Extension method of the \( \text{SCO}_2 \) turbomachinery performance map for low speed prediction

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1. Introduction

Recently, there is a growing interest in distributed power. This change in demand for distributed power sources has led to the miniaturization of nuclear power plants, which are mainly for large-scale base loads. As a result, the concept of a small modular reactor (SMR) has been proposed.

However, the steam cycle used in the conventional nuclear power plant is not suitable for the miniaturization of the power plant because of large sized components. Therefore, the use of \( \text{SCO}_2 \) (supercritical carbon dioxide) instead of steam as the working fluid attracts much attention. The \( \text{SCO}_2 \) cycle has a higher efficiency than the steam cycle at moderate temperature, and the \( \text{SCO}_2 \) near the critical point behaves in an incompressible fluid, so the work required for compression is drastically reduced [1]. In addition, \( \text{SCO}_2 \) cycle operates at pressures above the critical point, and thus has a high density, thereby reducing the size of the turbomachinery. Therefore, KAIST researchers developed the KAIST-MMR by combining the concepts of SMR and the \( \text{SCO}_2 \) power cycle [2]. The KAIST-MMR can be transported to remote regions by truck or ship, which has the advantage of reducing cost for on-site construction.

![Fig. 1. Concept of KAIST-MMR [2]](image)

The components of KAIST-MMR were designed at the conceptual stage and modeled using modified GAMMA + code for load following and accident situations [3]. However, in the case of start-up modeling, where the rotation speed of the turbomachinery rises from near zero to the nominal state, it is rarely performed due to the unsteady nature at low rotational speeds [4].

Therefore, in order to model the start-up of KAIST-MMR, the performance map of the turbomachinery at low rpm is needed. To solve this problem, in this paper, a new extension method of \( \text{SCO}_2 \) turbomachinery map at low rpm was developed.

2. Performance map extended method

2.1 Target system

In this study, pre-generated off-design performance maps of the compressor and turbine of KAIST-MMR were extended to the low rpm region. The performance map of turbine and compressor of KAIST-MMR and the design points are as follows. The performance map’s range of the RPM is 0.5 to 1.4.

![Fig 2. Compressor pressure ratio and efficiency map of the KAIST-MMR](image)

![Fig 3. Turbine pressure ratio and efficiency map of the KAIST-MMR](image)

| KAIST-MMR Design value (RPM= 19300, \( \dot{m} = 180\text{kg/s} \)) |
|-----------------|-----------------|-----------------|
| **Compressor**  | **T_{in}**       | **T_{out}**      |
| **P_{in}**      | **500\text{°C}**| **60\text{°C}**  |
| **P_{out}**     | **8\text{MPa}**  | **19.93\text{MPa}** |

Table1. Design points of the KAIST-MMR turbomachinery

2.2 Extended mass flow rate and actual work

To obtain the data of the mass flow rate and actual work of the extended performance map, the authors selected the modified similarity law of the pumps proposed by Sexton [4]. He modified the exponent of the dimensionless number of the similarity law of the pump to reflect the compressibility effects of gas.

Eqs. (1) - (4) were used for the modified similarity law of the pumps, where \( \dot{m} \) is mass flow rate, \( N \) is rpm, \( W \) is actual work and subscript ‘lowest, 1 and 2’ mean the lowest and second lowest value.

\[
\frac{\dot{m}_{\text{lowest,1}}}{\dot{m}_{\text{lowest,2}}} = \left(\frac{N_{\text{lowest,1}}}{N_{\text{lowest,2}}}\right)
\]

\[
\frac{W_{\text{lowest,1}}}{W_{\text{lowest,2}}} = \left(\frac{N_{\text{lowest,1}}}{N_{\text{lowest,2}}}\right)^3
\]
The extended mass flow rate and the actual work were obtained using the modified pump's similarity law but mostly applied to air conditions. Therefore, a new turbomachinery performance map extension method that can reflect the characteristics of sCO₂ was required.

To solve this problem, Oh et al. proposed a new turbomachinery map extension method considering isobaric capacity of sCO₂ which changes rapidly during compression and expansion [5]. However, Oh’s method does not consider that the enthalpy and internal energy of real gas are not a function of temperature alone.

Therefore, the new map extended method use the isentropic relations of real gas as follows using isentropic exponents presented by Baltadjiev for calculating the sCO₂ properties [6]. He proposed a new relation considering that the compressibility coefficient of real gas is no longer constant and that enthalpy and internal energy of real gas are not a function of temperature alone.

\[
T_{out} = T_{in} \times \left(1 + \frac{(n_s - 1)}{2\text{Mach}^2}\right)^{\frac{m}{n_t}}
\]

\[
P_{out} = P_{in} \times \left(1 + \frac{(n_s - 1)}{2\text{Mach}^2}\right)^{\frac{m}{n_t}}
\]

\[
n_s = \frac{\gamma}{\beta_{P}P}, m_s = \frac{\gamma - 1}{\beta_{T}}
\]

Where \(T\) is temperature, \(P\) is pressure, \(\gamma\) is ratio of specific heat, \(\beta_P\) is isobaric compressibility, \(\beta_T\) is isothermal compressibility, \(\text{Mach}\) is Mach number and subscript ‘in’ and ‘out’ means inlet and outlet of the turbomachinery.

2.3 Use of trends in turbomachinery maps

The outlet pressure of the sCO₂ turbomachinery can be calculated from REFPROP by inputting the inlet entropy and the ideal outlet enthalpy outlet as shown in Figure 4. The REFPROP is transport properties database developed by National Institute of Standards and Technology (NIST) [7].

![Fig 4. Calculation of extended pressure ratio using REFPROP and properties](Image)

To obtain the ideal outlet enthalpy in Figure 4, the extended efficiency of the turbomachinery is required. Because both pressure ratio and efficiency are unknown, one value must be predicted. In this study, the extended efficiency was estimated to find an extended pressure ratio because extended efficiency set at the low rpm tends to be similar to the original efficiency set at the lowest rpm. As a result of drawing the efficiency of the same position of each rpm in the efficiency map of the conventional MMR turbomachinery, it is confirmed that the following trends are shown.

![Fig 5. Efficiency trend for the same position of each rpm on the turbomachinery efficiency map](Image)

Therefore, the extended efficiency is defined using the lowest rpm efficiency sets and the second lowest rpm efficiency sets as follows where \(\eta\) is efficiency, \(N\) is rpm and subscript ‘ext’ means extended value. However, the extended efficiency and pressure ratios estimated using the map’s trend are not exact values and need to be corrected in the next step.

\[
\eta_{ext} = \eta_{\text{lowest1}} - \left(\eta_{\text{lowest2}} - \eta_{\text{lowest1}}\right) \times \frac{\eta_{\text{lowest2}} - \eta_{\text{lowest1}}}{\eta_{\text{lowest2}} - \eta_{\text{lowest1}}} \quad (8)
\]

2.4 Corrected extended efficiency and pressure ratio

In order to obtain the extended efficiency and pressure ratio, the extended pressure ratio obtained before and the isentropic equation proposed by Baltadjiev and REFPROP were used. The detailed algorithm is shown in Figure 6. To obtain accurate results from the isentropic equation, real-time temperature and pressure of the turbomachinery are needed. However, since this is impossible, the average values were used using the properties of the inlet and outlet turbomachinery.

The average pressure and the average temperature were calculated using the algorithm in Figure 6. As a result, an extended efficiency and pressure ratio were obtained considering the average temperature and pressure.
2.5 Extended turbomachinery map

Each turbomachinery map was drawn using the extended pressure ratio and efficiency for each rpm lower than the original line. The extended performance map’s range of the RPM is 0.1 to 0.4.

\[
\frac{\Delta h_{\text{comp}}}{\eta_{\text{comp}}} = \frac{1}{\eta_{\text{comp}}(\text{Extended})} - \frac{1}{\eta_{\text{comp}}(\text{Original})} \\
\eta_{\text{comp}}(\text{Extended}) = \frac{\text{Extended efficiency}}{\text{Original efficiency}}
\]

\[
\eta_{\text{turb}}(\text{Extended}) = \frac{\text{Extended efficiency}}{\text{Original efficiency}}
\]

\[
\frac{\Delta h_{\text{turb}}}{\eta_{\text{turb}}} = \frac{1}{\eta_{\text{turb}}(\text{Extended})} - \frac{1}{\eta_{\text{turb}}(\text{Original})} \\
\eta_{\text{turb}}(\text{Extended}) = \frac{\text{Extended efficiency}}{\text{Original efficiency}}
\]

\[
\frac{\Delta h_{\text{turb}}}{\eta_{\text{turb}}} = \frac{1}{\eta_{\text{turb}}(\text{Extended})} - \frac{1}{\eta_{\text{turb}}(\text{Original})} \\
\eta_{\text{turb}}(\text{Extended}) = \frac{\text{Extended efficiency}}{\text{Original efficiency}}
\]

2.6 Compare with KAIST-TMD extended performance map

The KAIST TurboMachinery Design (KAIST TMD) code which can estimate the performance of turbomachinery at design and off-design point was developed by the KAIST research team. The KAIST TMD code designs turbomachines using the following mass conservation equations and Euler equations based on the 1-D mean line analysis [8]. Where \( \rho \) is density, \( A \) is flow area, \( V \) is flow velocity, \( U \) is impeller tip speed and subscript ‘o’ and ‘st’ mean stagnation and static condition.

\[
\dot{m} = \rho(h_{st}, P_{st})AV
\]

\[
\Delta h_{\text{turb}} = h_{o2} - h_{o1} = U_2 V_{\theta2} - U_1 V_{\theta1}
\]

However, since the actual work of turbomachinery was not produced through the isentropic process, appropriate losses must be considered for each turbomachinery. In order to consider the exact losses, it is necessary to validated by comparing the turbomachinery experimental performance data and the results calculated by selecting the loss model set suitable for each turbomachinery.

Therefore, KAIST-TMD code was validated using experimental data considering various loss models for radial turbines and compressors [9, 10] which was used in MMR. To ensure the validity of the results of the new map extension method, the turbomachinery performance maps at the low rpm drawn by KAIST-TMD were compared with those of the new extended method presented in this paper.
Second, the extended pressure ratio was obtained by estimating the extended efficiency in consideration of the trend of the existing MMR turbomachinery efficiency map. However, the estimated extended efficiency and pressure ratio are similar to the actual extended value but is not accurate and needs to be corrected.

Third, the corrected extended efficiency and pressure ratio were calculated using the Baktadjievs's isentropic relation, which takes into account the characteristics of supercritical carbon dioxide, and the properties obtained before. The average property values of the turbomachine inlet and outlet are used for accurate calculation.

Finally, to validate the extended map, the results of the extended map developed in this study with the extended map results of KAIST-TMD validated with experimental data was compared. As a result, it was confirmed that there is almost no difference between the extended map of KAIST-TMD and the new extended map, which means that the result of the new extension method is reliable.

In this study, the \( \text{sCO}_2 \) turbomachinery map extension method at low rpm is described. However, \( \text{sCO}_2 \) compressors are often used near the critical point, unlike the design point of the target system in this study, MMR. Therefore, the extended method of \( \text{sCO}_2 \) compressor map at low rpm and near the critical point will be further studied.

### REFERENCES


