Observation of Variance Overestimation in Monte Carlo Multi-Physics Simulation

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1. Introduction

The underestimation of variance due to the inter-cycle correlation of fission source sites has been an issue of Monte Carlo (MC) community. There have been researches to quantify the amount of variance bias and to understand the phenomenon [1][2], and nowadays the underestimation of variance is well known.

Recently, another type of variance bias, overestimation of the variance, is observed in MC multi-physics simulation. In this paper, the overestimation phenomenon induced by thermal/hydraulics feedback and equilibrium xenon feedback is studied on BEAVRS benchmark using MCS [3].

2. Methods and Results

2.1 Subcycle

It is well known that the MC fission source sites have positive inter-cycle correlation which leads to underestimation of variance [1]. To reduce the correlation, MCS employs the subcycle method, the idea of subcycle identical to the batch method [2]. We accumulate the tallies at L subcycles as shown in the following equation

$$Q^i = \frac{1}{L}\sum_{l=1}^{L} Q^{i,l} \ , \qquad (1)$$

where $Q^{i,l}$ is the tally quantity at subcycle l of cycle i, and Q^i is the tally quantity of cycle i which is sum of $Q^{i,l}$.

2.2 Underestimation of Variance

The underestimation effect due to inter-cycle correlation of fission source, and the effect of subcycle are studied with BEAVRS [4] benchmark quarter core geometry at hot zero power state as shown in Fig. 1.

Two cases of simulations were performed by employing different numbers of subcycles:

Case 1: 300 inactive cycles/600 active cycles/1 subcycle/10,000 histories per subcycle

Case 2: 5 inactive cycles/20 active cycles/300 subcycles/10,000 histories per subcycle.

The core radially-integrated axial flux distribution was tallied for 20 equally-divided meshes in active core region. To estimate the real and apparent standard deviation, 50 independent simulations were performed for each case.

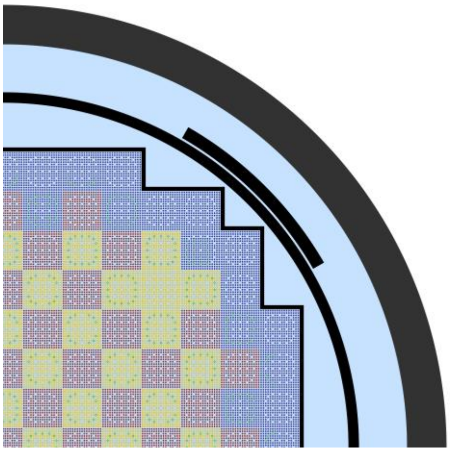

Fig. 1. BEAVRS Benchmark configuration.

Fig. 2. shows the underestimation ratio of axial flux distribution. The standard deviation is underestimated maximumly 10 times comparing to the real standard deviation due to inter-cycle correlation for typical MC simulation which employs 1 subcycle. On the other hands, the underestimation ratio with 300 subcycles is almost one for every mesh.

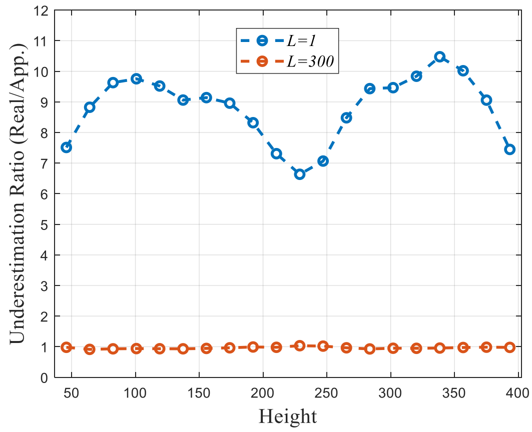

Fig. 2 Underestimation ratio of axial flux distribution.

2.3 Multi-Physics Feedbacks

Fig. 3 presents the calculation flow of MCS when feedbacks are used. As shown in the figure, MCS performs the feedbacks at the end of every transport cycle. Thermal/hydraulics feedback by using closed channel TH solver TH1D [5] and equilibrium xenon feedback are selectively used in this paper, and the temperature dependent cross-sections are treated by on-the-fly interpolation [6] and multipole method [7].

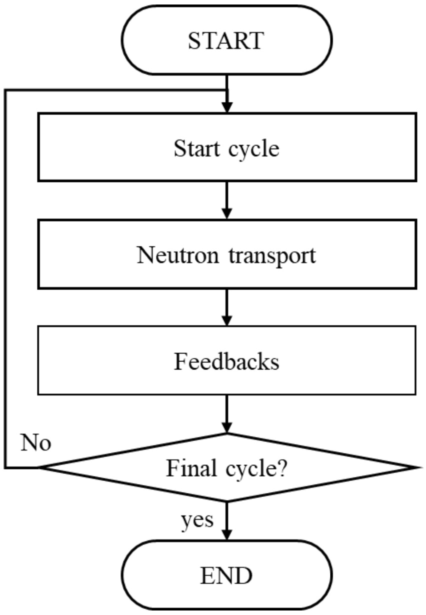

Fig. 3. Calculation flow of MCS.

2.4 Overestimation of Variance

The cell-wise feedback is adopted on BEAVRS benchmark quarter core at hot full power states. There are 20 axial cells per fuel pin, in total 254,760 fuel cells in the quarter core. Four simulations were performed with various combinations of feedback options:

Case 1: No feedback.
Case 2: thermal/hydraulics.
Case 3: Equilibrium xenon.
Case 4: thermal/hydraulics + equilibrium xenon.

All four cases ran with 5 inactive cycles, 20 active cycles, 300 subcycle, and 10,000 histories per subcycle, and 50 independent simulations were performed to estimate the real and apparent standard deviations.

Fig. 4 shows the axial flux distribution of four cases averaged from 50 simulations. As shown in the figure, the power distribution is flattened when feedbacks are used. The overestimation ratios are presented in Fig. 5. The effect of thermal/hydraulics feedback or equilibrium xenon feedback alone seems not huge. However, the overestimation ratio is noticeably large when two feedbacks are employed together. The significantly large overestimation is due to the small real standard deviation as shown in Figs. 6-7. When two feedbacks are used together, the real standard deviation is twice smaller than the other cases while all cases show similar apparent standard deviation.

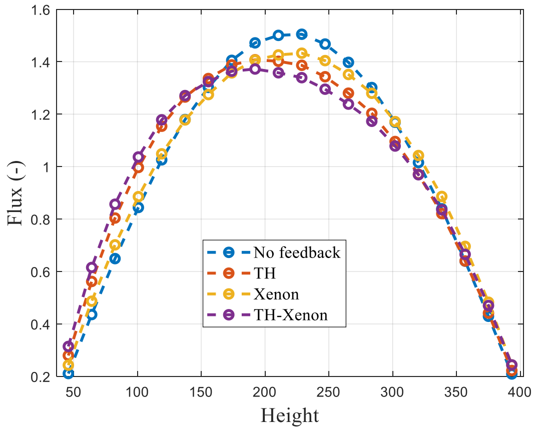

Fig. 4. Axial flux distribution.

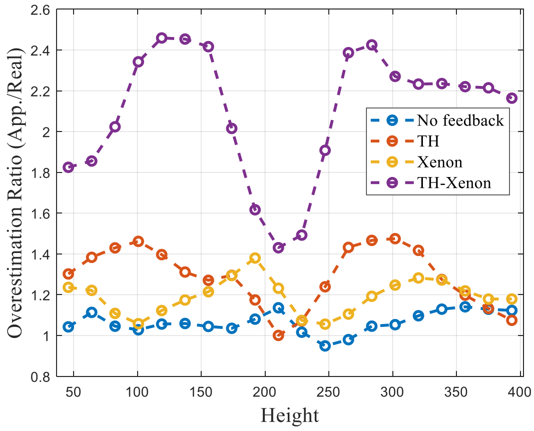

Fig. 5. Overestimation ratio of axial flux.

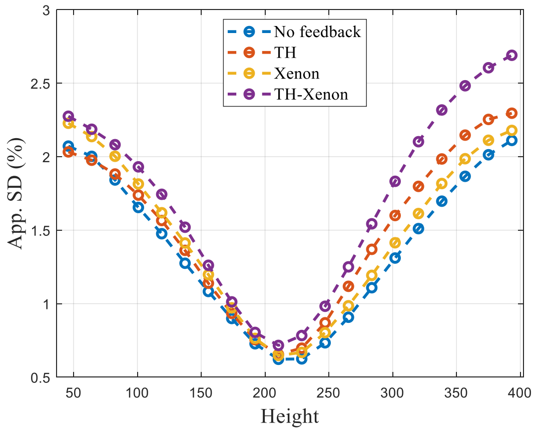

Fig. 6. Apparent standard deviation of axial flux.

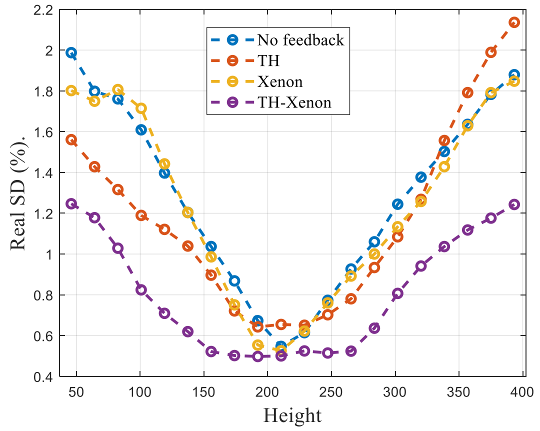

Fig. 7. Real standard deviation of axial flux.

In addition to the above observation, auto-correlation coefficients (ACCs) are estimated for case 4. To estimate the ACCs more faithfully, an additional simulation was performed with 4 inactive cycles, 5,000 active cycles, 300 subcycles and 10,000 histories per subcycle. Fig 8 presents the auto-correlation coefficients of axial flux at the 5^{th} mesh from bottom which presents the highest overestimation ratio as shown in Fig. 5. The magnitude of lag-1 ACC is about -0.6, and it rapidly decreases as lag increase. The negative correlation can be explained with Doppler feedback and xenon density. The cells have higher power at cycle i than cycle i-1 will have higher temperature and higher xenon density. Since the higher temperature and higher xenon density are both negative feedbacks to the neutron population, the power at cycle $i+1$ will be lower. In the same way, the power at cycle $i+2$ will be higher again and it will be repeated. Therefore, there is oscillation of ACCs around zero as shown in the figure.

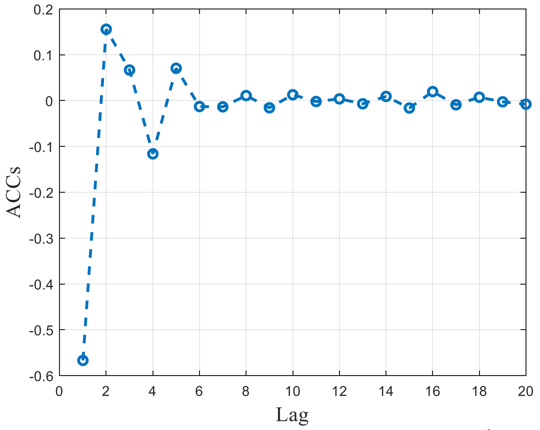

Fig. 8. Auto-correlation coefficient of axial flux at 5^{th} mesh.

3. Conclusions

The variance overestimation phenomenon was newly observed in Monte Carlo multi-physics simulation with cycle feedback algorithm. The overestimation ratio of axial flux distribution was calculated on BEAVRS benchmark for various combinations of the thermal/hydraulics feedback and equilibrium xenon feedback. The results showed that either thermal/hydraulics feedback or equilibrium feedback can lead to the overestimation of variance and two feedbacks together have a synergy effect that leads to larger variance overestimation. In the end, the inter-cycle correlation was estimated in terms of auto-correlation coefficients. It was shown that the correlation is strongly negative with lag-1, slightly positive with lag-2, and then decreases in magnitude and oscillates around. The strong negative correlation with lag-1 explains the phenomena of variance overestimation observed.

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REFERENCES

[1] H.J. Shim and C.H. Kim, "Real Variance Estimation Using and Intercycle Fission Source Correlation for Monte Carlo Eigenvalue Calculation," *Nuclear Science and Engineering*, 162, 2009.
[2] E.M. Gelbard and R.E. Prael, "Computation of Standard Deviation in Eigenvalue Calculations," *Progress in Nuclear Energy*, 24, 237, 1990.
[3] H. Lee, W. Kim, P. Zhang, et al., "Preliminary Simulation Results of BEAVRS Three-dimensional Cycle 1 Wholecore Depletion by UNIST Monte Carlo Code MCS," *Proc. M&C*, Jeju, Korea, April 16-20, 2017.
[4] N. Norelik, B. Herman, B. Forget, and K. Smith, "Benchmark for Evaluation and Validation of Reactor Simulations (BEAVRS), v1.0.1," *proc M&C*, Sun Valley, Idaho, 2013.
[5] M. Ryu, Y.S. Jung, H.H. Cho, and H.G. Joo, "Solution of the BEAVRS Benchmark using the nTRACER Direct Whole Core Calculation Code," *Journal of Nuclear Science and Technology*, 52, 961-969, 2015.
[6] H. Lee and D. Lee, "On-The-Fly Interpolation for Thermal Scattering in MCS," *Trans. KNS*, Jeju, Korea, May 17-18, 2018.
[7] C. Josey, P. Ducru, B. Forget, and K. Smith, "Windowed Multipole for Cross Section Doppler Broadening," *Journal of Computational Physics*, 2015.