Analysis of Nodalization Uncertainty for Higher-order Numerical Scheme under RBHT Experimental Conditions

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1. Introduction

One of phenomena that can occur in a pressurized water reactor (PWR) is the reflood phase during a large break loss of coolant accident (LOCA). The reflood is particularly interesting for the code assessment as it requires the system code to accurately predict specific fuel heat transfer and two-phase phenomena [1, 3]. During the reflood phase, several different heat transfer regimes (or modes) such as single-phase liquid convection, subcooled nucleate boiling, subcooled film boiling, transition boiling, dispersed flow, and single-phase vapor convection exist in the core. Sometimes all modes of heat transfer appears simultaneously [2]. That is why predicting the thermal-hydraulic phenomena accurately occurring during the reflood phase is regarded as an extremely difficult problem.

The existing nuclear system analysis codes such as RELAP5, MARS-KS and TRACE employ the 1st order numerical scheme in both space and time discretization. The 1st order numerical scheme is very robust and stable. However, it can yield excessive numerical diffusion problems. Thus, non-conservative results can be predicted for analyzing transients with steep spatial or temporal gradient of physical parameters. Thus, better predictive capability and more reduced computational cost are required for the advanced nuclear system analysis code.

In this study, the RBHT (Rod Bundle Heat Transfer) experiment is modeled by MARS-KS code. The authors conducted the uncertainty tests of the number and configuration of node for this experiment.

2. Methods

2.1 Higher-order Numerical Scheme in MARS-KS

In many nuclear system analysis codes, the 1st order upwind scheme is used for solving the governing equations due to simplicity and good stability. In the previous study [4], the first and the second-order upwind schemes, Lax-Wendroff scheme and centered differencing scheme were compared in terms of accuracy, stability and computational efficiency. Only Lax-Wendroff scheme are used for the analysis of nodalization uncertainty due to the numerical stability in this study. The governing equations are typically discretized as shown in eq. (1) on staggered grid with a semi-implicit scheme.

$$\frac{f_i^{n+1}-f_i^n}{\Delta t}+\frac{\langle f^n u^{n+1}\rangle_{i+1/2}-\langle f^n u^{n+1}\rangle_{i-1/2}}{\Delta x}=R_i^{n+1} \quad (1)$$

where f=ρψ, ρ is density of fluid, ψ=1 for the mass equation, ψ=u (velocity) for the momentum equation, ψ=e (internal energy) for the energy equation, Δt and Δx are the time and space steps, n and i are the temporal and spatial indices, R is the mass transfer term, the momentum source or heat source term plus the pressure term for the mass, momentum and energy equations, respectively. The angle brackets denote the fluxes as below.

$$\langle f^n u^{n+1}\rangle_{i+\frac{1}{2}}=u_{i+\frac{1}{2}}^{n+1}\left(f_i^n+\phi\left(1-\nu_{i+\frac{1}{2}}\right)\frac{f_{i+1}^n-f_i^n}{2}\right) \text{ if } u_{i+\frac{1}{2}}^{n+1}\geq 0$$
$$=u_{i+\frac{1}{2}}^{n+1}\left(f_{i+1}^n-\phi\left(1-\nu_{i+\frac{1}{2}}\right)\frac{f_{i+1}^n-f_i^n}{2}\right) \text{ if } u_{i+1/2}^{n+1}\leq 0 \quad (2)$$

$$\langle f^n u^{n+1}\rangle_{i-\frac{1}{2}}=u_{i-\frac{1}{2}}^{n+1}\left(f_{i-1}^n+\phi\left(1-\nu_{i-\frac{1}{2}}\right)\frac{f_i^n-f_{i-1}^n}{2}\right) \text{ if } u_{i-\frac{1}{2}}^{n+1}\geq 0$$
$$=u_{i-\frac{1}{2}}^{n+1}\left(f_i^n-\phi\left(1-\nu_{i-\frac{1}{2}}\right)\frac{f_i^n-f_{i-1}^n}{2}\right) \text{ if } u_{i-1/2}^{n+1}\leq 0 \quad (3)$$

where $\nu_{i+\frac{1}{2}}=\frac{u_{i+1/2}^{n+1}\Delta t}{\Delta x_{i+1/2}}$ is the Courant number. ϕ, ν is determined by the numerical schemes as shown in Table I.

Table I. ϕ, ν for the numerical schemes

Numerical scheme for the spatial	
1st order upwind scheme	$\phi=0$
2nd order upwind scheme	$\phi=3, \nu=0$
Lax-Wendroff scheme	$\phi=1$
Centered differencing scheme	$\phi=1, \nu=0$

For applying the higher-order numerical scheme on the boundary volume, the Lax-Wendroff scheme is applied for maintaining the order of accuracy and numerical stability [5]. In the 2nd order numerical schemes, the numerical dispersion problem can occur. Thus, to remove spurious oscillations of the 2nd order numerical scheme, the Van Albada (VA) flux limiter, which shows good performance in the study of Dean Wang et al. [6], is applied to the 2nd order numerical scheme.

2.2 RBHT Experiment

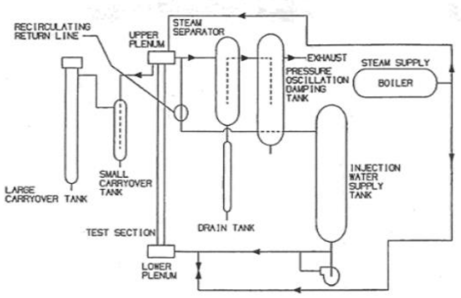

Fig. 1. Schematic of RBHT facility [3]

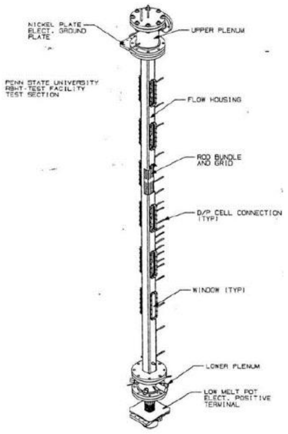

Fig. 2. Isometric view of test section [3]

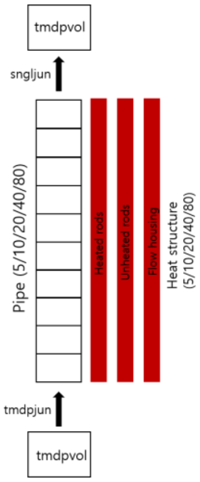

Fig. 3. Nodalization of test section

The RBHT (Rod Bundle Heat Transfer) facility was designed by the team of Penn State University with a special focus on development and validation of the reflood model. This experimental facility consists of a test section, coolant injection, steam injection systems, steam separator and steam collection tanks as shown in Fig. 1 [1-3]. The test section consists of the heated rod bundle, flow housing, lower and upper plena as shown in Fig. 2. The heated rod bundle simulates a small portion of a 17x17 PWR reactor fuel assembly.

The test section of the RBHT facility is modeled for the simulation in MARS-KS code. 45 heated rods, 4 unheated rods, flow housing, lower and upper plena of the test section are modeled as shown in Fig. 3. The lower and upper plena are represented by a time-dependent volume as the pressure boundary conditions. The heated and unheated rods are modeled as a pipe component with heat structures. The heat structures in the test section are modeled as 45 heated rods, 4 unheated rods and the flow housing wall. The reflood model is applied in the heated rods.

2.3 Nodalization Uncertainty

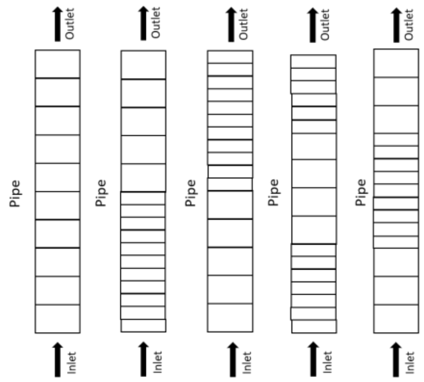

Fig. 4. 5 Cases for analysis of nodalization uncertainty

For the nodalization uncertainty analysis, the uncertainty for the number and configuration of nodes are compared. The number of nodes in the heated and unheated rods were changed; 5, 10, 20, 40 and 80. The axial nodes of the heat structures are identical with that of the pipe component. The radial nodes are fixed as 9 for the heated rods, 2 for the unheated rods and 4 for the flow housing wall. The node configuration of the pipe is determined as shown in Fig.4. The number of nodes is 20 for comparison of the node configuration uncertainty.

The nodalization uncertainty is evaluated with MARS-KS code having both the 1st order upwind scheme and the Lax-Wendroff scheme. The simulation results are compared with the experimental data of RBHT Test 0945.

3. Results

3.1 Difference with Reference Result

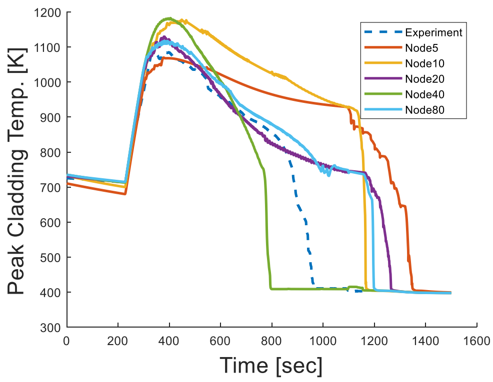

Fig. 5. 1st order upwind scheme results (PCT) depends on the number of nodes

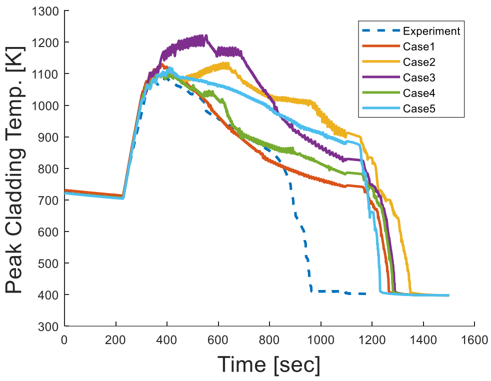

Fig. 6. 1st order upwind scheme results (PCT) depends on the configuration of nodes

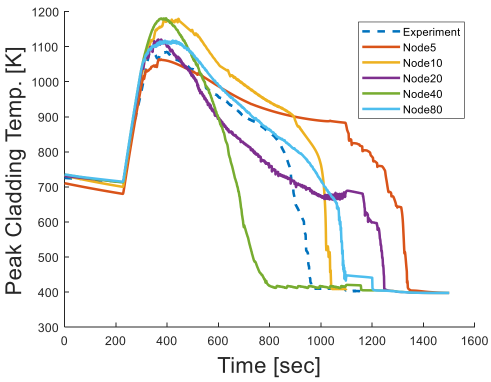

Fig. 7. Lax-Wendroff scheme results (PCT) depends on the number of nodes

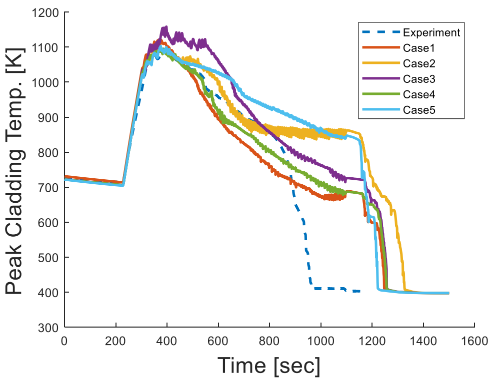

Fig. 8. Lax-Wendroff scheme results (PCT) depends on the configuration of nodes

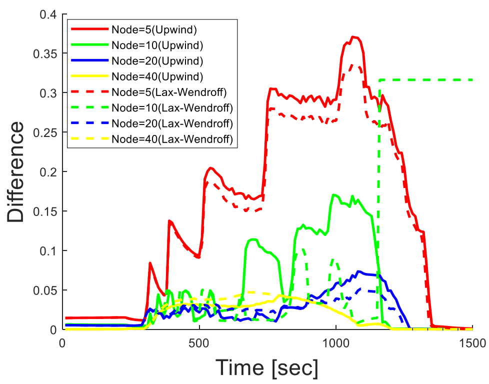

Fig. 9. Comparison for difference of 1st order upwind scheme or Lax-Wendroff scheme with the reference results

Figs. 5-8 show the results of PCT (Peak Cladding Temperature) depending on the number and configuration of nodes when using the 1st order upwind scheme and the Lax-Wendroff scheme. When using the same number and configuration of nodes, the 1st order upwind scheme and Lax-Wendroff scheme show different results as shown in Figs. 5-8.

Fig. 9 shows the difference of PCT using the 1st order upwind scheme or the Lax-Wendroff scheme with the reference result. The reference result is calculated by MARS-KS code with the Lax-Wendroff scheme using 80 uniform nodes. The difference is given by the following equation:

$$\text{Difference} = \left\| \frac{T_{ref} - T_{code}}{T_{ref}} \right\| / N \quad (1)$$

where $\|\cdot\|$ is the L_2 norm and T_{ref} is the reference results. And T_{code} are the solutions calculated by MARS-KS code with the 1st order upwind scheme or the Lax-Wendroff scheme and N is the node number.

Fig. 9 indicates that the difference is reduced when using the Lax-Wendroff scheme. This implies that the spatial accuracy can be improved in a situation where the dramatic changes in heat transfer and flow regimes occur. However, in case of 10 nodes with Lax-Wendroff scheme, the difference increases after about 1100sec. This is because the calculation of MARS-KS with the Lax-Wendroff scheme is failed at 1100sec.

3.2 Nodalization Uncertainty

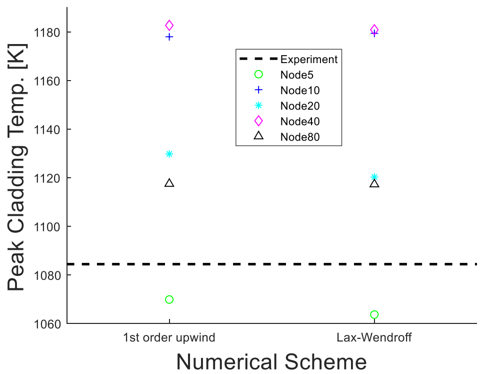

Fig. 10. Reflood peak comparison of 1st order upwind scheme and Lax-Wendroff scheme for the number of nodes

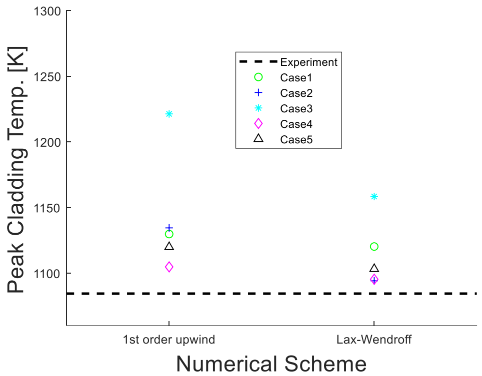

Fig. 11. Reflood peak comparison of 1st order upwind scheme and Lax-Wendroff scheme for the configuration of nodes

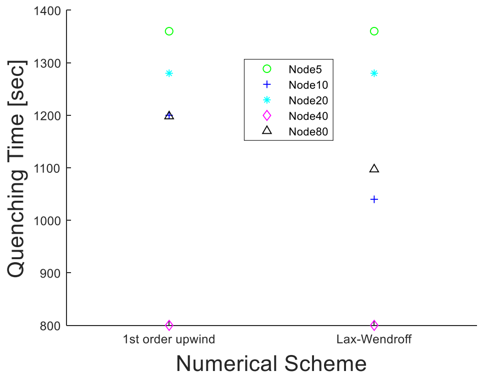

Fig. 12. Quenching time comparison of 1st order upwind scheme and Lax-Wendroff scheme for the number of nodes

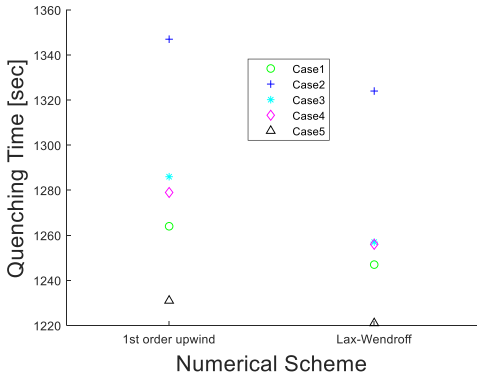

Fig. 13. Quenching time comparison of 1st order upwind scheme and Lax-Wendroff scheme for the configuration of nodes

Figs. 10-11 show comparison of the reflood peak using the 1st order upwind scheme or the Lax-Wendroff scheme for the number and configuration of nodes. There is no significant change in the uncertainty of the reflood peak depending on the number of nodes as shown in Fig. 10. However, the uncertainty of the reflood peak depends on the configuration of nodes are reduced significantly as shown in Fig. 11.

Figs. 12-13 show comparison of the quenching time using the 1st order upwind scheme or the Lax-Wendroff scheme for the number and configuration of nodes. The uncertainty of the quenching time depends on the configuration of nodes and is reduced significantly as shown in Fig. 12.

3. Conclusions

In this study, RBHT reflood experiment is modeled with the revised MARS-KS code for the node number and configuration uncertainty tests. When using the Lax-Wendroff scheme, the difference is reduced under RBHT experimental conditions, which is a situation in dramatic changes in the heat transfer and flow regimes. The reflood peak and quenching time are compared when using the 1st order upwind scheme and the Lax-Wendroff scheme. The uncertainty depends on the configuration of nodes in the reflood peak and quenching time is reduced using the Lax-Wendroff scheme.

For further works, the nodalization uncertainty tests for some integral effect tests and APR1400 accidents will be conducted. The effects of higher-order numerical schemes will be analyzed in a situation where several thermal-hydraulic phenomena occur simultaneously or dramatic changes in the heat transfer and flow regimes to study the predictive capability change with different numerical schemes.

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