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Preliminary Study on Monte Carlo Simulation for Fault Tree Quantification using Importance Sampling and Copula

Kyungho Jin [a], Gyunyoung Heo [a*]
[a]*Kyung Hee University, 1732 Deogyeong-daero, Giheung-gu, Yongin-si, Gyeonggi-do, 17104, Korea*
[*]*Corresponding author: gheo@khu.ac.kr*

1. Introduction

In probabilistic safety assessment (PSA), the top event probability that indicates a core damage of nuclear power plants (NPPs) is evaluated by fault tree analysis. One way to quantify fault trees is to determine minimal cut sets (MCSs), which are the minimal combinations of basic events occurring a top event. This method is widely used in the applications of PSA because the MCSs of a top event provide the useful information (e.g. which components or systems contribute significantly to the risk of NPPs.).

On the other hand, the Monte Carlo (MC) simulation is an alternative way to quantify fault trees. The method of quantifying a top event using MC has the disadvantage that it cannot derive MCSs, but it is possible to derive the top event probability without assumptions such as rare event approximation (REA) used in the MCS method.

Recently, the challenges of the quantification for the PSA models have emerged because the scope of PSA is extended to multi-units. There are some issues to apply both MSC and MC methods directly to multi-unit PSA (MUPSA) models. As the size of a PSA model increases, the number of failure logic and basic events included in the fault trees of the top event increases exponentially. As a result, it takes a long time to derive the MCSs of MUPSA model and numerous trials are required to obtain reasonable results in MC simulations. For this reason, it gets motivated to develop a practical method for quantifying fault trees of multiple units.

Research on the derivation of the MCSs for MUPSA model is now being conducted in the Multi-Unit Risk Research Group (MURRG) project. Therefore, this paper focuses on the advanced fault tree quantification method using MC simulations. In fact, the quantification of MUPSA models using MC simulations has also been studied in [1]. Since this research mainly focused on the development of a method for quantifying MUPSA model, there is still a need to reduce the computational cost in MC simulations.

In this paper, the importance sampling, one of the variance reduction techniques, is introduced to perform MC simulations more efficiently based on previous studies[2][3]. Furthermore, a joint probability distribution using copula is applied to dependent failures instead of common cause failure (CCF) modelling. The probability of dependent failures is evaluated via MC simulations of a copula to reduce the number of basic events in the PSA models. It is expected that these two approaches can reduce the cost of MC simulations for quantifying large fault trees.

2. Methods

2.1 Direct MC Simulation

The direct MC simulation is basically based on the following theoretical backgrounds. The expectation of function $g(x)$ is evaluated as follows:

$$E\big(g(x)\big) = \int g(x)f(x)dx \tag{1}$$

where $E\big(g(x)\big)$ is the expectation of $g(x)$ and $f(x)$ is a probability density function of x. The integral in Eq. (1) can be approximated using MC simulations.

$$E\big(g(x)\big) = \frac{1}{N}\sum_{i=1}^{N} g(x^i) \tag{2}$$

where N is the number of simulations and x^i is the i-th sample generated from $f(x)$. In quantifying a fault tree using MC simulations, $g(X)$ is the occurrence of a top event with d basic events, $X = (x_1, x_2, \dots, x_d)$. Generally, a random number is generated from a standard uniform distribution, $u_d^i \sim U(0,1)$. The state of basic events (i.e. TRUE or FALSE) is determined by comparing the probability of x_d^i with u_d^i. If u_d^i is less than the probability of x_d^i, then the state of x_d^i is set to TRUE, otherwise x_d^i is FALSE. The state of the top event for each simulation $g^i(X^i)$ can be determined by solving a failure logic among basic events. The simulation is repeated until *i* is equal to *N*. More detailed methods for direct MC on fault tree quantification are given in [4].

However, as in a general PSA model, the state of a basic event is less likely to be TRUE because the failure probability of a basic event (or component) is not high enough. The low probability of the basic event results in increases of the number of MC simulations to obtain sufficient TRUE state of basic events. Therefore, the importance sampling method as a variance reduction can be used to effectively perform MC simulations for rare events. Section 2.2 briefly introduces the MC simulation with importance sampling on fault tree quantification based on [2][3].

2.2 MC Simulation with Importance Sampling

The basic idea of the importance sampling is to generate samples from a proposal distribution $f^*(x)$ rather than an original distribution $f(x)$. Then the expectation is weighted to have original feature of $f(x)$.

Therefore, in fault tree quantifications, it is possible to reduce the total number of simulations by performing sampling from the distribution $f^*(x)$ that causes the basic event to fail more frequently. Eq. (3) describes a method of calculating the expectation of the function $g(x)$ by assigning the importance weight, $w(x)=\frac{f(x)}{f^*(x)}$. Eq. (4) represents a method of approximating Eq. (3) using MC simulation.

$$E\big(g(x)\big) = \int g(x)\frac{f(x)}{f^*(x)}f^*(x)dx \qquad (3)$$

$$E\big(g(x)\big) = \frac{1}{N}\sum_{i=1}^{N} g(x^i)\,w(x^i) \qquad (4)$$

In Fig. 1, the difference between the two methods for the random variable x and its cumulative distribution function (CDF), $F(x)$ is described. The importance sampling is intended to be applied to the output of CDFs, not to the area of random variable. This approach makes it possible to apply regardless of the type of probability distribution of individual basic events.

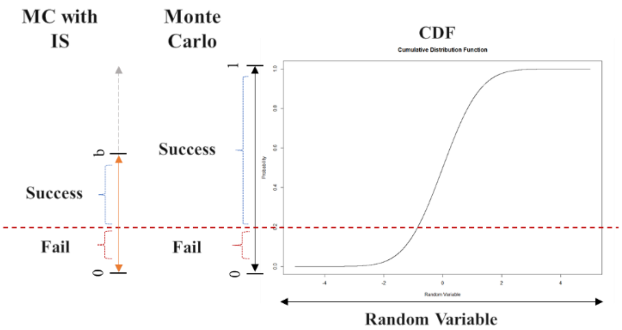

Fig. 1. The concept of direct MC simulation and the importance sampling method.

As shown in Fig. 1, the direct MC simulations determine the state of the basic event by generating random numbers from a uniform distribution between 0 and 1 ($u_d^i \sim U(0,1)$). However, the proposal distribution of the importance sampling generates random numbers between 0 and b ($u_d^i \sim U(0,b),\ b < 1$) to include TRUE state of the basic event more efficiently. The expectation is adjusted by the importance weight represented in Eq. (3) and (4) to address the original characteristics. The algorithm of MC simulation with importance sampling for the quantification of a fault tree is given in Table I.

Table I: The algorithm of MC simulation with importance sampling for the quantifications of a fault tree.

Start	***i* = 1 to *N***
I	Generate $u_d^i \sim U[0,b]$ for each basic event
II	Determine a state of each basic event, x_d^i (TRUE or FALSE)
III	Determine a state of a top event, $g(X^i)$ (TRUE or FALSE)
IV	Weight the basic events that cause the top event
V	Go to step I until i = N
End	Calculate the probability of a top event using Eq. (4)

2.3 FT quantification w/o CCF modelling

While, in the previous section, the authors have tried to reduce the number of trials required for MC simulation using the importance sampling method, in this section, the joint probability distribution using a copula is introduced to evaluate dependent failures without CCF modelling for the reduction of the number of basic events.

The CCF is defined as the failure of multiple components simultaneously or within a short time due to common causes. In the PSA model, the basic parameter model (BPM) or alpha factor model (AFM) [5] are used to model dependent failures in a fault tree by decomposing a basic event into an independent and common cause failures. Obviously, the number of the basic event to be addressed increases.

Therefore, this paper introduces a joint probability distribution between dependent components via copulas. In this framework, the dependent failure probability is estimated using MC simulations from a copula without CCF modelling. The study on how to evaluate the CCF probability using copulas has already been performed [6].

Copula is a multivariate probability distribution with uniform marginal distribution. The joint probability distribution can be constructed as follows [7].

$$\mathrm{F}(x_1, x_2, \dots, x_k) = \mathrm{C}(\mathrm{F}_1(x_1), \mathrm{F}_2(x_2), \dots, F_k(x_k)) \qquad (5)$$

where, $\mathrm{F}(x_1, x_2, \dots, x_k)$ is a joint probability distribution for random variable $X = (x_1, x_2, \dots, x_k)$, C is a copula and $F_k(x_k)$ is a marginal distribution of x_k. If the proper marginals and copula functions are given, it is possible to estimate the probability of the dependent failure using MC simulations.

Fig. 2 shows the traditional CCF modelling with the independent failure denoted by Q_A^1, Q_B^1 and common cause failure denoted by Q_{AB}^2. Since the reference [6] provides how to extract the copula parameter from the CCF factors, the dependent failure probability without CCF modelling can be achieved as shown in the bottom of Fig. 2.

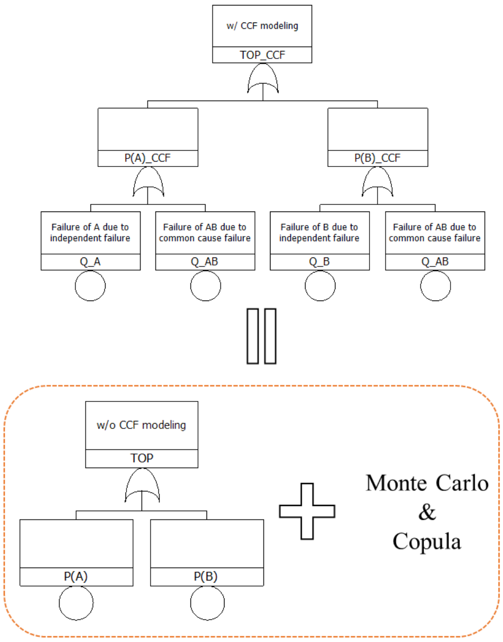

Fig.2. The concept of the estimation of the dependent failure probability without a CCF modelling using a copula and MC simulation.

3. Case Studies

In this chapter, a simple fault tree is constructed to compare the results of a direct MC and the MC with the importance sampling. In addition, the direct MC method is applied to a copula using the example case given in [6] to confirm that the same top event probability can be derived without a CCF modeling. The example fault tree is shown in Fig. 3.

The fault tree in Fig. 3 consists of a total of 9 basic events of which AC represents a common cause failure between the component A and C. Table II shows the probability of each basic event used in example studies and the sampling interval $U(0, b)$ of the uniform distribution for the importance sampling depending on the example cases.

Table II: The probability of the basic events included in the fault tree in Fig.3.

Basic Events	**Probability**	**Remarks**	$U[0, b]$	
A, C	1.94E-02	Independent	Case 1	[0, 0.8]
			Case 2	[0, 0.8]
AC	6.20E-04	CCF	Case 1	[0, 0.8]
			Case 2	[0, 0.1]
B, D~H	3.00E-04	Independent	Case 1	[0, 0.8]
			Case 2	[0, 0.1]

All sampling interval of the case 1 was setup as [0, 0.8] to show how the sampling interval contributes to the results. The sampling interval of the basic events except for A and C in the case 2 was reduced as [0, 0.1] to obtain TRUE state more efficiently. For the basic events A and C, the sampling interval was not narrowed because the probability of failure was relatively high.

3.1 Quantifications of the Simple Fault Tree with IS

First, the results of estimating the top event probability using the sampling interval of the case 1 given in Table II are represented in Fig. 4.

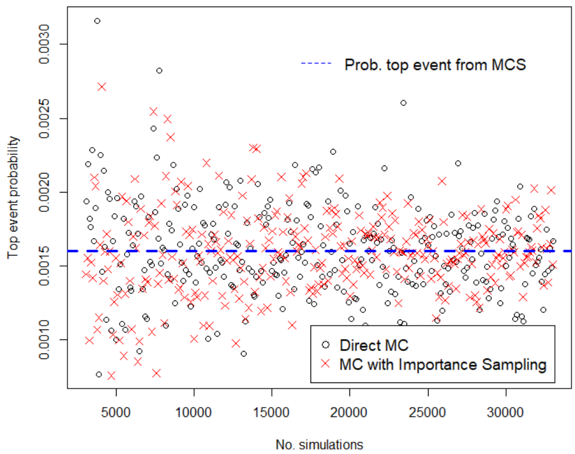

Fig. 4. Case 1: Top event probability depending on the number of simulations when b = 0.8 for all basic events.

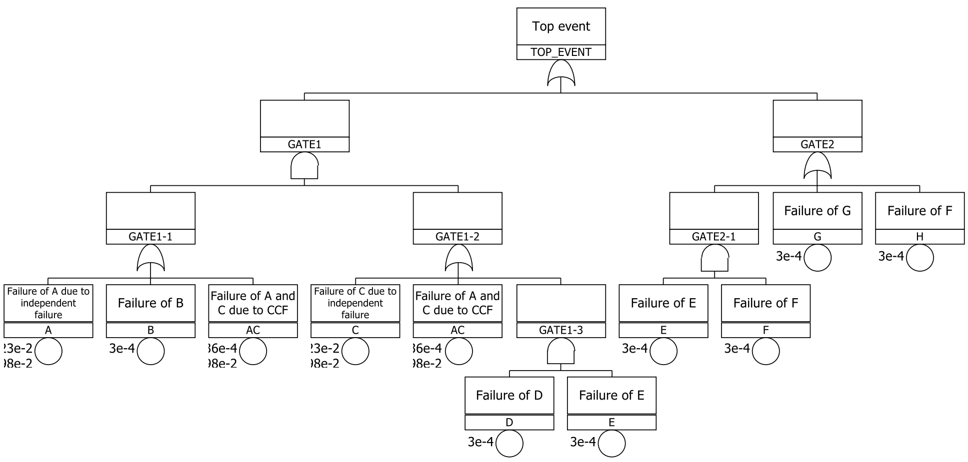

Fig. 3. The configuration of the fault tree used in example studies.

The blue line in Fig. 4 is the top event probability using the MCS method. When the sampling interval of the uniform distribution is relatively high (e.g. [0, 0.8]) compared to the component failure probability, it can be seen that it does not have the advantage of reducing variance even if the importance sampling was performed. On the other hand, Fig. 5 shows the advantage of the importance sampling using the narrowed sampling interval (Case 2).

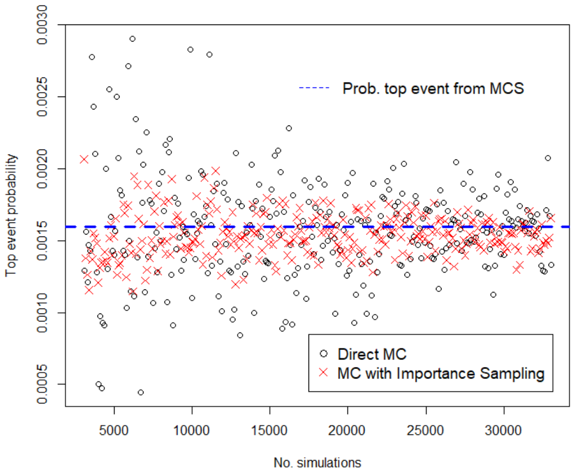

Fig. 5. Case 2: Top event probability depending on the number of simulations when b = 0.1 except for the basic events A, C.

As shown in Fig. 5, if the sampling interval of the basic events with low probability is adjusted, it can be confirmed that the variance reduction works well. In other words, the desired top event probability can be achieved with fewer number of simulations than that of direct MC method. The difference of the number of simulations between two methods will increase as the fault tree model becomes larger and the probability of a basic event becomes lower.

3.2 FT Quantification without CCF Modeling

Information on the CCF and copula parameters used in this paper is cited in [6]. As shown in the reference, the sampling between the component A and C was performed using the normal copula with the parameter 0.1778, which can lead to the same top event probability in the CCF modelling. In other words, the basic events A and C were not sampled from a uniform distribution, but a normal copula. Here, since the importance sampling was not applied to the copula model, only direct MC results were compared. Table III shows the results of the direct MC between w/ and w/o CCF modelling when the number of simulations is 1.0E+06.

Table III: The top event probability w/ and w/o CCF modelling.

	Top Event Probability
w/ CCF	1.60E-03
w/o CCF	1.59E-03

As shown in Table III, the top event probability between two methods is almost equal. In other words, the top event probability can be derived without CCF modelling when the fault tree is quantified using MC simulations and copulas. Although the example of only 2 train CCF model was compared, it is expected that if the PSA model becomes larger and more basic events subject to common cause failure are included in the PSA model, this method can be useful to reduce the quantification cost of MC simulations.

4. Conclusions

In this paper, the MC with the importance sampling method was briefly introduced based on the several references and an approach to reduction of the number of basic events using a copula was proposed to reduce the quantification cost of the multi-unit PSA model. It was confirmed that the variance reduction of the importance sampling is valid in MC simulations. The top event considering dependent failures was derived using a copula model without CCF modelling. As a future work, the study on how to accelerate quantification of fault tree in the MUPSA model will be performed by applying the importance sampling to the copula model.

Acknowledgement

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