A Study on Pin-wise Macroscopic Depletion of PWRs in the Conventional Two-step Reactor Analysis

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1. Introduction

As computing power is dramatically increasing, high-fidelity multi-physics and multi-dimensional reactor analyses are receiving lots of attention for advanced reactor designs, which require detailed information, e.g., pin-wise power profile. Although the direct whole-core transport calculation is quite promising, it needs a lot of computational costs. Another possible alternative option is the pin-by-pin core calculation with a low-order operator such as the diffusion one.

In the conventional two-step core analysis, pin-wise core analyses are based on pin-wise spatial homogenization in the conventional single fuel assembly (FA) transport analysis with the reflective boundary condition. Consequently, resulting pin-wise group constants (GCs) are always subjected to unavoidable errors due to the unphysical boundary condition. To overcome this inevitable error of conventional two-step procedure, authors recently introduced the GPS method [1], which is a combination of the generalized equivalence theory (GET) [2,3] and super-homogenization method (SPH) [4,5]. We showed that the GPS method substantially improves accuracy of the pin-wise PWR analyses for given core states [1].

In this paper, feasibility of a two-step pin-wise macroscopic depletion scheme is investigated before application of the GPS method. The 2-D MOC-based DeCART2D [6] code was used for the lattice depletion and whole-core transport calculations. Pin-wise depletion analyses were performed using an in-house NEM-based pin-wise nodal code with the hybrid CMFD (HCMFD) algorithm [7].

2. Pin-wise Macroscopic Depletion

Based on the assumption that the burnup of a pin is proportional to pin-power per heavy metal, the calculation for pin-wise macroscopic depletion can be carried out by the following equation:

$$\Delta B_i = \Delta B_c \frac{P_i}{G_i} / \frac{P_c}{G_c}, \quad (1)$$

where

ΔB_c = core average burnup increment in one step,
P_i = power in i_{th} region,
P_c = total power in the core,
G_i = heavy metal loading in i_{th} region,
G_c = total heavy metal loading in the core.

2.1 Xenon and Samarium Depletion Model

For accurate consideration of Xe and Sm number densities, the standard microscopic depletion of Xe and Sm is considered in this work. The well-known decay chains of I-135, Xe-135, Pm-149, and Sm-149 are used as follows Eqs. (2) to (5):

$$\frac{d}{dt}N_I(t) = \gamma_I \sum_{g=1}^{G} \Sigma_{f,g}\phi_g - \lambda_I N_I(t) \quad (2)$$

$$\frac{d}{dt}N_{Xe}(t) = \lambda_I N_I(t) + \gamma_{Xe}\sum_{g=1}^{G}\Sigma_{f,g}\phi_g - \lambda_{Xe}N_{Xe}(t) - \sum_{g=1}^{G}\sigma_{Xe,a,g}N_{Xe}(t)\phi_g \quad (3)$$

$$\frac{d}{dt}N_{Pm}(t) = \gamma_{Pm}\sum_{g=1}^{G}\Sigma_{f,g}\phi_g - \lambda_{Pm}N_{Pm}(t) \quad (4)$$

$$\frac{d}{dt}N_{Sm}(t) = \lambda_{Pm}N_{Pm}(t) - \sum_{g=1}^{G}\sigma_{Sm,a,g}N_{Sm}(t)\phi_g \quad (5)$$

where

N_i = Nuclei number density of isotope *i*
λ_i = Decay constant of isotope *i*
γ_i = Effective yield of isotope *i*
$\sigma_{i,a}$ = Microscopic absorption XS of isotope *i*

In this study, both equilibrium and transient Xe-Sm models are implemented. For the equilibrium Xe-Sm model, the steady-state solutions of Eqs. (2) to (5) are used, while the analytic solutions are adopted for the transient model.

Once the number densities of Xe and Sm are calculated, the macroscopic absorption cross section for a fuel pin is adjusted with their number densities.

$$\Sigma_{a,g}^{Xe-Sm} = \Sigma_{a,g} + \Delta\Sigma_{a,g}^{Xe} + \Delta\Sigma_{a,g}^{Sm} \quad (6)$$

where

$\Sigma_{a,g}^{Xe/Sm}$ = Macroscopic absorption XS with Xe-Sm effect
$\Sigma_{a,g}$ = Macroscopic absorption XS without Xe-Sm effect
$\Delta\Sigma_{a,g}^{Xe} = \sigma_{Xe,a,g}N_{Xe}$ and $\Delta\Sigma_{a,g}^{Sm} = \sigma_{Sm,a,g}N_{Sm}$

2.2 Predictor-Corrector Scheme

The predictor-corrector scheme is widely used in the fuel depletion calculation. As a combination of an explicit and an implicit technique, the predictor-corrector method proceeds by extrapolating a function to fit the derivative from the previous point to the new

point (predictor step), then using this to interpolate the derivative (corrector step).

In this study, the fully weighted predictor-corrector (FWPC) method in Fig. 1 is adopted. At burnup step B_n, the given state including number density N_n of Xe and Sm are depleted to obtain the predicted number density N_{n+1}^p while assuming the pin-power P_n and flux ϕ_n to be constant during burnup interval ΔB. The predicted pin-power P_{n+1}^p and flux ϕ_{n+1}^p are determined with the predicted number density N_{n+1}^p. As the assumptions for having constant pin-power and flux are very rough especially at the Gd-mixed fuel, the predicted values should be corrected. The corrected pin-power P_n^c and flux ϕ_n^c are evaluated by Eq. (7) with a weight factor, ω.

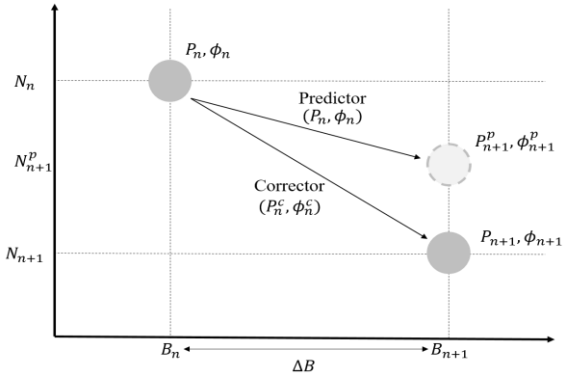


Fig. 1. Predictor-Corrector Scheme.

$$v_c^{t+1} = \omega v^t + (1-\omega) v_p^{t+1} \tag{7}$$

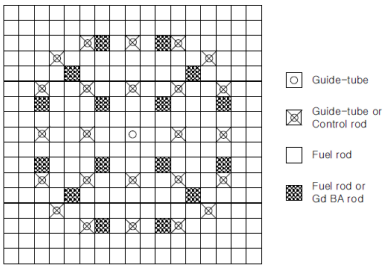
where the subscripts p and c indicate predictor and corrector, and the weight value is 0.5 in the current work.

3. Numerical Results and Discussions

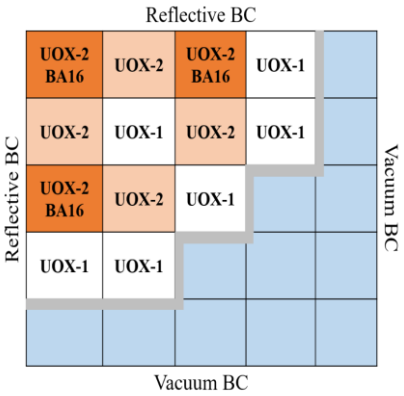
To evaluate the feasibility of pin-wise macroscopic depletion analysis, a small PWR [1] in Fig. 2 was considered as a test problem. In the small PWR, there are three typical 17x17 fuel assembles, UOX-1: 2.0 wt%, UOX-2: 3.3 wt%, UOX-2 with 16 BA (Gd-mixed) fuel pins. The total power of 2-D small PWR is set as 0.54265 MWth by assuming that the core height is 1 cm. For the consistency of node-size, the baffle-reflector regions are also treated with 2-group constants for pin-size nodes.

Pin-wise 2-group GCs are evaluated by infinite lattice calculations and the power of each lattice is set as the average FA power of the small PWR, 0.041742 MWth. In other words, the lattice depletions are performed with the core-average specific power of the PWR core. Figure 3 shows depletion results of three FA types in the small PWR. All lattices are depleted until 60 MWd/kgHM, and relatively fine burnup steps are set until 30 MWd/kgHM. Because of the Gd-mixed fuel, 'UOX-2 BA16' FA has a smaller burnup step size (0.5 MWd/kgHM) than one of the other two FA types (1.0 MWd/kgHM).

From the lattice depletions with given conditions (power, fuel/coolant temperature, CBC, etc.), the FA pin-wise 2-group GCs, including pin discontinuity factors (PDFs), yields and microscopic absorption XS for some nuclide described in Section 2.1, are tabulated with the pin-wise burnup. These FA pin-wise GC libraries are used to determine the pin GCs at given burnup condition by interpolation or extrapolation. As a feasibility study, other historical effects except burnup are not considered in this study.



a) UOX fuel assemble configuration



b) Core layout of the small PWR

Fig. 2. Configurations of the small PWR benchmark.

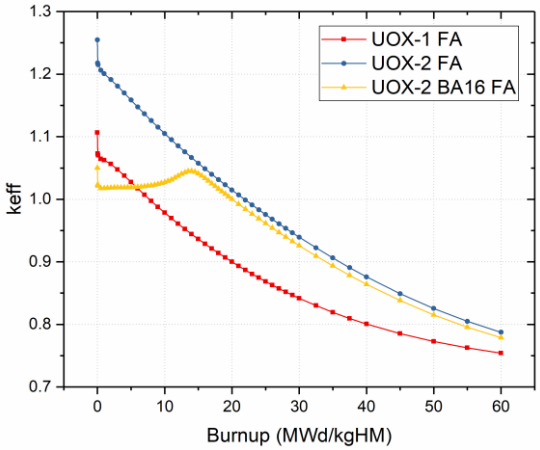


Fig. 3. Depletion results of each FA.

Figure 4 shows the reactivity change during the pin-wise macroscopic depletion of the small PWR core with two different Xe-SM models. For the equilibrium Xe-Sm model, there is 200 ~ 300 pcm reactivity difference at the early burnup steps where the number densities of Xe and Sm cannot reach the equilibrium state. For the transient Xe-Sm model, however, the reactivity difference due to the number densities of Xe and Sm clearly disappears. After the equilibrium, both two models have a quite similar depletion pattern within ~1 pcm reactivity difference. In the reference DeCART2D depletion calculation, a transient Xe-Sm option is also utilized. In Fig. 4, there is an upswing in the reactivity difference between 7.5 and 10 MWd/kgHM due to the fast depletion of Gd. It is mentioned that the pin-power and flux in the Gd-loaded fuel pins are far from constant when Gd burns quite quickly.

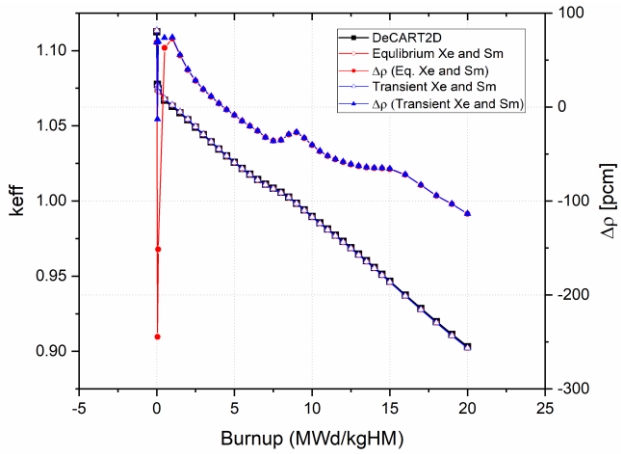


Fig. 4. Depletion results of two-step core analysis.

Figures 5 to 7 show FA-wise pin-power, pin-burnup and number densities of Xe and Sm of the small PWR at a few burnups. Since 2-group GCs are generated from the single FA lattice calculation, relatively larger pin-power errors occur along 1) interface between clearly different fuel assemblies and 2) peripheral fuel assemblies facing the baffle-reflector. These pin-power differences affect the pin burnup distribution. Therefore, pin burnup %error has a similar trend with pin-power %error.

For the number densities of Xe and Sm, there are large errors in the Gd-mixed fuel at 5 MWd/kgHM. Similar to the reason of upswing in the reactivity difference, the change of pin-power and flux in the Gd-mixed fuel pin is not constant. They change quite faster than other fuels. In that sense, both equilibrium and transient Xe-Sm number density models have limitation for the Gd-loaded pins. For the other burnup conditions, Gd-mixed fuel pins have relatively low number density errors since most of Gd are depleted. However, similar to the pin-power %error, larger errors occur along the interface between clearly different fuel assemblies and peripheral fuel assemblies facing the baffle-reflector. This is because the pin-power errors at each burnup lead difference in the accumulated pin burnup. Consequently, the solution reaches different states in terms of GCs, flux and etc.

1.639 0.61 (0.22) -0.32 (0.12) -5.06 (1.24) -6.52 (1.88)	**1.705** 1.15 (0.57) 0.97 (0.72) 2.45 (0.78) -3.49 (2.50)	**1.085** -1.30 (0.42) -1.25 (0.67) -2.82 (0.66) 14.07 (2.41)	**0.506** -2.64 (1.04) -2.70 (1.09) -3.11 (1.42) -3.28 (1.24)
1.705 1.15 (0.57) 0.97 (0.72) 2.45 (0.78) -3.49 (2.50)	**1.240** -1.69 (0.53) 0.33 (0.16) -5.83 (2.17) -7.54 (2.73)	**1.109** -1.99 (0.45) -2.47 (0.49) 2.36 (0.77) 3.50 (1.33)	**0.394** -3.95 (1.62) -3.83 (1.62) -5.06 (1.83) -10.95 (1.84)
1.085 -1.30 (0.42) -1.25 (0.67) -2.82 (0.66) 14.07 (2.41)	**1.109** -1.99 (0.45) -2.47 (0.49) 2.36 (0.77) 3.50 (1.33)	**0.523** -3.42 (1.51) -3.29 (1.43) -5.10 (2.09) -6.18 (1.91)	
0.506 -2.64 (1.04) -2.70 (1.09) -3.11 (1.42) -3.28 (1.24)	**0.394** -3.95 (1.62) -3.83 (1.62) -5.06 (1.83) -10.95 (1.84)	**Ref. FA Power** Pin Power Pin Burnup Number density of Xe Number density of Sm	

*Format: max. % error (RMS % error)

Fig. 5. FA-wise pin information at 5 MWd/kgHM.

1.751 1.29 (0.86) 0.55 (0.27) 1.28 (0.24) -6.45 (5.71)	**1.573** 1.96 (0.58) 1.01 (0.61) 3.19 (1.07) -7.03 (5.58)	**1.180** -1.59 (0.36) -1.42 (0.47) 3.00 (0.87) -7.13 (2.62)	**0.558** -2.93 (1.21) -2.67 (1.07) -4.41 (1.88) -4.16 (1.84)
1.573 1.96 (0.58) 1.01 (0.61) 3.19 (1.07) -7.03 (5.58)	**1.148** -3.35 (1.17) -1.27 (0.43) -7.63 (2.93) -11.04 (4.54)	**1.063** -1.61 (0.45) -2.11 (0.41) 3.24 (1.08) 4.04 (1.94)	**0.419** -4.07 (1.83) -3.91 (1.64) -7.52 (2.35) -7.93 (1.95)
1.180 -1.59 (0.36) -1.42 (0.47) 3.00 (0.87) -7.13 (2.62)	**1.063** -1.61 (0.45) -2.11 (0.41) 3.24 (1.08) 4.04 (1.94)	**0.518** -3.74 (1.93) -3.42 (1.53) -7.58 (2.86) -7.94 (2.38)	
0.558 -2.93 (1.21) -2.67 (1.07) -4.41 (1.88) -4.16 (1.84)	**0.419** -4.07 (1.83) -3.91 (1.64) -7.52 (2.35) -7.93 (1.95)	**Ref. FA Power** Pin Power Pin Burnup Number density of Xe Number density of Sm	

*Format: max. % error (RMS % error)

Fig. 6. FA-wise pin information at 10 MWd/kgHM.

1.377 1.51 (0.63) 0.88 (0.40) 1.22 (0.74) -8.50 (7.99)	**1.361** 2.88 (0.80) 1.60 (0.57) 3.76 (1.38) -8.86 (7.60)	**1.273** 2.46 (0.79) 0.85 (0.24) 3.85 (1.17) -6.78 (4.64)	**0.686** -4.37 (1.52) -3.01 (1.17) -6.70 (2.64) -5.91 (2.95)
1.361 2.88 (0.80) 1.60 (0.57) 3.76 (1.38) -8.86 (7.60)	**1.108** -4.51 (1.59) -2.64 (0.91) -8.58 (3.54) -11.95 (5.63)	**1.116** 2.71 (0.84) -1.61 (0.45) 4.30 (1.50) -5.56 (3.14)	**0.520** -6.38 (2.21) -3.95 (1.77) -11.51 (3.27) -10.38 (3.18)
1.273 2.46 (0.79) 0.85 (0.24) 3.85 (1.17) -6.78 (4.64)	**1.116** 2.71 (0.84) -1.61 (0.45) 4.30 (1.50) -5.56 (3.14)	**0.602** -6.44 (2.57) -3.67 (1.86) -11.63 (4.03) -10.43 (3.41)	
0.686 -4.37 (1.52) -3.01 (1.17) -6.70 (2.64) -5.91 (2.95)	**0.520** -6.38 (2.21) -3.95 (1.77) -11.51 (3.27) -10.38 (3.18)	**Ref. FA Power** Pin Power Pin Burnup Number density of Xe Number density of Sm	

*Format: max. % error (RMS % error)

Fig. 8. FA-wise pin information at 20 MWd/kgHM.

As shown in Figs. 5 to 7, pin-power and pin burnup %error at peripheral fuel pins facing the baffle-reflector are quite noticeable and they increase with burnup in general. However, it should be noted that the relatively large errors are located at the outmost fuel pins, where the normalized pin-power is quite low, as shown in Fig. 8.

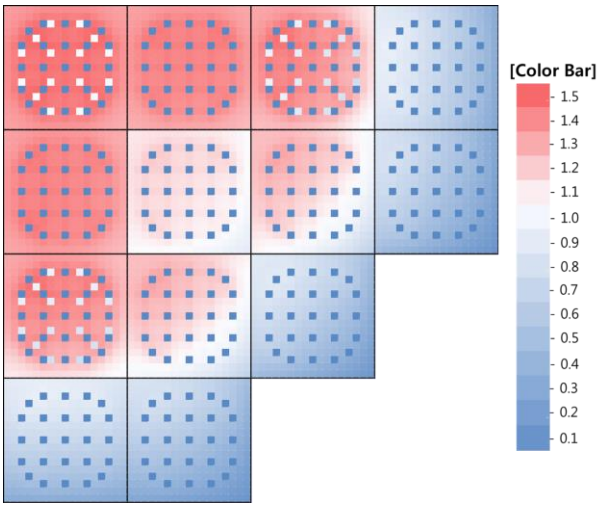


Fig. 8. Normalized pin power distribution at 20 MWd/kgHM.

Figure 9 shows the peak normalized pin-power in the small PWR and %error of the peak power as a function of burnup. It is noted that the error in peak pin power is usually quite less than 1%. Also, the maximum %error of peak pin power is only about 1.3% at 0.5 MWd/kgHM and corresponding normalized pin-power is 2.149.

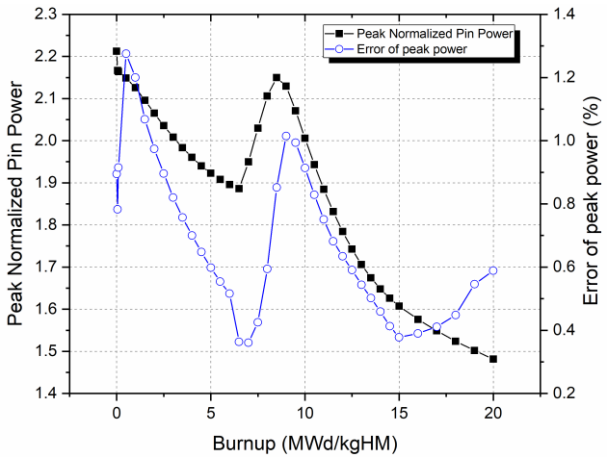


Fig. 9. Peak normalized pin-power and %error of peak.

4. Conclusions

In this paper, a two-step pin-wise macroscopic depletion method of PWR has been proposed and its feasibility evaluated against a small PWR problem. From the conventional lattice calculations, burnup-wise pin XSs and DFs are generated and tabulated. For the macroscopic depletion calculation, number densities of Xe and Sm are explicitly determined to adjust macroscopic absorption XS in each pin. In addition, conventional predictor-corrector scheme is adopted for efficient depletion calculation. Based on the numerical results on a small PWR, it is observed that the pin-wise macroscopic depletion method provides fairly accurate prediction in terms of the reactivity and pin-wise power profiles, leading to a maximum ~100 pcm error in eigenvalue and ~1.3% for the peak pin power. Nevertheless, it was found that relatively large pin-power errors occur along the interface between clearly different environments, such as different FA and baffle-reflector region. It is expected that the conventional GET-based pin-wise depletion calculation can be further improved by correcting burnup-dependent group constants.

ACKNOWLEDGEMENT

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