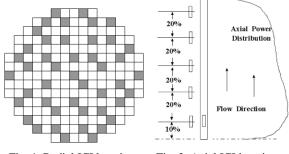
Sorted Compressive Sensing for Reconstruction of Failed In-Core Detector Signals

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1. Introduction

In nuclear reactor core, in-core instruments are installed to measure power distributions. Each in-core instruments (ICIs) have axially five detector levels. Fig. 1 shows radial configuration of ICIs (dark parts represent ICIs installed). Axial locations of detectors are shown in Fig. 2.





Reactor core monitoring system calculates core average axial power distributions and monitors whether core is approaching the operating limit conditions. During normal operation, some detectors may get failed. The accuracy of the calculated power distribution highly depends on the number of detectors available, and rejection of the failed detector requires costly process. To preserve full core coverage and appropriate tilt measurements, the reconstruction of failed detector signal is desirable.[1,2,3]

We propose a constructive method to reconstruct the failed in-core detector signals by using compressive sensing(CS) without any assumptions.

2. Sorted Compressive Sensing (ℓ_1 -reconstruction)

2.1 Compressive Sensing

CS is a signal processing technique to reconstruct a signal from far fewer measurements than required by the Shannon-Nyquist information criteria. We can get successful reconstruction, although the signal is sparse, which means most of the elements of the frequency domain signal are zero or negligible. [4,5]

The simple form of CS can be described as

$$y = Ax (1)$$

In this equation, the *N*-dimensional sequence, *y* is called measurement vector and formed by encoding frequency domain signal *x* into an *M*-dimensional measurements through a linear transformation by the $M \times N$ measurement matrix *A* (where m < n). The vector *x* is a discrete signal and called *k*-sparse if *x* has

at most $k \ll N$ nonzero entries. CS aims to reconstruct a signal x called k-sparse from y = Ax by solving the following ℓ_0 -minimization:

$$\min\{\|x\|_0 : Ax = y\}.$$
 (2)

This ℓ_0 -minimization is a combinatorial optimization problem and is considered as NP-hard. On the other hand, and ℓ_2 -minimization (least-squares method) is a viable method, but it is known cannot find the sparse solution. Hence, ℓ_0 and ℓ_2 -minimization are replaced by the ℓ_1 minimization:

$$\min\{\|x\|_1 : Ax = y\}. \tag{3}$$

2.2 ℓ_1 -Reconstruction

With partitioning the measured and lost signals, equation (3) can be regrouped:

$$Ax *= y \Rightarrow Ax *= b, Bx *= u. \tag{4}$$

where *b* is known, *u* is unmeasured vector, and x * is the optimal solution in the frequency domain. [6]

We suggest a more effective reconstruction method than conventional CS by sorting the data before reconstruction.[7] Because CS has higher accuracy when the signal x is sparser, we introduce to sort the measured signal first. In this case, we have to get the indices for that sorting because indices of the missing data are unknown. Since in-core detector signal is measured in the steady-state operation, the measured values are not that much different from the previous step in intact detector condition, as shown Fig. 5. We can get sort indices of previous step data and sort the measured data out using that indices without loss of generality. Fig. 4 shows the in-core detector measurements with failed detector. The data is listed in an arbitrary order. The sorted power distributions of intact(previous step) and lost detector conditions are shown in Fig. 5.

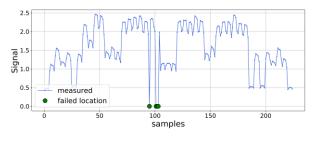


Fig. 4. ICI measurement with failed detectors

Fig. 6 shows frequency plot of original(left, not sorted) and sorted data(right). Apparently, the sorted data has less high frequency components and becomes sparser.

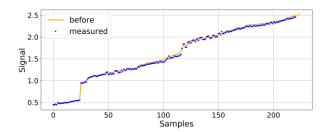


Fig. 5. Sorted ICI data using index of previous step

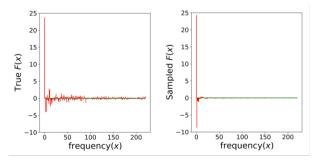


Fig. 6. Frequency components of the measurements

2.3 Demonstration and results

To demonstrate the performance of the developed method, we eliminated the signal at the locations of [95,101,102,103]. Fig. 7 shows the reconstructed result using conventional CS. We can see considerable reconstruction errors at the failed detector positions. Fig. 8 is the result of sorted ℓ_1 -reconstruction. The signals at the failed detector positions are perfectly reconstructed. The reason for this successful performance is from increasing the sparseness and greatly reduced number of frequency components by using sorted CS.

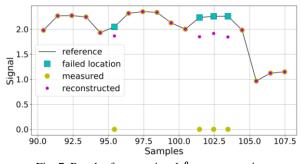


Fig. 7. Result of conventional ℓ_1 -reconstruction

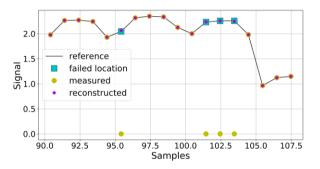


Fig. 8. Result of sorted ℓ_1 -reconstruction

Table I summarized the mean square errors(MSE) of CS reconstruction by increasing the number of failed detectors and the data is randomly selected from 15 measured data set from BOL to EOL. MSE results represent average of 10 cases of randomly eliminated positions for each case.

Table I: Mean Square Error of Reconstruction Results

# of failed	MSE of	MSE of
detectors	conventional CS	sorted CS
4	0.004658	0.000597
5	0.006193	0.000723
10	0.013935	0.001044
11	0.016753	0.001694

3. Conclusions

We propose a new method of reconstructing failed incore detector signals by using the sorted compressive sensing. The main idea of the method is to sort the signal to reduce its frequency components and to make the signal sparser. The demonstration shows that we can get almost perfect reconstruction of the failed detector signals without any assumption like replacing the lost signal with the symmetric position value. The developed technology can be successfully applicable to real-world reactor operation and monitoring.

Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Science, ICT & Future Planning(NRF-2016R1A5A1013919).

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