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Probabilistic Estimation of Fatigue Lifetime with Time-Series Data and Markov Chain Monte Carlo Simulation

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1. Introduction

In general, traditional fatigue life estimation models have been estimated from numerous end-of-life data points, which are usually represented on a strain/stress ~ life (S~N) curve plot [1-3]. One data point on the S~N curve implies one failure time data of a single fatigue test. Therefore, to construct the entire S~N curve with probabilistic scatter band requires almost hundreds of fatigue tests. However, it may not always be possible to conduct hundreds of expensive and time-consuming fatigue experiments for each and every different testing case. Additionally, aforementioned S~N curve approach does not consider the time-dependent behavior of material because it is based on the simple end-of-life (i.e., failure time) data. In other words, the conventional S~N curve approach has disregarded entire data history during the fatigue experiment except for the last information. It could be a type of *data dissipation* when only few fatigue data sets are available.

Based on this aspect, we propose a fatigue life prediction method based on the time-series experimental data. A statistical technique of Markov-Chain-Monte-Carlo (MCMC) simulation is applied to manage the time-series data. We apply the proposed method on two stress-controlled fatigue tests under different environmental conditions, such as in-air or pressurized water reactor (PWR) water. Finally, the results of the time-series data based model are compared with the conventional end-of-life data based model, which is estimated from precedent literature.

2. Time-Series Data Based Life Estimation

To demonstrate the time-series data based modeling approach, we used two independent fatigue data sets. Table 1 summarized the associated loading and environmental conditions of the fatigue tests. All tests were loaded symmetrically (i.e., R = -1), and 316 stainless steel (SS) is used as testing materials. The constant amplitude portion of the stress-controlled tests was intended to achieve an equivalent constant strain amplitude of 0.5%, but the observed strain amplitude was not 0.5% because of varying amplitude due to strain hardening/softening.

Figure 1 shows the two observed ratcheting strain histories obtained from the stress-controlled test cases presented in Table 1. The input cyclic loading consisted of the initial 12 cycles of variable stress amplitudes followed by constant cycle stress amplitudes. In the first 12 cycles of variable loading, the stress increased from 106 to 216 MPa. In the remaining constant-loading cycles, the stress was controlled at 216 MPa to achieve an intended strain amplitude of 0.5% [4].

Table 1. Symmetric (R = -1) type fatigue loading cases and associated environment for time-series data based modeling.

Test Case	**Loading**	**Environment**
ET-F43	Initial 12 variable amplitude and then constant amplitude stress-controlled fatigue test, with an equivalent intended strain amplitude of 0.5%	300 °C,in-air
EN-F44		300 °C, PWR water

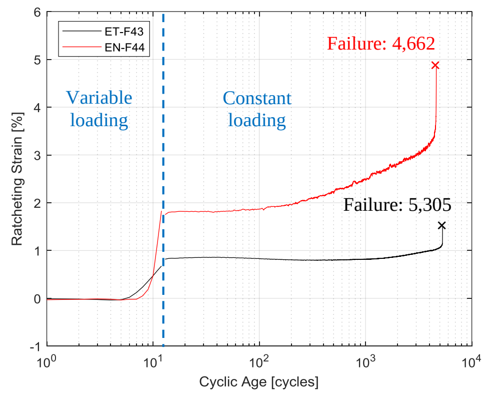

Fig. 1. Cyclic age vs. ratcheting strain data for stress-controlled data sets (data from [4]).

Basically, it is ideal to assume the probability of a system state may depend on its entire life history (or state history). However, for simplification, the *Markov chain* process assumes that the probability of next system state depends exclusively on its current state level [5]. In this case, we use the *ratcheting strain rate* in constant loading region to classify the system state levels.

Figure 2 shows an example histogram of the ratcheting strain rate in constant loading region for the ET-F43 data. We used the default auto binning algorithm built in the MATLAB (Ver. 2018a) function *histogram*, and defined the state levels by corresponding each non-zero histogram bin in ascending order with a state. In this case, the ET-F43 data can be classified by the total state of 32 and the EN-F44 data can be classified by the total state of 50.

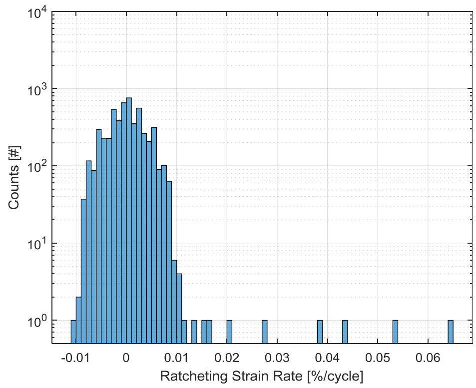

Fig. 2. Histogram of ratcheting strain rate for ET-F43 data.

Using the associated time-series state (i.e., the ranking number of ratcheting strain rates) profile, we can calculate the state transition probability P_{ij} as follows:

$$P_{ij} = P(\text{next state } j \mid \text{current state } i) \qquad (1)$$

The definition of P_{ij} is a conditional probability of state transitioning from a state i to j. In this case, i and j should not be larger than 32 because there are only 32 states in case of ET-F43 data. Figure 3 shows the calculated state transition matrix for ET-F43 data.

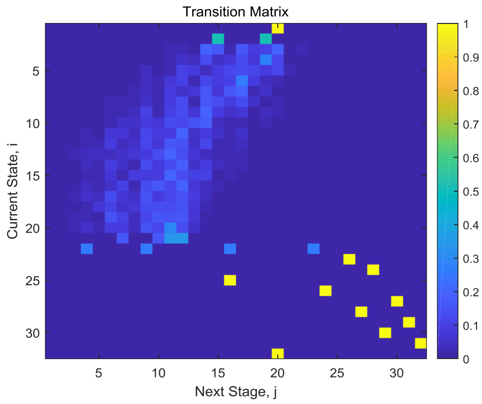

Fig. 3. Probability matrix of state transition for ET-F43 data.

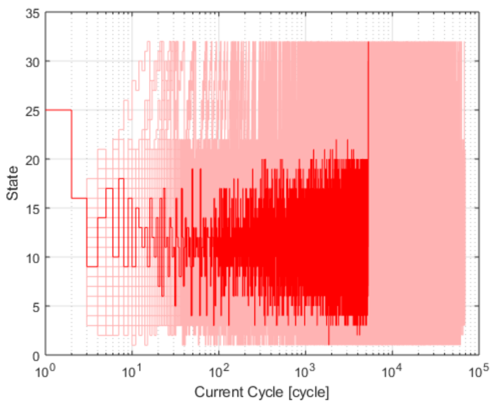

Fig. 4. Time-series state profile (in red line) and 1,000 MCMC simulated state profiles (in light red lines) of ET-F43 data.

Then, according to the assumption of the Markov chain process, it is possible to predict the next state probabilistically based on the information of the current state level. In this step, we performed 1,000 iterations in a Monte Carlo simulation to evaluate the uncertainty of state transitioning, as shown in Figure 4.

Thereafter, the Monte Carlo-simulated 1,000 state profiles were used for estimating the corresponding ratcheting strain rate profiles, and then the associated scatter in time-series ratcheting strain evolutions. Figure 5 shows the resulting ratcheting strain histories and their scatters for ET-F43 data through the MCMC simulation. The estimated uncertainty or scatter band of observed ratcheting strain can be used for estimating the cumulative distribution function (CDF) of fatigue lifetime.

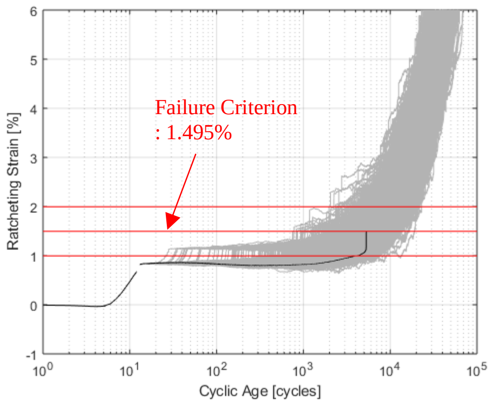

Fig. 5. Original ratcheting strain profile (in black line), estimated 1,000 MCMC ratcheting strain profiles (in grey line), and failure strain limits used for CDF calculation data (in red line) for ET-F43.

Using these probabilistic time-series data and a given failure criterion, we can estimate the failure probability of the test specimens. For the stress-controlled test cases, in this work, the MCMC-based CDFs are predicted by considering a failure criterion of each ratcheting strain data limit. For example, the failure criterion of 1.495% ratcheting strain is used for ET-F43 data (see Fig. 5). Similarly, the failure criterion of EN-F44 ratcheting strain is 4.835% (see Fig. 1). Figure 6 shows the resulting empirical CDFs calculated from the MCMC ratcheting strain simulation for ET-F43 data.

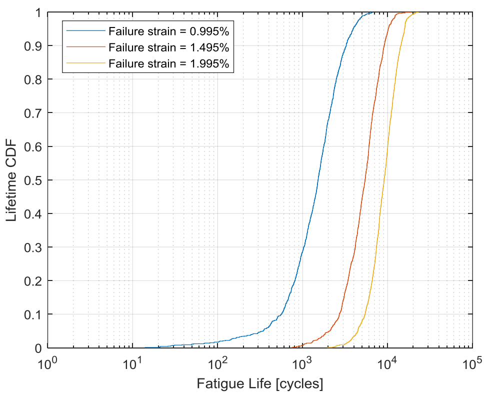

Fig. 6. Empirical CDFs estimated from MCMC simulated time-series ratcheting strain data for ET-F43 case.

3. Comparison with End-of-Life Data Based Model

The resulting lifetime CDF of time-series data based model can be compared with the respective CDF estimated directly based on end-of-life data. With the limited availability of time-series test data, this end-of-life CDF comparison will help to judge the accuracy of the overall MCMC method. If the MCMC method is found to reasonable, it can be extended for probabilistic life estimation by conducting few fatigue tests, which are prototypical to the actual loading and environment. This method will help to avoid the dependence on traditional stress/strain vs. life (or S~N) data, which might have been obtained under completely different loading and environment conditions compared to the actual loading and environmental conditions of interest.

For this comparison, the lifetime CDFs were estimated based on hundreds of end-of-life SS fatigue data [3] and the Weibull-Bootstrap method [6]. Table 2 shows the comparison of end-of-life and time-series data based model conditions and lifetime CDF quartiles. Figure 7 illustrates the lifetime CDF quartiles derived from Table 2 for the case of in-air and PWR-water conditions. It is shown that the MCMC simulated life distribution quartiles are comparable to the Weibull-Bootstrap predicted CDF for the in-air case, but it differs in case of PWR-water environment. The discrepancy in the PWR water case could be due to the following reasons:

- The discrepancy could have been caused by completely different types of loading (i.e., displacement-controlled vs. stress-controlled).
- The discrepancy could also be due to the type of stainless-steel grades used. For example, the end-of-life test data (used for Weibull-Bootstrap CDF prediction) comprise different stainless-steel grades such as 316, 304, etc., whereas the MCMC data are only based on 316 SS grade.
- The discrepancy could be due to the heat treatment, and heat of Weibull data set which is different compared to the heat treatment and heat of specimens used for the MCMC test cases.
- The Weibull data set was generated from fatigue tests conducted at temperatures in a range of 100-325 °C, whereas the MCMC test case specimens were tested at 300 °C.
- The PWR water test water chemistry and the strain rates under which the PWR water tests were conducted in a range of 10^{-5} to 0.3 %/s for the end-of-life test data reference, which are taken from precedent literature [3]. The underlying end-of-life data set are based on both slower strain rate (less than 0.1%/s) and higher strain rates (greater than or equal to 0.1%/s) fatigue tests. Under a lower strain rate and PWR water condition, the (cyclic) fatigue lives could be substantially lower than those under higher strain rate PWR-water condition. Note that the MCMC models are based on PWR-water tests, which were performed close to an equivalent strain rate of 0.1%/s.

Table 2. Comparison of end-of-life and time-series data based model conditions and lifetime CDF quartiles.

		End-of-life data based model	**Time-series data based model**
Data type and source		Hundreds of end-of-life data on S-N plot from [3]	Single time-series data set of cycle versus ratcheting strain (ET-F43, EN-F44)
Loading type		Strain/stroke-controlled	Stress-controlled
Material grade		316 SS, 304 SS, etc.	316 SS
Strain amplitude		Various strain amplitude data, but model used 0.5% amplitude	Intended strain amplitude of 0.5%
Data strain rate		10^{-5} to 0.3 %/s	Strain rate of 0.1%/s
Data temperature		100 to 325 °C	300 °C
Failure criteria		25% load drop	Ratcheting strain limit of data set
Life distribution type		Weibull distribution [6]	Empirical CDF from MCMC simulation
Quartiles (in-air) [cycles]	25%	3,741	3,714
	50%	6,291	5,423
	75%	9,475	7,256
Quartiles (PWR-water) [cycles]	25%	761	4,007
	50%	1,567	4,651
	75%	2,769	5,406

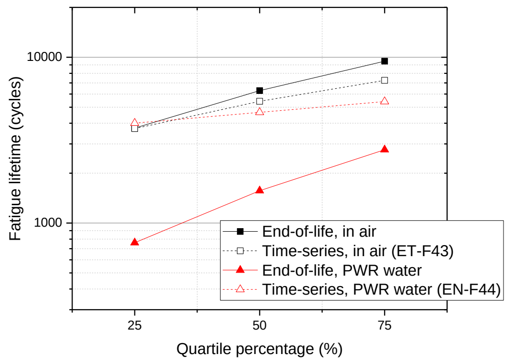

Fig. 7. Quartile comparison of end-of-life and time-series data based model lifetime CDFs based on Table 2.

With a lower strain rate, the environment effect becomes much more significant compared to that at higher strain rate (the discussed PWR water MCMC model case). Hence, additional PWR-water environment cases (under different strain rates, especially on lower side) are required to justify the above analogies as discussed. This area is one of our future studies in plan.

Nevertheless, the comparison results of the CDFs based on MCMC simulation and direct end-of-life data demonstrate the potential of MCMC-based probabilistic life estimation. The MCMC-based approach can facilitate the probabilistic life estimation under any loading and environmental condition in practical field, and at the same time, it requires just few test data sets. Note that the Weibull CDF predictions are based on hundreds of end-of-life data points. It is nearly impossible to conduct sufficiently large tests for each and every combination of actual reactor component loading and environmental conditions.

4. Conclusions

This study suggests a probabilistic modeling approach which can consider time-series fatigue data based on the MCMC simulation. For the purpose of demonstration, we develop the probabilistic fatigue lifetime models using the two independent fatigue data sets tested under different loading and environment conditions. The resulting fatigue models are compared with the traditional end-of-life data based models from precedent literature. Although some discrepancy is observed in the results of lifetime CDF comparison between time-series and end-of-life data based models, the comparison results highlight the potential of the MCMC-based probabilistic life estimation approach.

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