Evaluation of Mesh Configuration Uncertainty for Higher-order Numerical Scheme in 1D Nuclear System Analysis Code

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1. Introduction

In the last few years, analyses of thermal-hydraulic behavior in reactor systems have been conducted by using best-estimate codes. In order to provide realistic predictions of nuclear power plant (NPP) systems, the best-estimate codes employ numerous numerical methods and physical models. Therefore, the importance of assessing the best-estimate code capability to predict complex and wide range phenomena in reactor systems becomes evident. However, the existing nuclear system analysis codes such as RELAP5, MARS-KS and TRACE employ 1st order numerical scheme in both space and time discretization. 1st order numerical scheme is very robust and stable. However, it can yield excessive numerical diffusion problems. The existence of strong numerical diffusion in codes with 1st order numerical scheme is well known. Thus, non-conservative results can be predicted for analyzing transients with steep spatial or temporal gradient of physical parameters. Furthermore, the 1st order numerical scheme showed that calculated results depend on mesh size and number [1, 2]. Therefore, 1st order numerical scheme is not desirable for the analysis of accident conditions. So, better predictive capability and more reduced computational cost are required for the advanced nuclear system analysis code.

Therefore, this study presents the simulation results of several cases such as the subcooled flow boiling and Edwards pipe problem using the developed code and MARS-KS code for the concept validation and evaluating the mesh configuration uncertainty. The mesh configuration uncertainty for these cases are evaluated for the higher-order and the 1st order numerical schemes to understand the mesh uncertainty of the spatial distribution and the total number of meshes.

2. Methods

2.1 TWICE code

Fig. 1 shows algorithm of the developing in-house code using the higher-order numerical schemes and the moving mesh method. This code is called TWICE code (Transient Water system analysis code with ICE method) [1, 3-9]. This code basically mimics MARS-KS solver and algorithms [3-7]. The governing equations for two-phase and two-fluid model are implemented, which are identical to MARS-KS code. Additionally, the spatial discretization, the 1st and 2nd order upwind scheme, centered differencing scheme, Lax-Wendroff scheme, moving mesh PDE, and the monitor functions, which are proposed by Huang et al., can be used optionally for solving the governing equations.

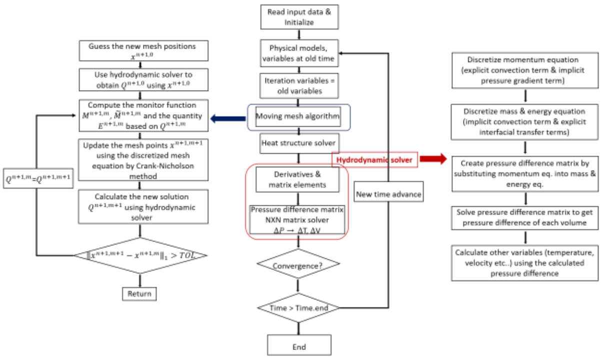

Fig. 1. Fraction of counts lost with voltage and charge

For the application of higher-order numerical scheme on the boundary volume, the Lax-Wendroff scheme is used to maintain the order of accuracy and numerical stability [10]. In 2nd order numerical schemes, numerical dispersion problem can occur. Thus, to remove spurious oscillations of the 2nd order numerical scheme, the Van Albada (VA) flux limiter, which shows good performance in the study of Dean Wang et al. [8], is applied.

In the previous study [11], although the 2nd order upwind scheme and the centered differencing scheme can improve the accuracy, the stability of these schemes is poor compared to the 1st order upwind scheme and Lax-Wendroff scheme as shown in Tables I and II. Table I and II show the comparison of error and order of accuracy depending on the number of mesh and the stability conditions for each numerical schemes, which are the results for the simulation of the single-phase pipe flow with temperature pulse in the previous study [11]. The error for each numerical schemes was calculated by assuming that the error behaves like the first or second order dependence to the number of meshes. The error is given by the following equation:

$$\text{Error} = \left\| \frac{T_{exact} - T_{code}}{T_{exact}} \right\| / N \qquad (1)$$

where ‖ · ‖ is the L_2 norm and T_exact and T_code are the exact solution and the solution calculated by TWICE code, respectively, and N is the mesh number.

For practical applications, the 2nd order upwind scheme and centered differencing scheme are not appropriate. Therefore, in this study, TWICE code was validated and the mesh configuration uncertainty with the 1st order and Lax-Wendroff scheme was preliminarily evaluated under the conditions of several cases.

Table I: Comparison of error and order of accuracy depending on the number of mesh in the previous study [11]

	1^{st} order upwind scheme	2^{nd} order upwind scheme	Lax-Wendroff scheme	Centered differncing scheme
No. mesh = 20	0.01213	0.01174	0.01121	0.01118
No. mesh = 40	0.00745	0.00411	0.00407	0.00395
No. mesh = 80	0.00409	0.00102	0.00106	9.95e-04
Order of accuracy	0.7842	1.7624	1.7013	1.7452

Table II: Stability conditions

	1^{st} order upwind scheme	2^{nd} order upwind scheme	Lax-Wendroff scheme	Centered differncing scheme
Stability condition	$\upsilon \leq 1.0$	$\upsilon \leq 0.2$	$\upsilon \leq 1.0$	Unconditionally unstable

2.2 SUBO Experiment

SUBO (SUbcooled BOiling) test facility was constructed by KAERI (Korea Atomic Energy Research Institute) to investigate the subcooled boiling phenomena by measuring the local bubble parameters [12, 13]. SUBO Test facility consists of pipes and rod type heater. The test section is a vertically arranged annulus with an in-direct heater rod at the channel center. The water is injected from the bottom to the top of the test section. The nodalization of the SUBO facility is shown in Fig. 2. For evaluating the mesh configuration uncertainty, the number of the test section meshes is chosen for 4 to 20. The mesh size of the pipe is randomly determined. Each case for the same mesh number is repeated 400 times with varying mesh sizes. The mesh configuration uncertainty are evaluated by TWICE code with the 1st order upwind scheme and Lax-Wendroff scheme. The mesh configuration uncertainty bands for the exit void fraction are compared.

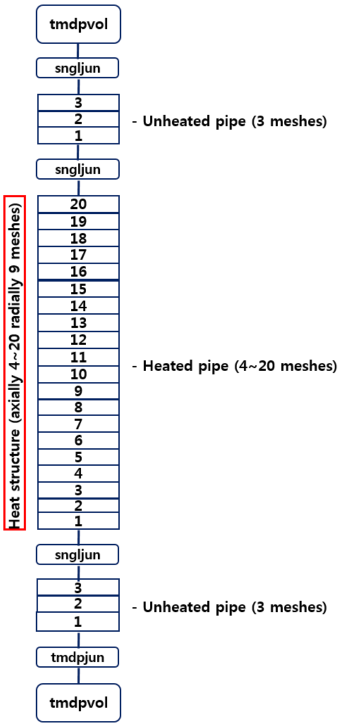

Fig. 2. Nodalization of SUBO experiment

2.3 Edwards Pipe Problem

The Edwards pipe problem is used to verify the blowdown behavior (including flashing) from a pressurized water charged pipe [14]. The nodalization is shown in Fig. 3. In this case, the higher order numerical schemes are directly implemented in MARS-KS code to reduce the calculation time of the mesh configuration uncertainty bands. It makes the revised MARS-KS code possible to model more various cases and conditions. In this case, the Henry-Fauske critical flow model is used as a default model in MARS-KS code. The mesh configuration uncertainty bands for the void fraction at the center of the pipe are compared with MARS-KS code using the 1st order upwind scheme and Lax-Wendroff scheme.

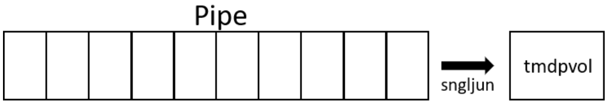

Fig. 3. Nodalization of Edwards pipe problem

3. Results

3.1 SUBO Experiment

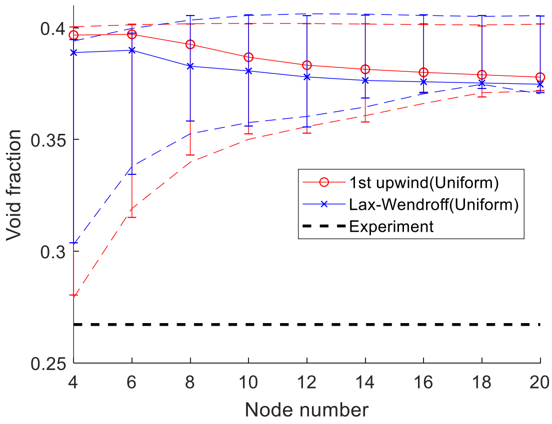

Fig. 4. Mesh configuration uncertainty bands for 1st order upwind scheme and Lax-Wendroff scheme

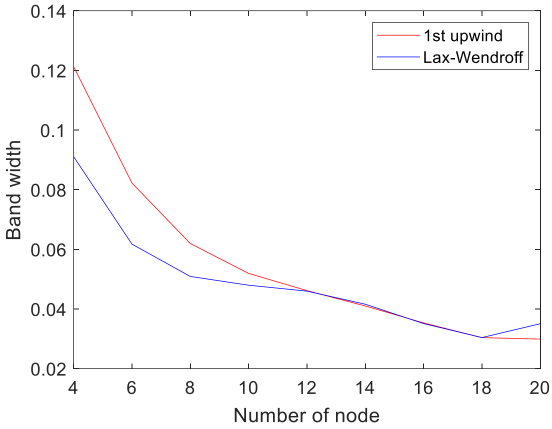

Fig. 5. Comparison for width of mesh configuration uncertainty bands

Mesh configuration uncertainty were evaluated with TWICE code by distributing the mesh size randomly with the 1st order upwind scheme and Lax-Wendroff scheme, respectively [2]. Fig. 4 shows mesh configuration uncertainty band for the exit void fraction of SUBO with TWICE code for the 1st order upwind scheme and Lax-Wendroff scheme. The red and blue bands mean the mesh configuration uncertainty band for 1st order upwind scheme and Lax-Wendroff scheme, respectively. From the figure, the mesh configuration uncertainty band width decreases with the increase of the number of mesh in TWICE code for the 1st order upwind scheme. This trend of TWICE code with Lax-Wendroff scheme is similar to the 1st order upwind scheme but the band width is narrower implying that the node configuration uncertainty is reduced. A comparison for the width of the mesh configuration uncertainty band is shown in Fig. 5. In this figure, the width of Lax-Wendroff scheme is narrower than the 1st order upwind scheme when the number of mesh is less than 12. This is because the order of the spatial and temporal accuracy in the Lax-Wendroff scheme is higher.

3.2 Edwards Pipe Problem

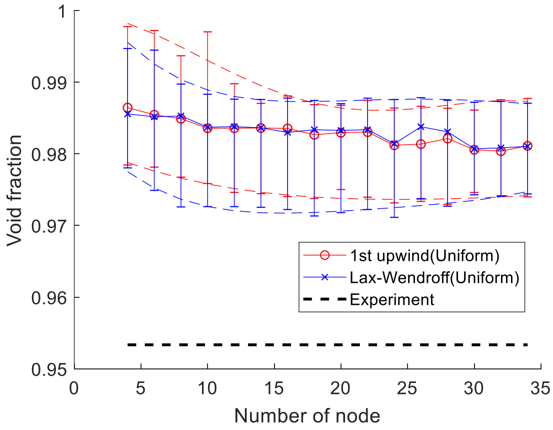

Fig. 6. Mesh configuration uncertainty bands for 1st order upwind scheme and Lax-Wendroff scheme

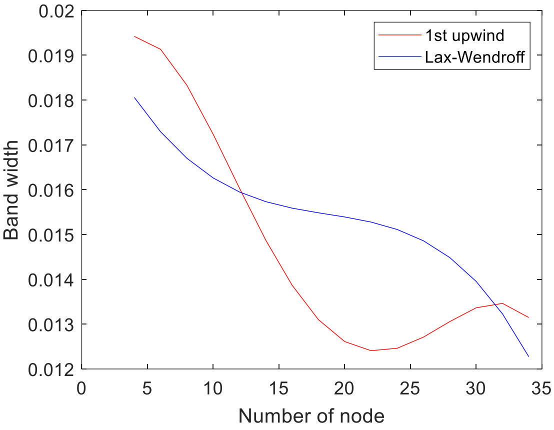

Fig. 7. Comparison for width of mesh configuration uncertainty bands

In this case, the mesh configuration uncertainty are evaluated with the revised MARS-KS code, in which the higher-order numerical schemes are directly implemented. Fig. 6 shows the comparison of the mesh configuration uncertainty bands for 1st order upwind scheme and Lax-Wendroff scheme in MARS-KS code. The red and blue bands are the mesh configuration uncertainty bands for 1st order upwind scheme and Lax-Wendroff scheme, repectively. The mesh configuration uncertainty band width decreases with the increase of the number of mesh in MARS-KS code for the 1st order upwind scheme and Lax-Wendroff scheme. However, the decreasing width is small compared to the previous case. This means that this problem is less sensitive to the number and size of meshes.

Fig. 7 shows the comparison for the width of the mesh configuration uncertainty bands. In this figure, the width of Lax-Wendroff scheme is slightly narrower than the 1st order upwind scheme when the number of mesh is less than 12. However, as the number of mesh increases, the width of the bands is repeatly crossed. In this case, the void fraction is changed from 0 to 1, and the flow regimes change dramatically in a short time. Since the mesh configuration uncertainty is smaller compared to the uncertainty from various correlations determined by the flow regime, the mesh configuration uncertainty is less affected by the number of meshes.

3. Conclusions

In this study, a couple of cases such as SUBO experiment and Edwards pipe problem are modeled with TWICE code and the revised MARS-KS code for the mesh configuration uncertainty tests. The mesh configuration uncertainty bands for each case are compared when using the 1st order upwind scheme and the Lax-Wendroff scheme, respectively. The band width of the mesh uncertainty for the Lax-Wendroff scheme is narrower than the 1st order upwind scheme when the number of mesh is smaller since the Lax-Wendroff scheme is a higher order scheme in both tested problems. However, when the number of mesh is large, the band width of the mesh uncertainty becomes similar, which shows that the effectiveness of the higher order scheme reduces when the number of mesh is increased.

For further works, the mesh configuration uncertainty tests for the moving mesh method will be conducted. The effects of these numerical schemes will be analyzed in a situation such as reflood or dramatic changes in the heat transfer and flow regimes to study the predictive capability change with different numerical schemes.

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