1. Introduction

Since the neutron transport theories theoretically guarantee the best estimation for reactor core analyses, they have always been one of the correct answers in reactor physics field. As the computing power has increased dramatically in last few decades, the neutron transport theories are utilized in more and more studies nowadays. However, due to the extreme computational costs required for the transport calculations, they can hardly be applied for a whole core analysis, and not to mention the whole core transient analysis, which requires much more computational cost than steady-state analysis.

In these circumstances, the HCMFD (Hybrid Coarse-Mesh Finite Difference) algorithm for an efficient 3D whole core pin-by-pin diffusion analysis [1] and the GPS (GET Plus SPH) method for the cross-section corrections in pin-wise diffusion analysis [2] have been suggested. Based on these methods, it is expected that one can get a whole-core pin-wise solution with sufficient fidelity in a reasonably short computing time.

Previously, the application of the HCMFD algorithm had been limited to steady-state analyses. In this paper, the potential performances, in the aspect of computing time, of the HCMFD algorithm for the whole core transient analysis are demonstrated with control rod movement scenario.

2. Methodology

As introduced in detail in the reference [1], the one-node and two-node CMFD methods are nonlinearly coupled for the local-global iteration in the HCMFD algorithm. The global eigenvalue problem is solved by the one-node CMFD method, which enables an efficient parallel computing by domain decomposition. The local domains, fuel assemblies composed of pin-level fine meshes in a PWR analysis, are solved in parallel by the conventional two-node CMFD method based on pin-level nodal analysis, the nodal expansion method in this work.

A transient diffusion analysis starts from an initial steady state solution prepared prior to the transient analysis, and it is based on the following time-dependent neutron diffusion equation:

\[
\frac{d\phi}{dt} = \nabla \cdot (D \nabla \phi) - \Sigma_{f, g} \phi + \sum_{g' \neq g} \Sigma_{e, g' \rightarrow g} \phi_{g'}^e + \sum_{g' \neq g} \Sigma_{g' \rightarrow g} \phi_{g'}^s + \lambda_d C_d, \quad d = 1, \ldots, \text{max } d, \tag{1}
\]

where

\[k_0 = \text{multiplication factor at the initial steady state,}\]

\[d = \text{delayed neutron precursor family index,}\]

\[C_d = \text{delayed neutron precursor density.}\]

With the implicit Euler method applied, Eq. (3) can be derived from Eq. (1) for a current time step \( t \):

\[
L\phi_d(t) + R\phi_d(t) = \frac{X_{pm}}{k_0} (1 - \beta) F\phi_d(t) + S\phi_d(t) + \sum_{\text{max}} S_j(t) + \frac{\phi_d(t_{-1})}{\nu_\lambda \Delta t}, \tag{3}
\]

where

\[
L\phi_d(t) = \nabla \cdot (\nabla \phi_d),
\]

\[
R\phi_d(t) = \left( \sum_{g \neq g'} \phi_{g'} + \frac{1}{\nu_\lambda \Delta t} \right) \phi_d(t),
\]

\[
S\phi_d(t) = \sum_{g' \neq g} \sum_{g' \neq g} \phi_{g'} \phi_{g'}^s(t),
\]

\[
F\phi_d(t) = \sum_{g' \neq g} \nu_{g'} \Sigma_{g' \rightarrow g} \phi_{g'}^s(t),
\]

\[
S_j(t) = \lambda_d \phi_d(t).
\]

The current step delayed neutron precursor density \( C_d(t) \) in Eq. (3) is expressed as

\[
C_d(t) = C_d(t_{-1}) e^{-\lambda_d t} + \int_{t_{-1}}^{t} \frac{X_{pm} \beta_{g'}^e}{k_0} F\phi_d(t') e^{-\lambda_d (t-t')} dt', \tag{4}
\]

and the integration in Eq. (4) is linearly approximated in time as

\[
F\phi_d(t) = F\phi_d(t_{-1}) \frac{(t-t_{-1})}{\Delta t} + F\phi_d(t_{-1}) \frac{(t_{-1} - t_{-2})}{\Delta t}. \tag{5}
\]

By applying Eq. (5) into the Eq. (4), the current step delayed neutron precursor density is then expressed as

\[
C_d(t) = C_d(t_{-1}) f_{d} + \frac{X_{pm} \beta_{g'}^e}{k_0} F\phi_d(t_{-1}) f_{d} + \frac{X_{pm} \beta_{g'}^e}{k_0} F\phi_d(t_{-1}) f_{d}, \tag{6}
\]

where
Substituting Eq. (6) into Eq. (3), following fission source iteration form for the transient fixed-source problem (TFSP) is derived as

\[
L \phi_g^{(i)}(t) + R \phi_g^{(i)}(t) = \frac{Z_{ef}}{k_0} (1 - \beta) F \phi_g^{(i)}(t) + S \phi_g^{(i)}(t) \\
+ \sum_{i \neq g} S_i(t_{i-}) f_{l,i} \\
+ \sum_{i \neq g} \beta_i X_{i,g} \beta_i F \phi_g^{(i+1)}(t_i)\\n+ \sum_{i \neq g} \beta_i X_{i,g} \beta_i F \phi_g^{(i+1)}(t_i) + \phi_g^{(i-1)}(t) + \phi_g^{(i-1)}(t) e^{\gamma_i \Delta t},
\]

where \( I \) is the fission source iteration index.

In this work, the exponential transformation, Eq. (8), is also applied to improve the accuracy of the implicit Euler method. The time derivative of the neutron flux becomes as Eq. (9).

\[
\frac{d \phi_g(t)}{dt} = \psi_g(t) e^{\gamma_i \Delta t}
\]

Finally, the TFSP equation corresponding to the exponential transformation is obtained as

\[
L \phi_g^{(i+1)}(t) + R \phi_g^{(i+1)}(t) = \frac{Z_{ef}}{k_0} (1 - \beta) F \phi_g^{(i+1)}(t) + S \phi_g^{(i+1)}(t) \\
+ \sum_{i \neq g} S_i(t_{i-}) f_{l,i} \\
+ \sum_{i \neq g} \beta_i X_{i,g} \beta_i F \phi_g^{(i+1)}(t_i)\\n+ \sum_{i \neq g} \beta_i X_{i,g} \beta_i F \phi_g^{(i+1)}(t_i) + \phi_g^{(i-1)}(t) + \phi_g^{(i-1)}(t) e^{\gamma_i \Delta t},
\]

where \( I \) and \( i \) are the node index in global and local domains respectively.

The most notable advantage of HCMFD algorithm for a transient analysis is that the fission source iteration is mainly performed in global sense, assembly-wisely, while the local TFSPs and pin-level nodal kernels do not need to be solved frequently. Since the local detailed quantities are already converged once in the steady-state analysis, repeated updates of the local solutions are not very influential to the accuracy of the transient analysis. This will be discussed again with the numerical results in the following sections.

The overall flowchart of transient HCMFD algorithm is shown in Fig 1.
Once the local problems are updated, the local TFSPs are solved twice, initially once and once more with the updated nodal kernel information: local correction factors and partial currents on the local boundary. It is for a better convergence between the local problems and it actually reduces possible numerical instability during the nonlinear iterations between local and global problems.

3. Numerical Results

In this work, a 3-D OPR-1000 core was treated to figure out a rough computing time for a whole core transient analysis achieved by the HCMFD algorithm. Detailed core geometry is described in Table I, and loading patterns, assembly types are shown in Figs. 2. and 3. The pin-wise two-group cross-sections of each assembly type are generated using a neutron transport code DeCART-2D [3], and a 6-group typical set of delayed neutron constants for PWR are used.

A test transient problem was solved, where the perturbation is caused by symmetric control rod movements in 8 control element assemblies (CEAs), D06, D10, F04, F12, K04, K12, M06, and M10. In the test problem, all initially inserted control rods move together as shown in Fig 4. They are initially inserted into the active core region by 100.263 cm (5 axial meshes), withdrawn by 3 axial meshes in 3 seconds, then fully inserted in 1 seconds. The time-dependent position of control rods are discretized by 0.2 seconds to avoid an error caused by the time step size difference for the transient calculation.

Table I: Core geometry description

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Thermal power</td>
<td>2815.0MWth</td>
<td></td>
</tr>
<tr>
<td>Radial configuration</td>
<td>No. of assemblies</td>
<td>177</td>
</tr>
<tr>
<td></td>
<td>No. of reflector nodes</td>
<td>64</td>
</tr>
<tr>
<td>Axial configuration</td>
<td>Top reflector</td>
<td>20.95cm *1</td>
</tr>
<tr>
<td></td>
<td>Active core</td>
<td>20.0526cm *19</td>
</tr>
<tr>
<td></td>
<td>Bottom reflector</td>
<td>20.95cm *1</td>
</tr>
<tr>
<td>Assembly geometry</td>
<td>20.56cm*20.56cm (16 by 16 pins)</td>
<td></td>
</tr>
<tr>
<td>Total No. of coarse meshes (No. of local problems)</td>
<td>5,061</td>
<td></td>
</tr>
<tr>
<td>Total No. of fine meshes</td>
<td>1,295,616</td>
<td></td>
</tr>
</tbody>
</table>

The fission source convergence criterion was 10^-7, and the local and global problems were both solved by a BiCGstab (Biconjugate gradient stabilized) method [4]. All calculations were performed on Intel Xeon E5-2697 v3 2.60 GHz CPU with 40 physical cores. Parallel computing was performed using the OpenMP parallel algorithm [5]. All numerical results of this work are obtained by utilizing 40 cores.
First, the same test problem was solved with fixed time step size 0.2 seconds, and with varying the local update cycle, where it means the number of fission source iterations of global TFSP between consecutive local analyses. A large update cycle indicates a less number of local analyses in each time step. Table II shows the computing time, the average number of global fission source iteration per time step, the number of local updates per time step, and maximum power error. It is notable that the average number of fission source iteration is not very dependent on the update cycle if the local updates are sufficiently frequent. At the same time, the computing time is highly dependent on the number of local updates. It indicates that the local calculations take major part of the computational loads, so the overall computing time can be dramatically reduced if the number of local updates can be minimized, as in the cases with update cycle larger than 50.

Table II: Numerical results depending on the cycle

<table>
<thead>
<tr>
<th>Update cycle</th>
<th>Computing time (s)</th>
<th>Average No. of iterations</th>
<th>Average No. of local update</th>
<th>Max. total power error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3113.498</td>
<td>1084.1</td>
<td>1084.1</td>
<td>Ref.</td>
</tr>
<tr>
<td>2</td>
<td>1580.029</td>
<td>1084.1</td>
<td>542.3</td>
<td>0.000</td>
</tr>
<tr>
<td>5</td>
<td>665.329</td>
<td>1084.3</td>
<td>217.2</td>
<td>0.000</td>
</tr>
<tr>
<td>10</td>
<td>340.899</td>
<td>1084.6</td>
<td>108.8</td>
<td>0.000</td>
</tr>
<tr>
<td>20</td>
<td>189.156</td>
<td>1085.7</td>
<td>54.7</td>
<td>0.000</td>
</tr>
<tr>
<td>50</td>
<td>100.587</td>
<td>1110.0</td>
<td>22.7</td>
<td>0.000</td>
</tr>
<tr>
<td>100</td>
<td>72.499</td>
<td>1182.1</td>
<td>12.1</td>
<td>0.001</td>
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<tr>
<td>200</td>
<td>65.567</td>
<td>1519.7</td>
<td>7.8</td>
<td>0.007</td>
</tr>
<tr>
<td>300</td>
<td>71.320</td>
<td>1940.1</td>
<td>6.6</td>
<td>0.009</td>
</tr>
<tr>
<td>400</td>
<td>73.548</td>
<td>2163.2</td>
<td>5.5</td>
<td>0.009</td>
</tr>
<tr>
<td>500</td>
<td>75.792</td>
<td>2331.5</td>
<td>4.8</td>
<td>0.019</td>
</tr>
<tr>
<td>9999</td>
<td>33.155</td>
<td>1063.5</td>
<td>1.0</td>
<td>0.608</td>
</tr>
</tbody>
</table>

In the aspect of the accuracy comparing to the case with update cycle 1, only a limited level of power error was observed even with the update cycle 500, while a non-negligible error exists with only 1 local updates per time step. This trend can be seen also in the Fig. 5 that the power variations are very similar when there are more than one local updates. It shows that how frequently the local problems are updated is not a major factor that affects the accuracy of transient analysis, if they are updated more than one time. Considering the accuracy and computing time, the update cycle for the following analysis was chosen to be 100.

Table III shows the computing time with various time step sizes and with update cycle 100. Since the required number of fission source iterations are generally proportional to the power variation per time step, the average number of local updates are smaller with smaller time step size when the update cycle is fixed. The computing time for the 4-second transient is only 508 seconds with a sufficiently short time step, 0.01s.

4. Conclusions

It was demonstrated that a reasonably short computing time can be achieved by the HCMFD algorithm for a 3-D whole-core pin-by-pin transient analysis of a PWR core even with a sufficiently small time step size, ~500 seconds with 0.01s time step.

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REFERENCES