A Comparison of Cooling Schedule in Spent Nuclear Fuel Grouping Optimization by Simulated Annealing for Dry Cask Loading

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1. Introduction

In June 2017, Korea's unit 1 of the Kori nuclear power plant (NPP) has been permanently shut down, and there is a growing interest in transportation/storage (T/S) of spent nuclear fuel (SNF) as well as safely managing cask for it. In addition, there are thousands of SNF in the spent fuel storage pool (SFP), and safety limits such as criticality, surface dose and temperature, and fuel cladding peak temperature should be considered when they are loaded in the T/S casks.

The thousands of SNF are stored in each SFP. Thus, if the SNFs of each SFP are loaded into the T/S casks, tens to hundreds of casks are needed. if the SNF with enough cooling time is loaded into the cask, the safety limits of the cask are mostly satisfied. However, if the SNFs are loaded one by one, some casks may be loaded with a higher safety margin, while others may have very low safety margins. Therefore, in order to have similar high safety margins as a whole, it is necessary to optimize the fuel groups and the loading pattern to be loaded into each cask.

However, optimizing the selection of the SNF and LP for each cask is too big a problem, so that the time and computation consumption can be large. Therefore, the whole problem is divided into two steps in order to solve the whole problem efficiently. The first step is to optimize the grouping of the SNF by selecting the SNF for each cask, and the second step is to optimize the LP to load the selected SNF to meet the safety limits.

In this study, only the reactivity was considered. And in the two-step, the LP for each cask is fixed as a model that is empirically known to have relatively low criticality, and the model is that the fuel assembly (FA) with high burnup is placed on the inside, and the FA with low burnup is placed on the outside. The grouping method uses the simulated annealing (SA) algorithm, which is often used for optimization problems [1], and the efficiency tests were performed according to cooling schedules that affect the optimization efficiency of SA.

2. Simulated Annealing

The SA is a general-purpose combinatorial optimization algorithm that mimics the thermal annealing phenomenon. Due to flexible frame-work of this algorithm and advantage of achieving near optimal solution, SA algorithm has been applied to various combinatorial optimization problems. SA creates a new solution by applying a perturbation to the current solution and probabilistically evaluates whether to move from the current solution to the new solution.

The SA algorithm has been popularly adopted for the optimum FA LP search calculations in initial/reload core design of light water reactors. However, it has a major drawback of long computing time because it requires neutronics evaluation of tens of thousands of trial LPs during the optimization.

In Fig. 1., the i and j are variables of state, and X is the solution space of the problem. T_k is the temperature at kth repeated phase, Equilibrium(T_k) is a number of iterations until equilibrium is reached. Function Acceptance() determines whether or not to exchange.

```
Procedure Simulated Annealing
    Initialize auxiliary variables
    Repeat
        for i = 1 to Equilibrium( Tk )
            Perturbation( j from i ∈ X );
            Acceptance( i, j );
        end for
        k = k + 1;
        update Tk
    Until stop-criterion
end Simulated annealing
```

Fig. 1. Pseudocode of Simulated annealing algorithm.

This function can be considered in a variety of ways, in this study, it is considered using the Metropolis algorithm. The exchange probability is given by Eq. (1) in this function.

$$Acceptance(i,j)=\begin{cases}1 & E(j)\le E(i)\\ \exp\left(-\dfrac{E(j)-E(i)}{T}\right) & otherwise\end{cases} \quad (1)$$

In Eq. (1), E() is the evaluation function, and T is the control parameter called temperature. This probability depends on a control variable called temperature, and the higher the temperature, the more likely it is to move to a new solution, even if that is not good. With this stochastic valuation methodology, SA has proven to find the best solution without falling into a localization solution.

2.1 Cooling Schedule

One of the most important features of SA is choosing a cooling schedule, and there have been many attempts to derive or suggest a good schedule.

The Eq. (2) is the proposed geometric schedule when introducing SA in the journal [1].

$$T_k = \alpha^k \times T_0, 0 < \alpha < 1 \quad (2)$$

In the Eq. (2), k, 'time' is the step count, and T_0 is the initial temperature with k = 0.

Particularly theoretically important is the logarithmic cooling scheme introduced by S. Geman and D. Geman [2],

$$T_k = \frac{c}{\log(k+d)} \quad (3)$$

Where d is usually set equal to one. The only existing theorem proves that if c is greater than or equal to the largest energy barrier in the problem, this schedule will lead the system to a global minimum state at the limit of infinite time. However, the schedule is completely impractical because the temperature gradually decreases.

Another schedule proposed by H. Szu and R. Hartley [3] is in Eq. (4),

$$T_k = \frac{T_0}{(k+1)} \quad (4)$$

This schedule is faster than logarithmic cooling schedule and then given a general D-dimensional Cauchy probability for generating the state.

In addition, the idea of reducing the length of the inner loop if the difference in equilibrium probabilities at successive temperatures is small has led to a cooling schedule with Eq. (5) suggested by E. H. L. Aarts [4].

$$T_{k+1} = T_k \times \left[1 + \ln(1+\delta) \times \frac{T_k}{3 \times \sigma(T_k)}\right]^{-1} \quad (5)$$

Where δ is a tunable parameter.

And, a cooling schedule provided by M. Lundy and A. Mees [5] is shown to progress according to the function in Eq. (6),

$$T_{k+1} = \frac{T_0}{(1 + k \times \beta \times T_0)} \quad (6)$$

3. Test Problem and Results

In order to analyze the efficiency of the cooling schedule, OPR-1000 type NPP and TN-32 T/S cask were chosen for this study. The reactor core of the NPP consists of a 177 (16x16) FA. In the study, it is considered 624 FAs with a discharge burnup and cooling time of 18-28 years of cycle 1 to 6 of the NPP. And the TN-32 cask is available to load 32 FAs, and its cross-sectional view is shown in Fig. 1 [6].

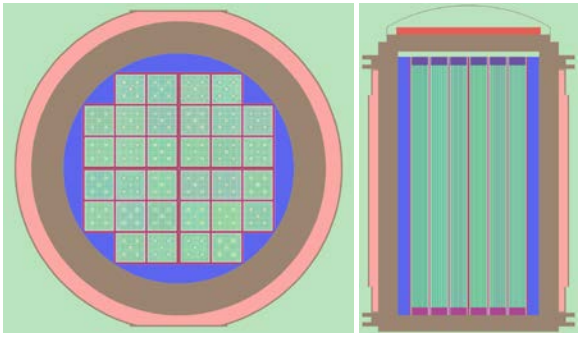

Fig. 1. The horizontal and vertical cross-sectional view of TN-32 cask.

The global problem is the case of loading the 19 casks with the 624 FAs. According to the two-step strategy described in the introduction section, the loading pattern of each cask is a model known to empirically have a small reactivity. The grouping optimization for distributing FAs to each cask is performed using the SA algorithm using the five cooling schedules to find out which schedule is more efficient. In addition, in order to limit the optimization time, the number of step count (k) and temperature are limited to 1000 and 10^{-8}. The coefficients of Eq. (2) – (6) are shown in Table I.

Table I: Cooling Schedule Description

Cooling schedule	Equation
Geometric 1	Eq. (2), α = 0.95
Geometric 2	Eq. (2), α = 0.90
Geometric 3	Eq. (2), α = 0.85
Boltzmann	Eq. (3), c = T_0
Cauchy	Eq. (4)
Aarts	Eq. (5), δ = 1
Lundy	Eq. (6), β = 1

The evaluation function of the SA algorithm is also called an objective function ($f_{Obj.}$), and this function in the study is the deviation of the mean and standard deviation of each factor as shown in Eq. (7) so that the mean and variance of factors of all groups are similar in order to make the characteristics of groups similar, and the factors are burnup, activity, decay heat, ϒ-power, neutron source, and ϒ source of FAs (7 factors of FA). In addition, the goal of the SA algorithm in this study is to minimize the value of the objective function.

$$f_{Obj.} = \sqrt{\frac{\sum_{f}^{N_{factor}} \sum_{g}^{N_{group}} \left[\left(m_g - \bar{m}\right)^2 + \left(\sigma_g - \bar{\sigma}\right)^2 \right]_f}{N_{factor} \times N_{group}}} \quad (7)$$

In Eq. (7), N_{factor}, N_{group}, m, and σ are a number of factors and groups and mean and standard deviation of each group and factor. In addition, before applying the value of each factor to the $f_{Obj.}$, it is preprocessed by min-max normalization as in Eq. (8) to reduce the effect of the range of it.

$$X_i = \frac{(x_i - x_{\min}) \times 100}{(x_{\max} - x_{\min})} \quad (8)$$

Table II shows the results of optimization of grouping using the SA algorithm with the cooling schedules shown in Table I. The results in Table II are obtained by averaging the results obtained by repeating the optimization attempt 10 times. Fig. 2-3 shows the temperature and value of $f_{Obj.}$ behavior with step (k) in grouping optimization.

In Table II and Fig. 2-3, all of the geometric schedules completed the optimization within an average of 5 minutes, and the means of the value of $f_{Obj.}$ were also derived with a mean of less than 0.2. Other schedules met the algorithm termination condition, step < 1000, so the algorithm ended before finding the optimal value and on average, it took about 15 minutes. the results show that geometric schedules cool faster than the other schedules. So, when the SA algorithm is ended up with two limits of the stop of the algorithm, the value of $f_{Obj.}$ of geometric schedules are lower than that of the other schedules. In addition, if the cooling rate (α) is too small in the geometric schedule, the value of $f_{Obj.}$ becomes larger because it is cooled too quickly and does not exit from the local optimal point. Therefore, it is effective to use the geometric schedule for the cooling schedule of the SA algorithm in terms of optimization and time. Considering the level of optimization among them, Geometric 1 is better than the other schedules with the cooling schedule.

Table II: Comparison Results of Each Schedule

Cooling schedule	f_{Obj}^{*}	Time [min]	Step count
Geometric 1	0.137661	4.80	338
Geometric 2	0.173685	3.44	166
Geometric 3	0.191529	2.36	112
Boltzmann	3.175624	15.59	1000
Cauchy	0.224216	16.02	1000
Aarts	0.224498	16.31	1000
Lundy	0.230182	14.14	1000

*$f_{Obj.}$: The objective function in Eq. (7) of the final step.

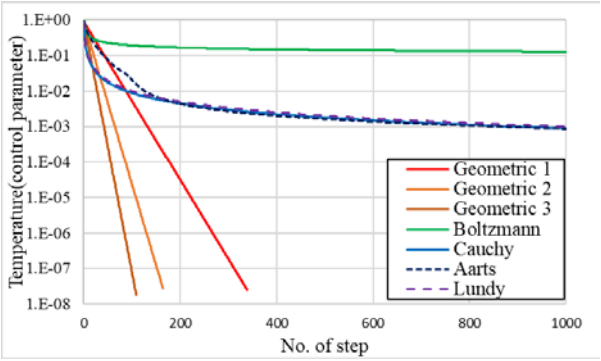

Fig. 2. Temperature (control parameter) behavior with the step in grouping optimization.

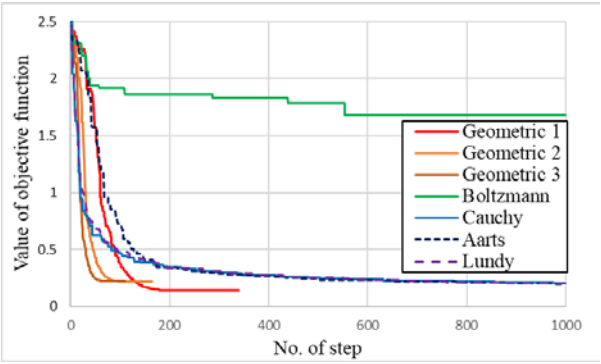

Fig. 3. Value of objective function behavior with the step in grouping optimization.

After optimization of grouping, the reactivity of cask for each group is calculated using MCS code, a Monte Carlo code developed by Computational reactor physics and experiment (CORE) laboratory at Ulsan national institute of science and technology (UNIST) [7]. By analyzing the distribution and variation of the reactivity of cask of each group in case of each schedule, it is found that the effective multiplication factor of each cask was less than 0.95, which is the reactivity limit of the T/S cask, and that the reactivity variation of all the casks can be made small.

Table III and Fig. 4 shows the variation and distribution of the effective multiplication factor of the cask for each group by selecting any one of the 10 results of the optimization of grouping in each schedule. In Fig. 4, the effective multiplication factor of all casks in each schedule is less than 0.95, and in Table III, the variation of them in Geometric 2 schedule is smallest. This result is different from the result that the value of $f_{Obj.}$ in the Geometric 1 schedule is the smallest in Table II. This is thought to be the effect of the $f_{Obj.}$ to uniformize the reactivity, surface dose, decay heat, and so on of all casks by similarly arranging the 7 factors of the groups.

However, in view of time and optimization in addition to the reactivity, it seems more efficient to optimize the grouping using the SA algorithm applying the Geometric 2 schedule.

Table III: Comparison Deviation of Effective Multiplication factors of Each Group in Each Schedule

Cooling schedule	*Dev. of k_{eff}*
Geometric 1	0.00907
Geometric 2	0.00852
Geometric 3	0.00961
Boltzmann	0.01448
Cauchy	0.01013
Aarts	0.01138
Lundy	0.01121

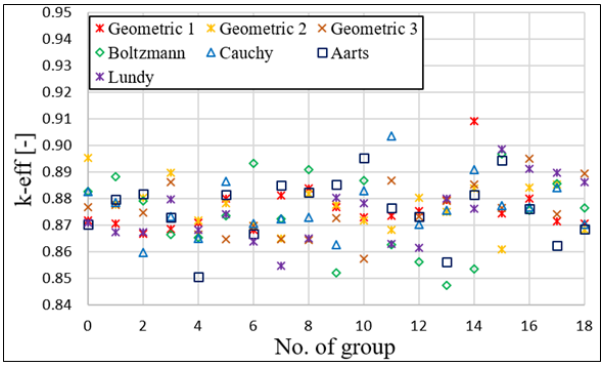

Fig. 4. Distributions of effective multiplication factors of each group in each cooling schedule.

4. Conclusion

Optimizing the fuel grouping and LP for each cask to meet the safety limits for thousands of SNF in hundreds of T/S casks requires a great deal of time and computation. Therefore, this whole problem is divided into a first step, grouping optimization and a second step, LP optimization. This 2step strategy effectively solves the global problem.

In this paper, only the reactivity of the cask is considered, and the LP is fixed as a model known to have a relatively low reactivity in the empirical model. The problem of grouping optimization is solved by using the SA algorithm which is mainly used for optimization problem-solving.

There are many factors that increase the efficiency of the SA algorithm. In this study, the efficiency tests are performed according to cooling schedules that affect the efficiency of SA algorithm, and geometric (cooling rate; 0.95, 0.90, and 0.85), Boltzmann, Cauchy, Aarts, and Lundy schedules is suggested as the cooling schedule of SA algorithm for the test.

As a test problem, 624 SNF with cooling time between 18-28 years and discharge burnup of 1 to 6 cycles of the OPR-1000 type NPP are loaded in 19 T/S casks. The cask is available to load 32 FAs. Here, the $f_{Obj.}$ of the SA algorithm is the deviation of the mean and standard deviation of 7 factors (burnup, activity, decay heat, ɤ-power, neutron source, and ɤ source of FAs), and optimization of grouping minimizes the value of this f_{Obj}.

As a result of the test, the geometric schedules are optimized within 5 minutes and the value of $f_{Obj.}$ in this case, is less than 0.2. In terms of time and optimization, this has shown that geometric schedules are more efficient than other schedules. The reactivity distributions of the casks in each schedule show that in all schedules, the reactivity of the T/S casks did not exceed the safety limit of 0.95, and Geometric 2 schedules have small deviations in the reactivity deviations of each schedule.

The difference between the result of the value of $f_{Obj.}$ and reactivity deviation is due to the fact that the 7 globally equalized factors considered in the $f_{Obj.}$ have different effects on the reactivity. Considering both time, optimization, and reactivity, Geometric 2 schedules seem to be the most effective.

5. Future Work

In this study, the efficiency test is performed considering only the reactivity. However, the safety assessment factors for T/S casks include surface dose and temperature, and fuel cladding peak temperature as well as reactivity. Therefore, the efficiency test of each schedule will be performed when considering other safety evaluation factors through the f_{Obj}.

In addition, $f_{Obj.}$ will be re-selected through the sensitivity test for each safety evaluation factor of the factors considered in the f_{Obj}.

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