Analytical stress calculation for UMo/Al dispersion fuel

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1. Introduction

UMo/Al dispersion fuel, where UMo alloy fuel particles are dispersed in an Al matrix, has been developed to lower uranium enrichment of fuel used in high-power research reactors in the world. The fueled zone in UMo/Al dispersion fuel is referred to as the fuel meat.

The main obstacle to achieve the successful qualification of the dispersion fuel is the breakaway swelling occurred in the fuel meat, which is caused by the growth of pores and their interconnection as the burnup increases. This breakaway swelling is the results of the degradation of mechanical strength due to the formation of an interaction layer (IL) and pores [1].

It is known that the stress exerted on the pore outside is one of the factors affecting the pore growth. From post-irradiation examinations (PIE), it was observed that large-sized pores at the IL-Al interface were not found at the lateral fuel meat edge where the fission rate and burnup were the highest, but they were found at the region apart from the fuel meat edge where the tensile stress was expected to be predominant by finite element analysis [2]. It implies that not only burnup or fission rate, but also the stress is one of the deciding factors on pore growth.

In this study, an analytic model has been developed to calculate the stress distribution in UMo/Al dispersion fuel. The solution was derived using the pressurized thick-wall sphere theory which is applied for a sphere composed of UMo fuel particle, IL shell, and Al shell. The results calculated by the analytic model were compared to solutions obtained by Abaqus commercial finite element analysis (FEA) package in order to validate the analytic solution scheme.

2. Model derivation

A derivation of the analytic model is aimed at calculating the stress distribution in UMo fuel, IL, and Al matrix regions, as shown in Fig. 1. From the results, it is possible to obtain the radial stress component at each interface (i.e., UMo/IL or IL/Al). This radial stress component can be compatibly used in the model to predict pore growth. In the following subsections, equations for analytic derivation, irradiation models used to calculate strain components, and calculation procedure are presented.

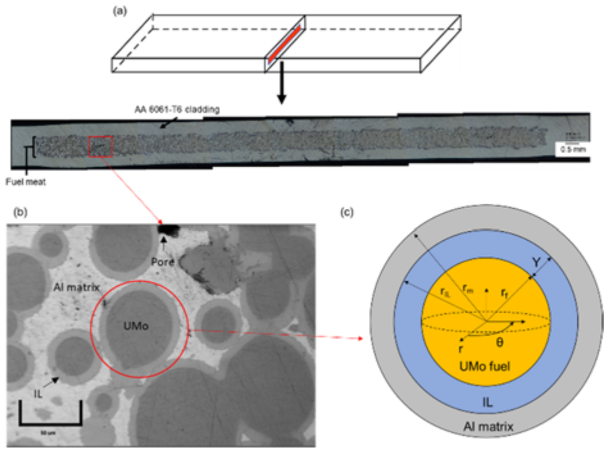


Fig. 1 Illustration of the hypothetical composite sphere model. (a) a schematic of plate-type fuel and an optical image of fuel meat cross-section, (b) an image showing irradiated fuel meat (V6022M from RERTR-4 [3]), and (c) schematics for hypothetical single composite for the domain of analytic derivation.

2.1 General equation

A system of equations is given to calculate stresses for the hypothetical composite sphere which is comprised of UMo fuel particle, IL shell, and Al shell. Three equations, including constitutive law for stress-strain relation, strain-displacement relation, and mechanical equilibrium are summarized as follows:

- Constitutive law:

$$\varepsilon_r^{total} = \frac{1}{E}[\sigma_r - 2\nu\sigma_\theta] + \int_0^t A_c\dot{f}[\sigma_r - 2\mu\sigma_\theta]d\tau + S_r + \eta_r + \beta_r \quad (1)$$

$$\varepsilon_\theta^{total} = \frac{1}{E}[(1-\nu)\sigma_\theta - \nu\sigma_r] + \int_0^t A_c\dot{f}[(1-\mu)\sigma_\theta - \mu\sigma_r]d\tau + S_\theta + \eta_\theta + \beta_\theta \quad (2)$$

- Strain-displacement relation:

$$\varepsilon_r^{total} = \frac{\partial u}{\partial r} \quad (3)$$

$$\varepsilon_\theta^{total} = \frac{u}{r} \quad (4)$$

- Force equilibrium:

$$\frac{\partial \sigma_r}{\partial r} - \frac{2}{r}(\sigma_\theta - \sigma_r) = 0 \quad (5)$$

where ε is the strain, σ is the stress in MPa, E is the Young modulus, ν is the Poisson ratio, t is the irradiation time in sec, A_c is the creep rate constant in

cm^3/MPa, $\dot{f}$ is the fission rate in fission/(cm^3-sec), μ is the Poisson's ratio affected by creep deformation, τ is the arbitrary time during strain increment in sec, S is the strain by fission-induced swelling, η is the strain by thermal expansion, and β is the strain induced by the IL formation. The subscripts r and θ stand for the radial and circumferential direction, respectively.

Combining Eq. (5) with Eqs. (1) and (2) yields the second-order differential equation with respect to the radial stress, σ_r, as follows:

$$2r\frac{\partial\sigma_r}{\partial r}+\frac{r^2}{2}\frac{\partial^2\sigma_r}{\partial r^2}=f(t) \tag{6}$$

where $f(t)$ is the stress component by irradiation-induced swelling, thermal expansion, and chemical volume expansion by IL. The time-dependent stress function $f(t)$ is given as follows:

$$\begin{aligned} f(t)=&\frac{E}{1-\nu}(\Delta S+\Delta\eta+\Delta\beta) \\ &-\frac{E}{1-\nu}\exp[-\int_0^t\lambda(\tau)d\tau]\cdot\int_0^t[(\Delta S+\Delta\eta+\Delta\beta)\cdot\frac{d}{d\tau}[\exp(\int_0^t\lambda(\tau)d\tau)]]d\tau \end{aligned} \tag{7}$$

where $\lambda(t)=\dfrac{EA_c\dot{f}(t)(1-\mu)}{1-\nu}$ and Δ denotes the difference of quantities between radial and circumferential strain (e.g., $\Delta S=S_r-S_\theta$).

The solution of the second-order differential equation is then obtained using integration factor, and stresses in radial and circumferential direction with two unknown coefficients are given as follows:

$$\sigma_r=C_1+\frac{C_2}{r^3}+\frac{2}{3}f(t)\cdot\ln(r) \tag{8}$$

$$\sigma_\theta=C_1-\frac{1}{2}\frac{C_2}{r^3}+\frac{1}{3}f(t)[2\ln(r)+1] \tag{9}$$

Two unknown coefficients are determined using the boundary condition and interfacial stress condition, as shown in Fig. 2. Two interfacial stress components (Π_1 and Π_2) are also unknown, but they can be obtained using the condition of continuity in the radial displacement of the interface at UMo/IL and IL/Al. These continuities are expressed as follows:

$$\begin{aligned} u_f(r_f)&=u_{IL}(r_f) \\ u_{IL}(r_{IL})&=u_{Al}(r_{IL}) \end{aligned} \tag{10}$$

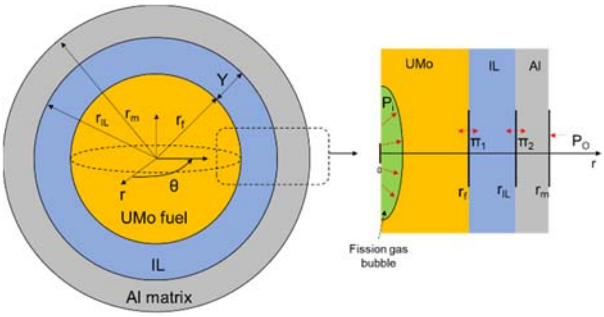


Fig. 2 Boundary and interfacial conditions for the hypothetical sphere-shell composite.

2.2 Material properties

Physical and mechanical properties for fuel meat constituents are summarized in Table 1. Since no data for physical and mechanical properties of IL are available, the property values of UAl_4 are employed.

Material properties of UMo and IL are assumed to be constant during irradiation. The Poisson ratio of Al matrix is set to 0.33, but it increases up to 0.5 due to the irradiation-induced stiffening.

Table 1 Material properties used in the calculation.

	UMo	IL	Al matrix
Physical property			
Density (g/cm^3)	17.1 (U10Mo) 17.3 (U7Mo)	6.10	$2.7/(1+\overline{\alpha}\cdot\Delta T)^3$ *
CTE (10^{-6})§	$7.91+1.21\cdot10^{-2}T$	16.5	$18.1+2.38\cdot10^{-2}T$ $-2.94\cdot10^{-5}T^2$ $+3.03\cdot10^{-8}T^3$
Mechanical property			
Young modulus (GPa)	67.7 (U-10Mo) 50.6 (U-7Mo)	134	$79.9-0.033T$
Poisson ratio (ν)	0.34	0.241	0.33
Poisson ratio under creep (μ)	0.34	0.241	0.5
Creep rate constant (10^{-25} cm^3/MPa)	500	400	50
Ref.		[3]	

* Temperature (T) is in K, and $\overline{\alpha}$ is the mean coefficient of linear thermal expansion.
§ Coefficient of linear thermal expansion.

2.3 Irradiation models

Several models for irradiation performance of UMo and IL are used to calculate strain by fission-induced swelling and volumetric strain by IL formation. Details of each model are available in [4] and references therein.

For the boundary condition, the internal pressure by fission gas bubbles inside UMo fuel particle and hydrostatic stress outside the spherical composite are used. The internal pressure by fission gas bubble is obtained using ideal gas law and known fission gas bubble swelling [5] while the hydrostatic stress is obtained using PRIME [3] which is recently developed for performance analysis of UMo/Al dispersion fuel.

2.3 Calculation scheme

It should be noted that the displacement and stresses are coupled during the calculation. The coefficients derived in the equations for stress and strain contain both known radial coordinates and unknown interfacial stresses. Once the unknown interfacial stresses are determined, the radial coordinates for each interface are updated, and all stresses expressed in Eqs. (8) and (9) for each region should be renewed. Thus, a numerical iteration follows with the tolerance (10^{-4}) to obtain the convergence in radial coordinates as well as stresses in each region. The calculation steps for the displacement and stress are summarized as follows:

1) Calculate strain components by fission-induced swelling, thermal expansion, and chemical volume changes by IL growth.
2) Calculate fission gas bubble pressure (P_i).
3) Repeat the iteration until radial displacement for each interface is converged. For the iteration number index (i), the convergence criterion for the interfacial displacement is given as follows:

$$\frac{\Delta u_j^{(i)}}{u_j^{(i)}} \leq 10^{-4} \tag{11}$$

where $u_j^{(i)}$ is the displacement at (i)-th iteration, $\Delta u_j^{(i)}$ is the displacement increment at the j-interface (e.g., j = UMo/IL, IL/Al). The number of iterations required to satisfy the tolerance of 10^{-4} is typically less than 20.
4) Proceed to the next time step with updated radial coordinates.

3. Results and discussion

The numerical simulation results obtained by Abaqus commercial FEA software were compared to the results from the analytic solution. Abaqus solutions provide the means of verifying the analytic solutions. Irradiation and fabrication data from two different irradiated plates in RERTR-4 and RERTR-9 were used in the calculation as summarized in Table 2. V6022M showed substantial IL growth with consuming Al matrix, while IL growth was suppressed in R3R108 by Si-addition in Al matrix.

For the Abaqus model to compute displacement and stress distribution, the finite element modeling for the single spherical composite, as shown in Fig. 3, was used. IL growth was implemented using time-dependent predefined filed variable during analysis.

Table 2 Fabrication and irradiation data for two irradiated fuel.

Plate ID	V6022M	R3R108
Test campaign	RERTR-4	RERTR-9
Fuel composition	U10Mo/Al	U7Mo/ Al-5Si
U-loading (gU/cm^3)	6	8
UMo fuel particle size (μm)	50	
Initial IL thickness (μm)	0.5	
Irradiation time (EFPD)	257	98
Fission density (10^{21} f/cm^3)	5.5	4.1
Ref.	[3]	

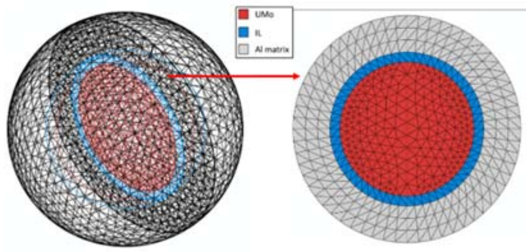
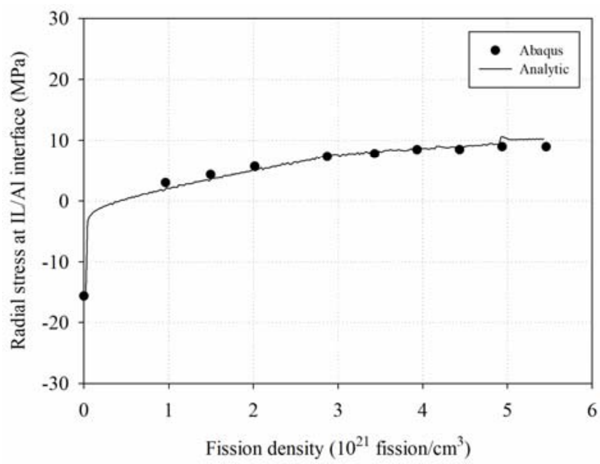
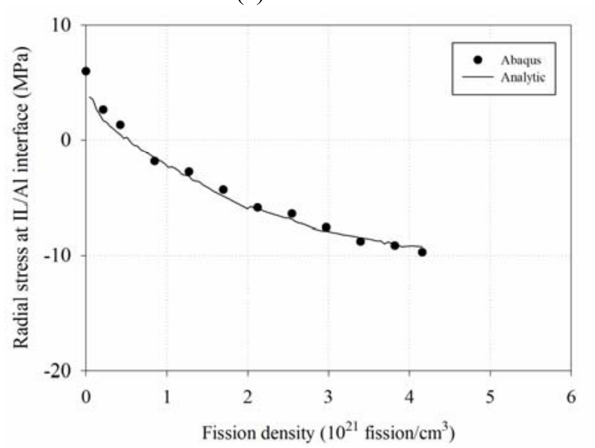


Fig. 3 Finite element used in Abaqus model.

For the benchmarking of results from the analytic model against Abaqus results, the radial stress at IL/Al interface was computed and compared as shown in Fig. 4. This stress was selected since it could be potentially coupled to the pore growth prediction. The results from the analytic model for samples of two plates were in good agreement with the Abaqus simulation results, implying that the interfacial stresses that are unknown during the calculation scheme were reliably calculated.



(a) V6022M



(b) R3R108

Fig. 4 Comparison of local stress at IL/Al interface calculated using the analytic model and Abaqus simulation.

The transition of the interfacial stress was noticeably different for two plates; compressive to tensile for V6022M while tensile to compressive for R3R108. The initial stress was determined by the hydrostatic stress exerted on the composite outside. The transition during irradiation was then determined by the consumption of Al matrix by IL growth. Since Al matrix has a higher coefficient of thermal expansion than UMo and IL, the volumetric expansion UMo/IL composite is restrained as the more Al matrix remains.

4. Conclusions

The analytic model was developed to calculate local stress distribution in UMo/Al dispersion fuel by solving the equations for a hypothetical composite sphere comprised of UMo, IL, and Al matrix. This composite was approximated as a pressurized thick wall-sphere and the stress distribution was analytically derived using boundary and interfacial conditions.

The benchmark of the analytic solution was performed against Abaqus simulation results for two plates. The results by the analytic model for the stress at IL/Al interface were agreeable with the Abaqus results. This agreement indicates that the stress at the IL/Al interface was reliably calculated by the analytic derivation, and it could be coupled to the prediction of stress-dependent pore formation observed at IL/Al interface.

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