# Low Power Transient Analysis for Subcritical PWR Core with Fixed Neutron Source via 3-D Nodal Diffusion Code RAST-K

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#### 1. Introduction

The fixed neutron source provides the minimum detector response for the reactivity monitoring and helps the reliable initial critical approach during the startup period of the pressurized water reactor (PWR) reload core. During this period, when the power level is in the source range, the fixed source imposes a negative reactivity to the steady-state subcritical core as [1]:

$$\rho_0 = -\frac{s_{fixed}}{p_0},\tag{1}$$

where  $s_{fixed}$  and  $p_0$  are the weighted averages of the fixed source and the fission source, respectively. This initial negative reactivity can be very crucial in the low power transient analysis.

Recently, a three-dimensional (3-D) nodal diffusion code RAST-K (v2.2) has been developed by Ulsan National Institute of Science and Technology (UNIST) and sponsored by Korea Hydro & Nuclear Power Co., Ltd. (KHNP) [2]. The RAST-K uses the cross section library generated by the lattice transport code STREAM [3]. The RAST-K has the capabilities of both the microscopic depletion and transient calculations with thermal hydraulics feedback. The multicycle depletion capability of RAST-K has been extensively tested for several types of PWR cores for verification and validation [4].

This paper describes a new capability of the RAST-K to calculate the fixed source distributions based on the spontaneous fissions of the reloaded fuels and the incorporation of the fixed source term in the steady-state and the transient calculations. The numerical results of the rod ejection accident for the PWR reload core at the hot zero power (HZP) show the importance of the fixed source in the low power transient analysis.

# 2. Incorporation of Fixed Source in RAST-K

## 2.1. Fixed Source Distribution

There exist two major fixed neutron sources in the PWR reload core, which are the spontaneous fission of actinides and the  $(\alpha,n)$  reactions of the light nuclides. When these neutron source can provide the minimum count rate for the

reactivity monitoring, the external source such as Sb-Be is removed from the reload core.

The node-wise spontaneous fission source distributions are calculated as:

$$S_{fixed,g}^{m} = \chi_g \sum_{i} \nu_i \lambda_i N_i^{m} , \qquad (2)$$

where *m* is the node index, *g* is the two-group index ( $\chi_1=1$  and  $\chi_2 = 0$ ), *i* is the index for the actinide,  $\lambda_i$  and  $\nu_i$  are the spontaneous decay constant and the neutron yield per spontaneous fission, respectively, obtained from the decay data of the ENDF/B-VII.1 library, and  $N_i^m$  is the actinide number density at node *m* in a reloaded fuel, which is obtained from the restart file of the RAST-K generated for the multi-cycle depletion. A non-trivial contribution from the ( $\alpha$ , n) reactions [5] will be considered in a further study.

To consider the uncertainty of the isotopic compositions of the reloaded fuels and the unquantified fixed sources, a uniform source multiplier was introduced as a fudge factor.

#### 2.2. Steady-State Calculation with Fixed Source

The steady-state two-group nodal balance equations with fixed source can be written for node *m* as:

$$L_1^m + \sum_{r,1}^m \phi_1^m - \nu \sum_{f,1}^m \phi_1^m - \nu \sum_{f,2}^m \phi_2^m = S_{fixed,1}^m , \qquad (3)$$

 $L_{2}^{m} + \Sigma_{a,2}^{m} \phi_{2}^{m} - \Sigma_{1 \to 2}^{m} \phi_{1}^{m} = S_{fixed,2}^{m} , \qquad (4)$ 

where

$$L_g^m = \sum_{u=x,y,z} \frac{1}{a_u^m} (J_{g,u}^{m,+} - J_{g,u}^{m,-}) \text{ for } g = 1,2, \qquad (5)$$

and the standard notations are used. The RAST-K solves Eqs. (3) and (4) by the non-linear nodal expansion method based on the unified nodal method (UNM) formulation [8], where the Bi-Conjugate Gradient Stabilized (BiCGSTAB) algorithm [6] is used to solve the matrix equation.

In the subcritical steady-state with fixed source, the neutron flux level is inversely proportional to the soluble boron concentration. Thus, the soluble boron concentration can be iteratively updated based on the secant method, so that the corresponding core power converges to a given power level.

### 2.3. Transient Calculation with Fixed Source

The time-dependent two-group nodal balance equations with fixed source can be written as:

$$\frac{1}{v_g} \frac{d\phi_g^m}{dt} = R_g^m + S_{fixed,g}^m \,, \tag{6}$$

where

$$R_1^m = (1 - \beta^m)\psi^m - \Sigma_{r,1}^m \phi_1^m - L_1^m + S_d^m, \qquad (7)$$

$$R_2^m = \sum_{1 \to 2}^m \phi_1^m - \sum_{a,2}^m \phi_2^m - L_2^m,$$
(8)

$$\psi^m = \nu \Sigma_{f,1}^m \phi_1^m + \nu \Sigma_{f,2}^m \phi_2^m, \tag{9}$$

and  $v_g$  is the average neutron speed in group g, and  $S_d^m$  is the delayed neutron source.

The exponential transformation is applied to Eq. (6) to improve the accuracy of the temporal discretization in the flux as:

$$\phi_g^m = \tilde{\phi}_g^m e^{\omega_g^m t} , \qquad (10)$$

where  $\omega_g^m$  is frequency for the exponential transformation, which will be defined later in Eq. (12). Then, Eq. (6) becomes:

$$\frac{1}{v_g} \frac{d\tilde{\phi}_g^m}{dt} = \left( R_g^m + S_{fixed,g}^m - \frac{\omega_g^m}{v_g} \phi_g^m \right) e^{-\omega_g^m t} \,. \tag{11}$$

To formulate the transient fixed source equation, the temporal discretization based on the theta method is applied to Eq. (10) as:

$$\begin{pmatrix} \frac{1}{v_g \Theta \Delta t} + \frac{\omega_g^{m,l}}{v_g} \end{pmatrix} \phi_g^{m,l} - R_g^{m,l}$$

$$= \left[ \left( \frac{1}{v_g \Theta \Delta t} - \frac{1 - \Theta}{\Theta} \frac{\omega_g^{m,l}}{v_g} \right) \phi_g^{m,l-1} + \frac{1 - \Theta}{\Theta} R_g^{m,l-1} \right] e^{-\omega_g^{m,l} \Delta t}$$

$$+ S_{fixed,g}^m \left[ 1 + \frac{1 - \Theta}{\Theta} e^{-\omega_g^{m,l} \Delta t} \right],$$

$$(12)$$

where  $\Theta$  is an arbitrary parameter on the interval [0, 1] and usually chosen as 0.5, and the frequency of the exponential transformation is determined as:

$$\omega_g^{m,l} = \frac{\log(\phi_g^{m,l-1}) - \log(\phi_g^{m,l-2})}{\Delta t}.$$
 (13)

## 3. Numerical Results

The configurations of PWR test problem is shown in Fig. 1. The initial core condition is assumed as end of cycle (EOC) and hot zero power (HZP) condition with control rod (CR) bank X being fully inserted, while the other CR banks are located at their insertion limit at HZP condition.

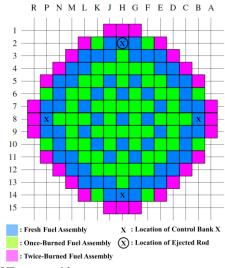


Fig. 1 PWR test problem

For the rod ejection accident, the rod cluster control assembly (RCCA) located at H2 (see Fig. 1) is ejected within 0.1 sec. The trainset analysis is performed by the RAST-K for various initial core power levels (1E-7%, 1E-6%, 1E-5%, and 1E-4%) with and without fixed source.

Figure 2 shows the node-wise fixed source distributions of the PWR reload core at EOC, where the distributions are high in the reloaded fuels. Table I shows both the effective multiplication factors and the boron concentrations of the initially subcritical reactor with fixed source for the various core power levels.

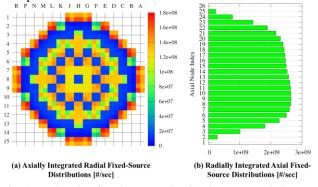


Fig. 2 Node-wise fixed source distributions; (a) axially integrated and (b) radially integrated fixed sources.

Table I Effective multiplication factor and boron concentration of initially subcritical reactor with fixed source for various core power levels

Core Power [%]	Effective	Boron	Critical Boron
	Multiplication	Concentration	Concentration
	Factor*	[ppm]	[ppm]
1E-7	0.86255	2607.57	
1E-6	0.98790	513.23	346.84
1E-5	0.99907	359.46	340.84
1E-4	0.99991	348.05	

\*The effective multiplication factor is estimated by the ratio of neutron production rate to loss rate from the subcritical steady-state calculation with fixed source.

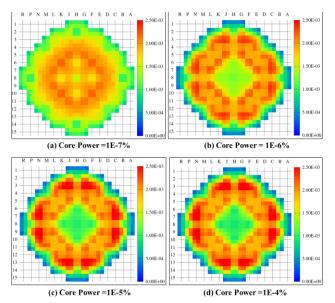


Fig. 3 Axially integrated fission source distributions (normalized by unity) of initially subcritical reactor for various core power levels.

Figure 3 shows that the fission source distributions (FSDs) for the various core power levels. As the core power level decreases, the FSDs in the center region of the core become higher. It should be noted that the ejected rod worth will be affected by both the shape of the FSDs and the boron concentration.

Figures 4 and 5 show that the core power change and the dynamic reactivity of the initially subcritical reactor during the rod ejection transient for the various initial power levels, respectively, while the initially critical reactor is assumed in Figs. 6 and 7. It is observed that when we consider the fixed source, both the initial reactivity and the ejected rod worth are different for various core power levels, while those are almost same when the initially critical reactor is assumed. It is also observed that the critical state assumption leads to the more conservative results (higher maximum core power) for the given initial power level.

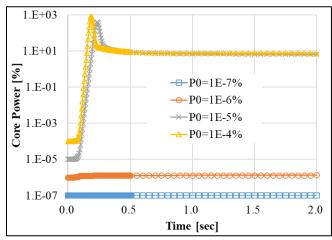


Fig. 4 Core power change during rod ejection transient of initially subcritical reactor with fixed source for various core power levels.

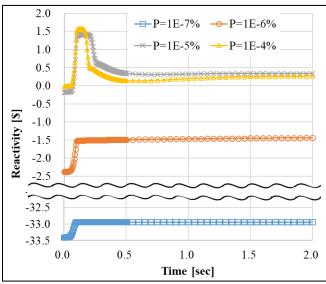


Fig. 5 Dynamic reactivity during rod ejection transient of initially subcritical reactor with fixed source for various core power levels.

## 4. Summary and Conclusions

To improve the accuracy of the low power transient analysis, the fixed source module is incorporated into the RAST-K. The rod ejection accident scenario at HZP was simulated for various the core power levels with and without the fixed source. The numerical results show that when we assume that the initial critical state, the trends of the core power and the reactivity for various core power levels were almost same, while totally different trends were observed by considering the fixed source. In reality, the low power transient will be surely affected by the fixed source and the initial core power level.

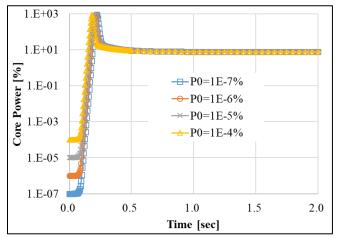


Fig. 6 Core power change during rod ejection transient of initially critical reactor with fixed source for various core power levels.

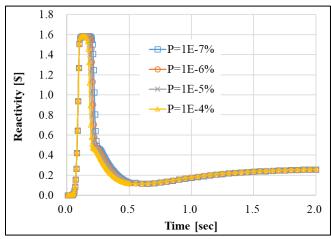


Fig. 7 Dynamic reactivity during rod ejection transient of initially critical reactor with fixed source for various core power levels.

It is also noted that the initial critical state assumption leads to the more conservative results for the rod ejection accident tested in this paper. It is mainly due to the initial negative reactivity being proportional to the fixed source intensity as in Eq. (1). As the fixed source increases, the reactor becomes more deeply subcritical for the same power level. Thus, the fixed source can be considered as an inherent safety feature of the PWR reload core in the low power transient.

The fixed source capability of the RAST-K can be also useful for the low power physics test. For example, the saturation time for the initial critical approach can be provided by the transient analysis of the boron dilution while the ex-core detector response is calculated based on the fixed source distributions.

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