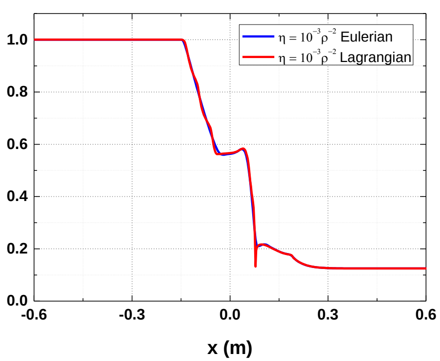


[Velocity in y direction (m/s)]

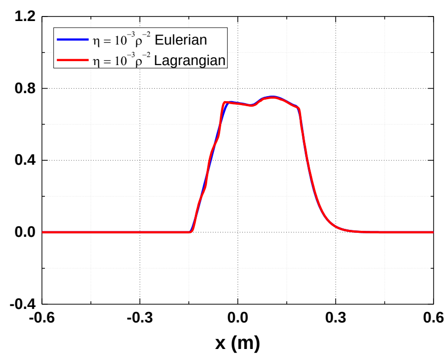


[Magnetic field]

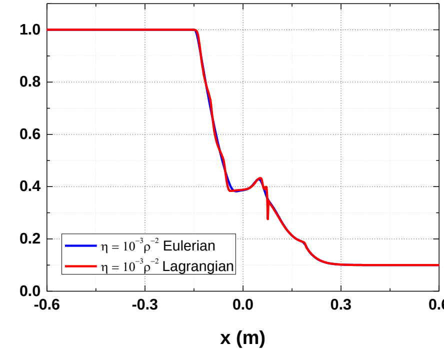
➢ Varying resistivity ($\eta = 10^{-3}\,\rho^{-2}$)



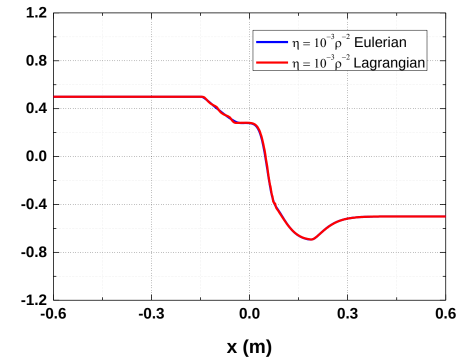
[Density (kg/m³)]



[Velocity in x direction (m/s)]



[Pressure (N/m²)]



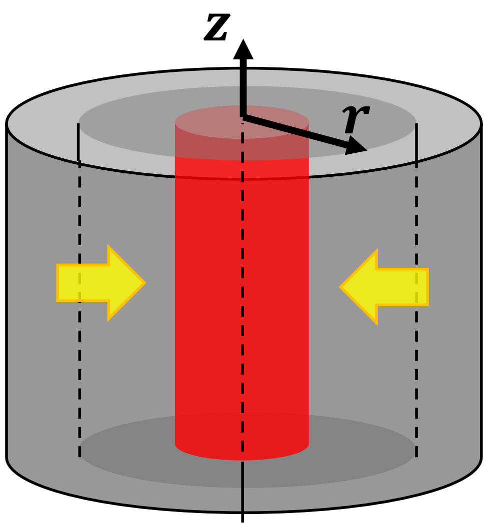
[Magnetic field]

V&V of Developed SPMHD model

❖ Magnetized Noh Z-pinch problem

- Magnetized Noh Z-pinch problem is an extension of the classic gas dynamics Noh problem.
- In this problem, current driven through a cylindrical column of the plasma induces the material to rapidly compress axially through $J \times B$ force.
- Recently, this problem proposed as a benchmark problem to determine whether pinch plasma can be simulated [15].
- At 30 nsec, some physical properties are compared and verified with the analytic solution.

$$\rho = 3.1831 \times 10^{-5} r^2 \ [g/cm^3]$$
$$v_r = -3.24101 \times 10^7 \ [cm/s]$$
$$B_\phi = 6.35584 \times 10^5 r \ [gauss]$$
$$p = C \times B_\phi^2 \quad (\beta = 8\pi \times 10^{-6})$$

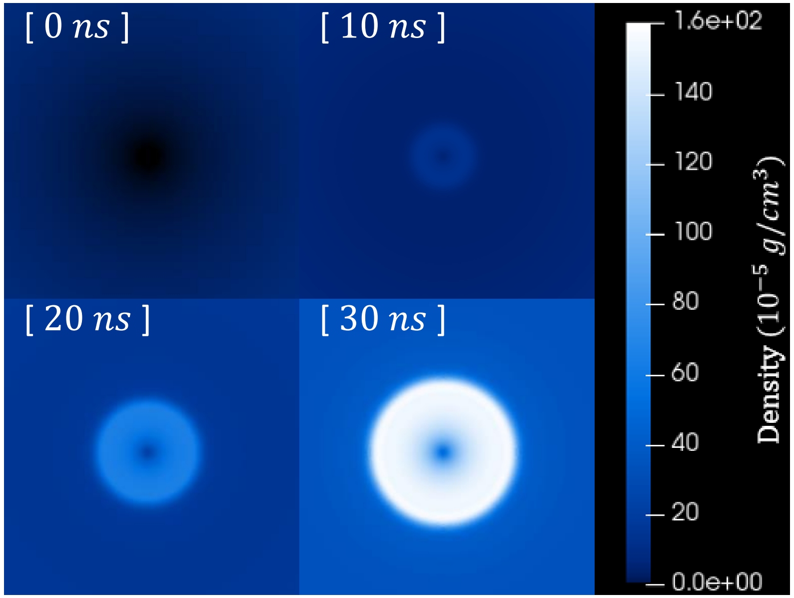


[Schematic diagram of Magnetized Noh Z-pinch problem]

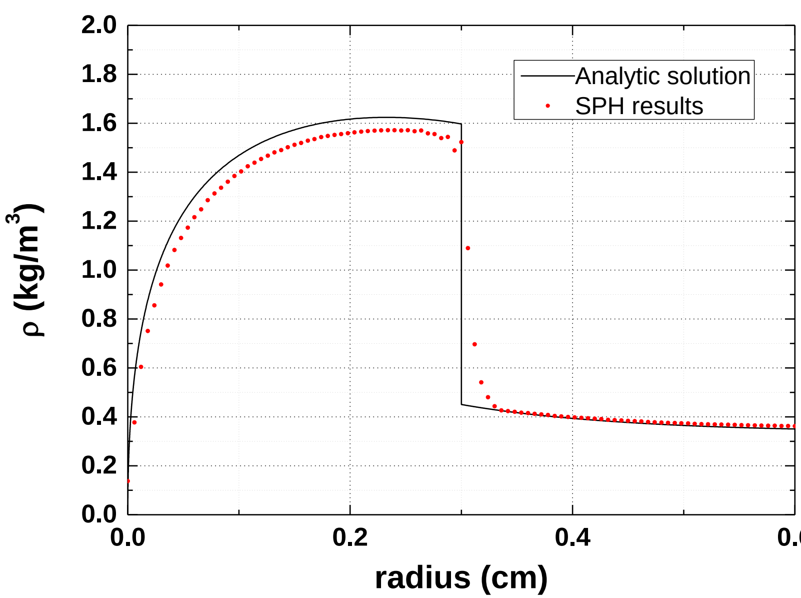
V&V of Developed SPMHD model

❖ Magnetized Noh Z-pinch problem

- Magnetized Noh simulation is performed to verify the implemented model in the pinch situation.
- The results for the 3 properties (density, pressure, and v_r) are compared with the theoretical values in the range of $r = 0 \sim 0.6\ cm$.
- The implemented model predicts the pinch plasma behaviors fairly well.



[Noh problem Density distribution]



[Density profiles at 30 ns)]

V&V of Developed SPMHD model

❖ Magnetized Noh Z-pinch problem

- Magnetized Noh simulation is performed to verify the implemented model in the pinch situation.
- The results for the 3 properties (density, pressure, and v_r) are compared with the theoretical values in the range of $r = 0 \sim 0.6\ cm$.
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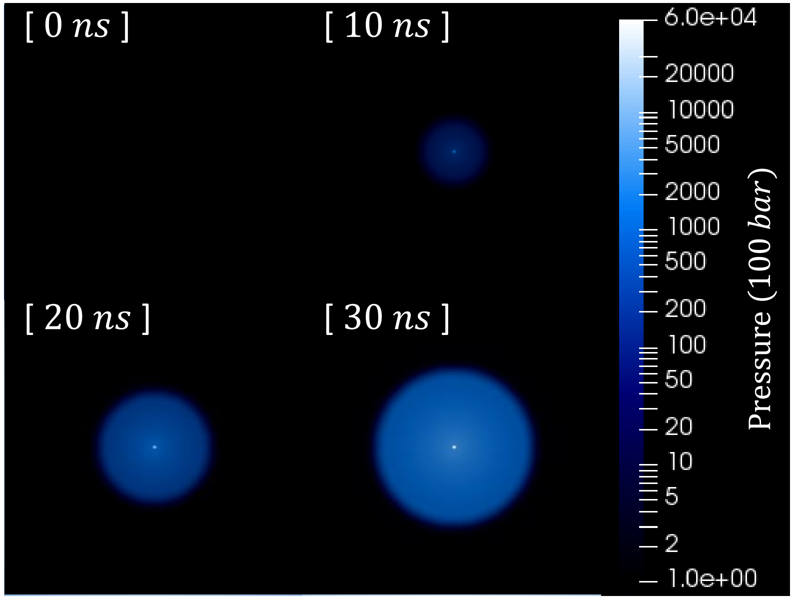
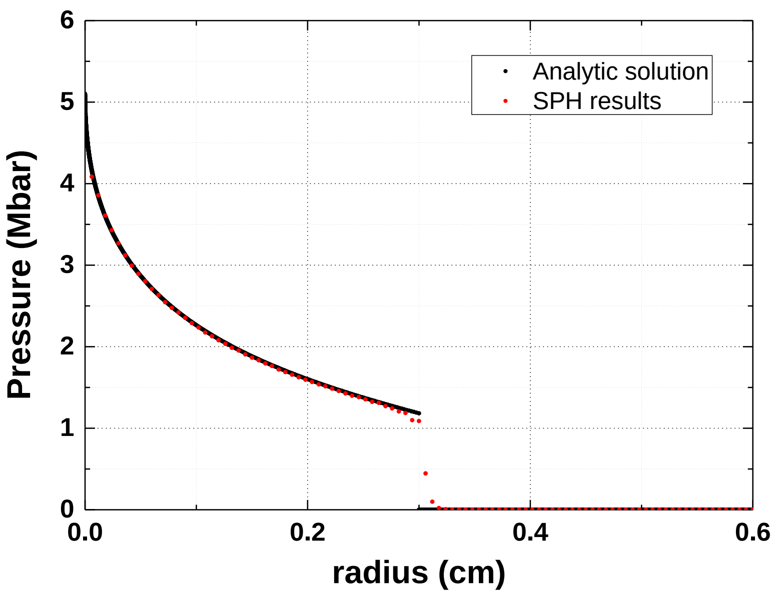


[Noh problem Pressure distribution]

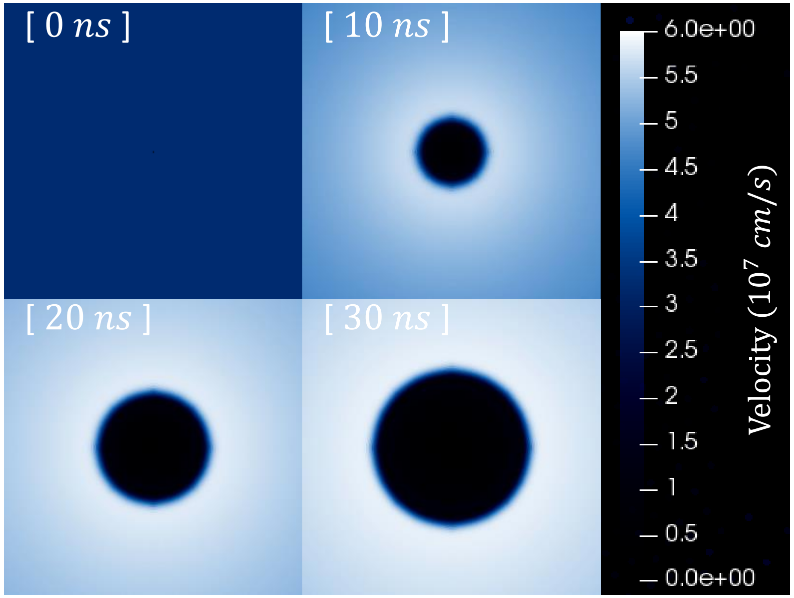


[Pressure profiles at 30 ns)]

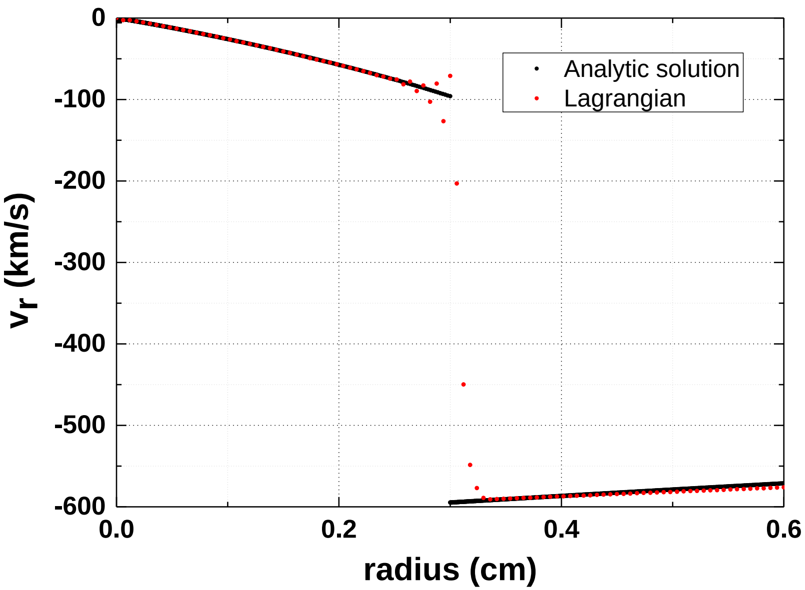
V&V of Developed SPMHD model

❖ Magnetized Noh Z-pinch problem

- Magnetized Noh simulation is performed to verify the implemented model in the pinch situation.
- The results for the 3 properties (density, pressure, and v_r) are compared with the theoretical values in the range of $r = 0 \sim 0.6\ cm$.
- The implemented model predicts the pinch plasma behaviors fairly well.



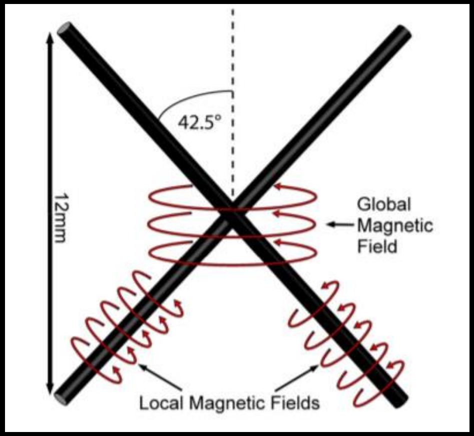
[Noh problem Velocity distribution]

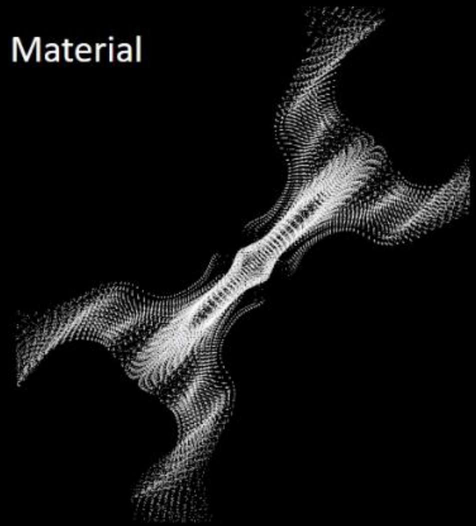


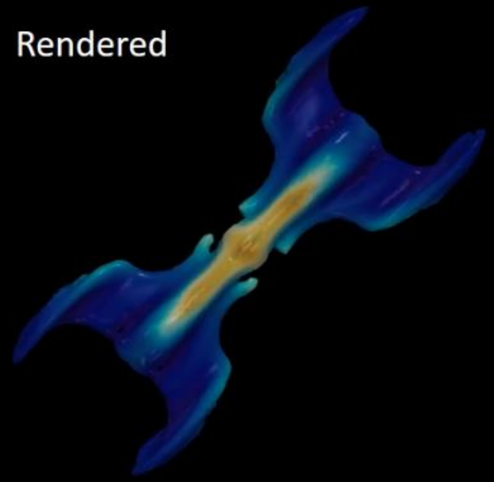
[Velocity profiles at 30 ns)]

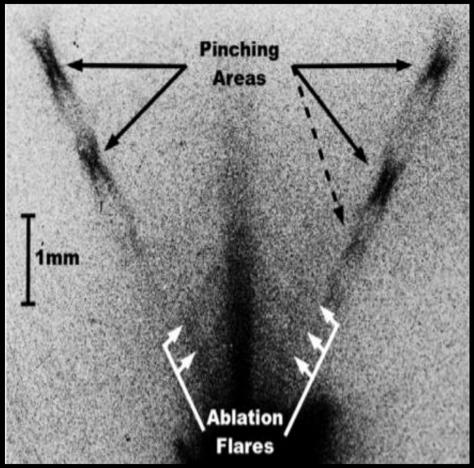
Preliminary simulation of X-pinch (on-going)

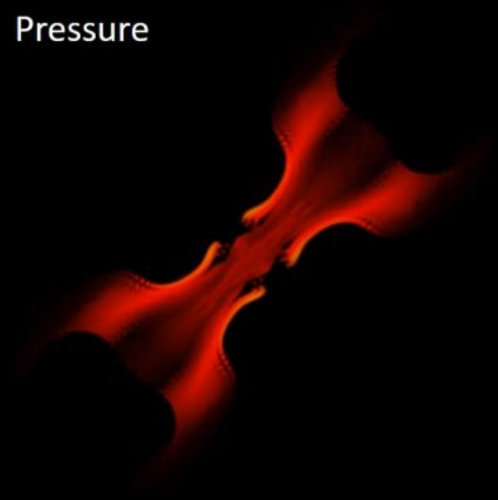
❖ X-Pinch Plasma Simulation

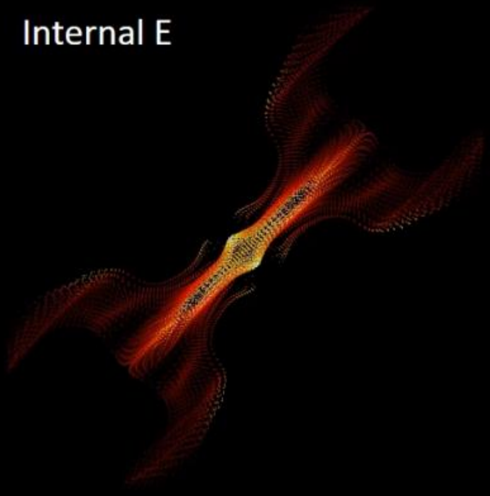












5. Summary

Summary of Study

❖ Summary

: Implementation of **resistive-MHD** based **SPH** model for Numerical Simulation of Pinch Plasma

- The resistive MHD based SPH model has been developed, and it has been verified and validated through various MHD simulations.
- Several **dissipation terms** that capture the shock and reduce numerical instability are incorporated.
- The **divergence B correction term** is incorporated to maintain the divergence constraint of the plasma ($\nabla \cdot \boldsymbol{B} = 0$).
- The **ASPH method** is incorporated to enable accurate calculations for uneven particle distribution.
- The implosion behavior of X-pinch plasma has been simulated with the developed SPH code. The simulation well produces the neck and beam shape, which are important features of X-pinch.
- In order to derive the exact physical values of X-pinch simulation, the following models must be supplemented. (future work)
  ① Correct EOS model
  ② Rigorous plasma resistivity model
  ③ Radiation model
  ④ Plasma ionization equilibrium equation

Thank you!

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