Leakage Feedback Method for Pin-by-Pin Core Calculation

Hyunsik Hong and Han Gyu Joo*
Department of Nuclear Engineering, Seoul National University, 1 Gwanak-gu, Seoul, Korea, 08826
**Corresponding author: joohan@snu.ac.kr*

1. Introduction

The conventional two-step core analysis systems including the nTRACER/RENUS [1] of SNU have been widely used for the design analyses of commercial reactor cores owing to its remarkably low computing costs. However, it is difficult to incorporate the actual core environment in the two-step because it employs the assembly-homogenized few-group constants (GCs) generated by a lattice transport for a single assembly with reflective boundary condition (SA). Although the B_1 leakage correction method had been adopted to mitigate the problem of missing the leakage effect, there still remains considerable errors in core reactivity and power distributions [2]. In this regard, leakage feedback method (LFM) [3] for on-the-fly GC correction was developed. The LFM iteratively updates the GCs using the leakage parameters and its performance was demonstrated for a group of PWR calculations.

On the other hand, the development of pin-wise two-step core analysis systems, which might be regarded as 1.5 step system, had been started by numerous research groups worldwide including SCOPE-2 of NFI [4]. The pin-wise two-step aims at filling the gap between the conventional two-step which yields nontrivial errors for highly heterogeneous cases and the direct whole core calculation which yields high-fidelity solutions, yet requires exhaustive computing resources. In the pin-wise two-step system at SNU, the pin-homogenized GCs are generated by the SA transport calculations employing the nTRACER code [5], and the pin-by-pin calculation based on either the diffusion or the simplified P3 (SP3) formulation is carried out by the SPHINCS-FD code [6] which employs the finite difference method at the pin cell level. To handle the pin-homogenization error, the SPHINCS-FD adopts the superhomogenization (SPH) method [7] together with assembly-wise discontinuity factors (ADF) [8].

The pin-wise two-step calculations agree with the transport solutions better than the conventional two-step calculations for various core problems. It is, however, noted that the 2-and 4-group solutions involve somewhat notable error. It comes from the difference in the spectra between the SA environment and the actual core environment. Therefore, obviously the pin-by-pin calculations require proper spectral correction as well.

This paper presents the extension of the LFM to the pin-by-pin calculations. In order to examine the performance, two PWR cores, the APR-1400 [9] and AP1000 [10], are analyzed. The reference solutions are generated by the nTRACER transport calculations.

2. Pin-wise Leakage Feedback Method

The essential part of the LFM is the parameterization of the relative change in the GCs in terms of the leakage-to-removal ratio (LR) which is defined as:

$$l_G = \left(\sum_{surf} J_G^{surf} A_{surf} \right) \Bigg/ \left(\int_{vol} \Sigma_{r,G} \phi_G dV \right). \quad (1)$$

The subscript G indicates the energy range. For two groups (2-G), the subscript denotes fast (F) and thermal (T) groups. For three or more groups, it denotes fast (F), intermediate (I), and thermal (T) ranges. The rationale of using the LRs and cross-group leakage dependency is given in Ref. [11].

In the case of pin-by-pin calculation, the relative difference in the cross section of a pin between *Core* and *SA* can be parameterized in terms of the absolute difference of LRs as follows:

$$\frac{\overline{\Sigma}^{Core} - \overline{\Sigma}^{SA}}{\overline{\Sigma}^{SA}} = \sum_G C_G \left(l_G - l_G^{SA} \right) = \sum_G C_G \Delta l_G \quad (2)$$

The superscripts *Core* and *SA* denote whether the group condensation is based on the actual problem (core) or the single assembly (SA) spectrum. The coefficients (C_G) are obtained by the fixed point least square fitting on data sets which incorporates various neighbor conditions of the core. The selection of the data sample is described in Section 2.3. It should be noted that ΔLR, not the LR, is used because each pin has a certain amount of leakage over surfaces even though the assembly has all reflective boundary condition.

2.1 Two-group LFM

In the case of the assembly-wise LFM, the spectral index shift (SIS) as well as the LR is used for the fuel assemblies near the core periphery. The SIS is defined as the difference in the fast-to-thermal flux ratio:

$$\Delta\Gamma^{Core} = \frac{\phi_F^{Core}}{\phi_T^{Core}} - \frac{\phi_F^{SA}}{\phi_T^{FA}} . \quad (3)$$

Due to the radial reflector, a peripheral assembly (PA) has a spectrum that is quite different from the interior assemblies (IAs). As described in Ref. [11], the SIS indirectly represents the spectral shift in the whole

fast range, and the spectral characteristics of the peripheral assemblies can be taken by the SIS. In the case of the pin-wise LFM with 2-G constants (or simply the 2-G LFM), the SIS is not used because it does not improve the solution while it deteriorates the stability of the scheme. In some cases, the 2-G LFM with the SIS was diverged or converged to erroneous solutions.

The parameterization of the GC with the two LRs is given as:

$$\overline{\Sigma}_G^{Core} = \overline{\Sigma}_G^{SA}\left(1+\alpha_G \Delta l_F + \beta_G \Delta l_T\right) \quad (4)$$

The 2-G LFM based on the LRs only does not show the convergence problem, but it cannot incorporate the spectral characteristic of PAs. Therefore, PAs require special coefficient sets which are different from IAs.

2.2 Three-group LFM

In the cases of three or more groups, the pin-wise LFM correction is performed with three LRs which represent the fast, intermediate, and thermal ranges. It is called the three-group (3-G) LFM. The LR_F and LR_I directly represent the spectral shift in the fast range, as the SIS of the assembly-wise LFM.

The parameterization of the GC with the three LRs is given as:

$$\overline{\Sigma}_G^{Core} = \overline{\Sigma}_G^{SA}\left(1+\alpha_G \Delta l_F + \beta_G \Delta l_I + \gamma_G \Delta l_T\right) \quad (5)$$

By employing the LR_I, the 3-G LFM can capture the spectral characteristics of various core configurations. The 3-G LFM does not require the special coefficient set for PAs. It would be worthwhile to note that the spectral correction based on group-wise leakage parameters provides the same benefit, like the method described in Ref. [12], but it requires the increase in the number of fitting samples which is proportional to the number of energy groups. Although refining energy groups remarkably decreases the spectral error, it is undesirable to spend too much computing resources to correct relatively small errors.

2.3 Treatment of the radial reflector

The GCs of the radial reflector with the shroud (or baffle) is generated by the transport calculation on fuel-reflector local problems which represent parts of the whole reflector. According to the shapes shown in Fig. 1 with the red box, the problems are named as edge, corner, and nook. Note that the SPH factors of the reflectors are based on the same configurations.

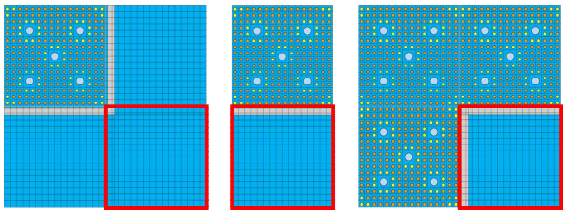

Fig. 1. Configurations of the corner (left), edge (center), and nook (right) type radial reflectors

To assess the need for the spectral correction on the reflector GCs, a series of test calculations is carried out for various configurations. The results indicate that the correction is not required to the pin-homogenized reflector GCs. In the case of the two-dimensional (2-D) APR-1400 initial core, for example, two sets of the reflector GCs are applied: one from the local problems, and the other from the whole core transport solution. Even though the GCs are condensed into 2 groups to increase the effect of spectral error, the difference in the solution caused by the reflector GCs was only about 2 pcm for reactivity and less than 0.2 % for pin power.

2.4 Fitting samples for LFM coefficients

The coefficient set to be used for the generation of the core GCs is obtained by least square fitting on data samples. In the case of the assembly-wise LFM, perturbed GCs and LRs for the data points are obtained by the transport calculations for two configurations shown in Fig. 2. The 2x2 checkerboard (CB) which mimics the core center is used for IAs and the 3x3 local problem which represents fuel-reflector region at the core periphery is used for PAs.

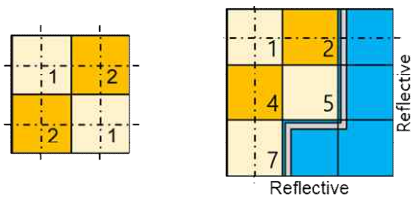

Fig. 2. Configurations of 2x2 checkerboard for IAs (left) and 3x3 local problem for PAs (right)

The same configurations are used for the 2-G LFM. The 2x2 CB and the 3x3 local problem provide the sample for IAs and PAs, respectively. It would be worthwhile to note that the coefficient set for the edge and corner type PAs should be generated separately from selected fuel assemblies (FAs). Among 5 FAs loaded in the local problem, the set for PA-edge is based on Positions 1 and 2 and for PA-corner is based on Positions 4 and 5.

In contrast, the 3-P LFM does not requires the 3x3 local problem. As noted in Section 2.2, the coefficient set based on the 2x2 CB can be applied regardless of the FA position in the core, including the periphery.

3. Performance Examination

Two PWR initial cores are selected for the pin-by-pin diffusion calculation with LFM. The APR-1400 core is taken to verify the solution improvement by the LFM, and the AP1000 core is taken to assess its applicability for the GEN-III+ reactors which have the advanced heterogeneous core design. The thermal condition is set to the hot temperature without feedback for APR-1400 and the hot-zero-power (HZP) for AP1000.

For the two cores, the pin-wise 2-and 4-GCs are generated by the 47-G nTRACER calculations with the transport corrected P_0 MOC option. The calculations are performed for the SAs for FAs and for the fuel-reflector local problems for the radial reflectors. Note that lower energy boundaries (eV) of the 4-G are 9.1188E+3, 3.9279E+0, 6.2506E-1, and 1.0000E-4 for group-1 to 4 [12]. The generation of the SPH factors and ADFs is based on the same configurations. Note that the SPH factors, ADF, and reflector GCs are kept unchanged. Each pin was subdivided into 2x2 meshes.

3.1 Two-dimensional core of APR-1400

The APR-1400 initial core shown in Fig. 3 has a total 9 types of FAs. It indicates that 9 points from the 2x2 CBs and 5x9 points from the 3x3 local problems can be obtained per a pin in a FA.

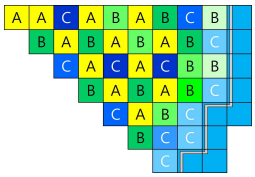

Fig. 3. Radial configuration of the APR-1400 PWR octant core model with surrounding assembly-sized reflector

The results of the pin-by-pin diffusion calculations for the APR-1400 with the 2-and 4-GCs are named as *NO LFM*, *LFM*, and *Core GC*. *LFM* denotes that the spectral correction is applied to the GCs. In the case of the 2-G, *LFM-IA* which refers the 2-G LFM without the coefficient set for PAs is also generated to assess the need for the PA set. *Core GC* denotes that the GCs are generated by the whole core transport with nTRACER so that the GCs do not contain the spectral error. The results are summarized in Table I.

Table I: Results of the APR-1400 2-D core (k-eff: 1.01410)

Grp.	Case	Δρ (pcm)	Abs. Pin ΔP (%)		Rel. Pin ΔP (%)	
			RMS	Max	RMS	Max
2	NO LFM	100.5	4.00	-9.50	3.88	-8.27
	LFM-IA	51.1	1.41	4.21	1.38	-3.20
	LFM	38.0	0.75	-2.03	0.75	2.11
	Core GC	5.8	0.49	-1.71	0.48	-1.37
4	NO LFM	37.8	1.29	-4.60	1.25	-3.74
	LFM	14.3	0.68	-2.31	0.65	-1.81
	Core GC	5.4	0.82	-2.67	0.80	-2.00

Due to severely large spectral error especially at the core periphery, the 2-G *NO LFM* shows significant error. The reactivity difference (Δρ) is larger than 100 pcm and the maximum absolute ΔP is 9.5 %. The 2-G *LFM-IA* shows some improvement, but the magnitude of the errors is still only about half of the *NO LFM* case. The agreement comparable to the 2-G *Core GC* can be obtained by the *2-G LFM*, with the special coefficient set for PAs. The absolute ΔP distributions shown in Fig. 4 clearly indicate the error reduction from the 2-G *NO LFM* to the *Core GC* case.

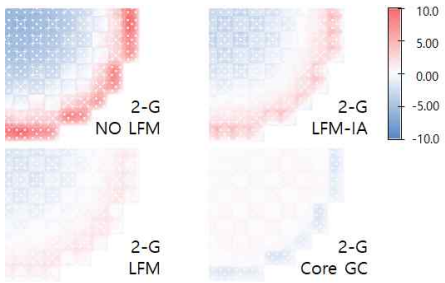

Fig. 4. Absolute pin ΔP (%) distribution of the APR-1400 PWR 2-D core (2-G diffusion vs. nTRACER)

In the case of the 4-G, however, the 4-G *LFM* with the coefficient set based on the 2x2 CB type samples yields the agreements comparable with the 4-G *Core GC*. The results indicate that less consideration on the fitting samples is alright for the 3-G LFM.

3.2 Two-dimensional core of AP1000

The AP1000 initial core is loaded with 157 FAs in 15x15 array as shown in Fig. 5. FAs are numbered from Region 1 through 5 by enrichment ranging from 0.74 to 4.80 %. The highest-enriched Region 5 is subdivided by 5A, 5B, and 5C according to the number of BPs. Therefore, a total of 7 types of FAs are in the core so that 7 points from the 2x2 CBs and 5x7 from the 3x3 local problems can be obtained per a pin in a FA.

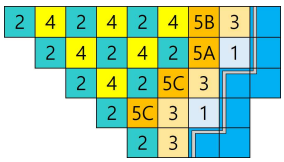

Fig. 5. Radial configuration of the AP1000 PWR octant core model with surrounding assembly-sized reflector

The results of the pin-by-pin diffusion calculations for the AP1000 are summarized in Table II and Fig. 6. The *NO LFM* cases show the error larger than the APR-1400 case due to the significant core heterogeneity. The pin power error is especially severe between the assemblies because of a large number of integral fuel burnable absorber (IFBA) loaded along the boundaries of Region 4 and 5 FAs. Despite of the severe heterogeneity, the *LFM* cases consistently show remarkable decrease of error. The Δρ is reduced from

129.3 to 52.0 pcm in the 2-G and from 19.0 to 2.5 pcm in the 4-G. The maximum of absolute ΔP is also reduced, from about 10.9 to 3.7 % in the 2-G and from 5.7 to 2.9 % in the 4-G.

Table II: Results of the AP1000 2-D core (k-eff: 1.00286)

Grp.	Case	Δρ (pcm)	Abs. Pin ΔP (%)		Rel. Pin ΔP (%)	
			RMS	Max	RMS	Max
2	NO LFM	129.3	3.93	-10.91	3.82	11.92
	LFM-IA	59.3	1.60	4.01	1.67	4.36
	LFM	52.0	1.40	3.65	1.56	5.71
4	NO LFM	19.0	1.44	-5.71	1.83	9.53
	LFM	-2.5	0.68	-2.88	1.14	6.40

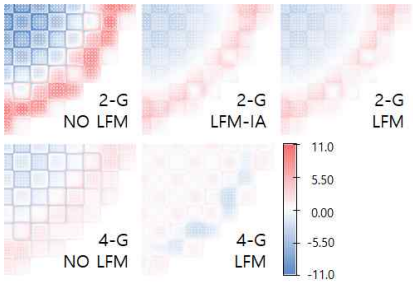

Fig. 6. Absolute pin ΔP (%) distribution of the AP100 PWR 2-D core (diffusion vs. nTRACER)

It would be worthwhile to note that the difference between the 2-G *LFM-IA* and *LFM* is only about 7 pcm in the reactivity and less than 0.4 % in the absolute ΔP. It is regarded because of the low-leakage core design, which is represented by 0.74 % natural UO_2 fueled Region 1 FAs at the periphery. The assembly-wise power distribution in Fig. 7 provides a reference to assess the effect of low-leakage design.

Fig. 7. Assembly-wise fission power distributions of an octant core, for the APR-1400 (left) and the AP1000 (right)

4. Conclusions

The leakage feedback method originally developed for spectral correction of the assembly-homogenized GCs was extended for pin-by-pin calculation. The refined LFM is named as the 2-G LFM with the 2-GCs and the 3-G LFM with three or more energy groups. The performance of the pin-wise LFM is tested for the two PWR cores.

Due to the insufficiency of the leakage parameters, the 2-G LFM requires at least two different types of the coefficient sets corresponding to IAs and PAs. On the other hand, the 3-G LFM can regenerate the GCs with the set corresponding to IAs only, and does not require an increase in the number of fitting samples by the refinement of the energy groups.

To examine the performance of the 2-G and 3-G LFM, the diffusion calculations are carried out for the APR-1400 and AP1000 2-D cores with and without the LFM. In the case of the 2-G LFM, the Δρ larger than 100 pcm is decreased to less than about 50 pcm, and the absolute ΔP within about 5 % can be obtained. It also shows the need for separate coefficient sets for PAs. In the case of the 3-G LFM, the change in Δρ is not much notable but the ΔP is decreased nearly by a half.

Based on the presented results, further studies will be performed to apply the LFM on more heterogeneous problems including depleted or rod-inserted cores, and also the LFM with the pin-by-pin SP3 will be tested.

REFERENCES

[1] J. I. Yoon and H. G. Joo, Two-Level Coarse Mesh Finite Difference Formulation with Multigroup Source Expansion Nodal Kernels, J. of Nuc. Sci. Tech., Vol. 45, no. 7, pp. 668-682, 2008.
[2] J. I. Yoon and et al., Development of the High-Performance Pin-by-Pin Calculation Code with Planar Parallelization, Trans. Am. Nuc. Soc., Vol. 119, Orlando, Florida, November 11-15, 2018.
[3] Y. S. Ban, Development of Advanced Leakage Feedback Method for Two-step Core Analyses, Ph.D. Thesis, Seoul National University, 2018.
[4] M. Tatsumi and A. Yamamoto, Advanced PWR Core Calculation Based on Multi-group Nodal-transport Method in Three-dimensional Pin-by-Pin Geometry, J. of Nuc. Sci. Tech., Vol. 40, No. 6, pp. 376-387, 2003
[5] Y. S. Jung, H. G. Joo and et al., Practical Numerical Reactor Employing Direct Whole Core Neutron Transport and Subchannel thermal/hydraulic solvers, Ann. Nuc. Energy, Vol. 62, pp. 357-374, 2013.
[6] H. H. Cho, H. G. Joo and et al, Preliminary Development of Simplified P_3 based Pin-by-pin Core Simulator SPHINCS, Trans. of KNS, Jeju, Korea, May 23-24, 2019
[7] A. Hebert, A Consistent Technique for the Pin-by-Pin Homogenization of a Pressurized Water Reactor Assembly, Nuc. Sci. Eng., Vol. 113, pp. 227-238, 1993.
[8] A. Yamamoto et al., Improvement of the SPH Method for Pin-by-Pin Core Calculations, J. of Nuc. Sci. Tech., Vol. 41, No. 12, p.1155-1165, 2004.
[9] H. Hong and H. G. Joo, Analysis of the APR1400 PWR Initial Core with the nTRACER Direct Whole Core Calculation Code and the McCARD Monte Carlo Code, Trans. of KNS, Jeju, Korea, May 18-19, 2017.
[10] F. Franceschini et al., Westinghouse VERA Test Stand - Zero Power Physics Test Simulations for the AP1000® PWR, CASL-U-2014-0012-000, Consortium for Advanced Simulation of LWRs, Rev. 1, March 6, 2014.
[11] Y. S. Ban and H. G. Joo, The Rationale of Leakage Parameters adopted in Leakage Feedback Method, Trans. of KNS, Yeosu, Korea, October 25-26, 2018.
[12] F. Tatsuya, T. Endo and A. Yamamoto, A new technique for spectral interference correction on pin-by-pin BWR core analysis, J. of Nuc. Sci. and Tech., Vol. 51, No. 6, pp. 783-797, 2014.