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Evaluation of Implicit Uncertainty Considering Double Heterogeneity

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1. Introduction

Neutronics parameter uncertainty induced by nuclear data uncertainty can be calculated by combining the sensitivity of the parameter and the covariance data of the cross section. The covariance data is given for the infinitely-diluted cross sections in the evaluated nuclear data files. On the contrary, because self-shielded multi-group cross sections, not infinitely-diluted cross sections, are conventionally used for the uncertainty analysis of the neutronic parameter, the uncertainty change caused by the resonance treatment should be considered as the implicit uncertainty.

In MUSAD (Modules of Uncertainty and Sensitivity Analysis for DeCART) [1], the implicit uncertainty can be calculated using Chiba method [2,3] which applies the self-shielding factor. The method has an advantage that the implicit effect can be easily calculated in the lattice code without any data from the resonance treatment code. However, it was found out that the uncertainty error in the case of the double heterogeneous fuel of VHTR is slightly larger than the homogeneous case when applying the method.

Thus, this paper presents the correction method for the implicit uncertainty considering the double heterogeneity effect and the verification results to the HTGR UAM benchmark problem [4].

2. Methods and Results

MUSAD basically uses the correction method proposed by Chiba for the implicit uncertainty. However, the method cannot be directly applied to DeCART [5] code which do not deal with the self-shielding factor. In section 2.1, the modification of the correction method for the DeCART will be described in detail. Then, an additional correction factor for the implicit uncertainty considering the double heterogeneity (DH) is proposed in next section and the verification results are provided in section 2.3.

2.1 Correction Method for Implicit Uncertainty

If a sensitivity of a general response, R , by an infinitely diluted cross section, σ , is expressed with the self-shielded cross section, $\tilde{\sigma}$, the sensitivity can be expressed as follows:

$$S = \frac{dR}{d\sigma}\frac{\sigma}{R} = \left(\frac{dR}{d\tilde{\sigma}}\frac{\tilde{\sigma}}{R}\right)\left(\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}}\right) = \tilde{S}\left(\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}}\right), \quad (1)$$

where, $\tilde{S}$ is the explicit sensitivity for the general response by the self-shielded multi-group cross section which is used in the DeCART code.

In addition, the sensitivity, Eq.(1), can be rewritten in terms of the background cross section, σ_b , as follows:

$$\tilde{S}\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}} = \tilde{S}\left(\frac{d\tilde{\sigma}}{d\sigma_b}\right)\left(\frac{d\sigma_b}{d\sigma}\right)\frac{\sigma}{\tilde{\sigma}}. \quad (2)$$

Contrary to the Chiba's derivation, the resonance integral on behalf of the self-shielding factor can be applied for obtaining the relation between σ and $\tilde{\sigma}$ in DeCART as follows:

$$\tilde{\sigma} = \frac{\sigma\sigma_b T}{\sigma_b - \sigma_a T}, \quad (3)$$

where, σ_a are the absorption cross section. In addition, the resonance integral, T (or I), for the self-shielded cross section can be reproduced from the Segev's interpolation [6] as follows:

$$T = \frac{I}{\sigma} = \left(\frac{\sigma_b}{\sigma_b + \eta}\right)^p, \quad (4)$$

where, η and p are coefficients determined from the two resonance integral table entries.

If applying the differentiation in terms of σ_b , Eq.(4) can be transformed as

$$\frac{dT}{d\sigma_b} = \frac{p\eta T}{\sigma_b(\sigma_b + \eta)}. \quad (5)$$

Then, $\dfrac{d\tilde{\sigma}}{d\sigma_b}$ in Eq.(2) can be replaced with σ_b , $\tilde{\sigma}$, and T from Eq.(3), Eq.(4), and Eq.(5). Finally, the sensitivity, Eq.(1), can be rewritten as follows:

$$S \cong \tilde{S}\tilde{\sigma}\left[\frac{1}{I} - \frac{p\eta}{I(\sigma_b+\eta)} + \frac{1}{\sigma_b}\right] = \tilde{S}\omega_1, \quad (6)$$

where, the approximation, $\frac{d\sigma_b}{d\sigma} \cong -\frac{\sigma_b}{\sigma_a}$, derived by Chiba was applied for the simplification and ω_1 is the correction factor for the implicit sensitivity induced by the self-shielding effect.

2.3 Implicit Uncertainty Correction Considering Double Heterogeneity

However, it is known that the DH fuel of VHTR have another spatial self-shielding effect. It means that the uncertainty is changed by the DH effect and it needs an additional correction.

We can define the relation between the effective cross section for the DH fuel region, $\hat{\sigma}$, and the shielded multi-group cross section, $\tilde{\sigma}$, as follows:

$$\hat{\sigma} = \gamma\tilde{\sigma}, \quad (7)$$

where, γ is the self-shielding factor by the DH. Because DeCART applies the renewal theory by Sanchez method [7], the self-shielding factor by the DH can be approximated as follows:

$$\gamma \approx \frac{\hat{\Sigma}}{\Sigma_{mix}}, \quad (8)$$

where, $\hat{\Sigma}$ is the macro effective cross section for the DH region calculated by the renewal theory. The detailed explanation for the effective cross section can be found in the reference [7]. In addition, Σ_{mix} is the volume weighted cross section for the mixture with the TRISO fuel as follows:

$$\Sigma_{mix} = p_0\Sigma_0 + \sum_{ik} p_{ik}\Sigma_{ik}, \quad (9)$$

where, p_0 and Σ_0 are the volume fraction and the total cross section for the base material of the fuel compact and p_{ik} and Σ_{ik} are the volume fraction and the total cross section for the *k* layer of the *i* type TRISO.

Applying the perturbation equation for Eq.(7), we can derive the sensitivity considering the self-shielding effect for the DH as follows:

$$\begin{aligned}\tilde{S}\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}} &= \hat{S}\left(\frac{d\hat{\sigma}}{d\tilde{\sigma}}\frac{\tilde{\sigma}}{\hat{\sigma}}\right)\left(\frac{d\tilde{\sigma}}{d\sigma}\frac{\sigma}{\tilde{\sigma}}\right) \\ &\approx \hat{S}\left(\frac{d\gamma}{d\tilde{\sigma}}\frac{\tilde{\sigma}}{\gamma}+1\right)\omega_1 = \hat{S}\omega_2\omega_1\end{aligned}, \quad (10)$$

where, ω_2 is the correction factor for the implicit effect of the DH and the value of $\frac{d\gamma}{d\tilde{\sigma}}$ can be calculated by the direct perturbation as follows:

$$\frac{d\gamma}{d\tilde{\sigma}} \approx \frac{\Delta\gamma}{\Delta\tilde{\sigma}}. \quad (11)$$

2.4 Numerical Results

The DeCART/MUSAD code system was modified to consider the implicit effect of the DH as described above.

The verification calculation was performed on MHTGR 350 Ex.I-1b proposed by IAEA CRP HTGR UAM [4] which is a DH fuel compact pin cell problem. DeCART/MUSAD uses the cross sections originated from the ENDF/B-VII.0 and the covariance matrix processed from the ENDF/B-VII.1. The reference results were made from McCARD [8] based on Monte Carlo method.

Table 1 shows the comparisons between the reference and the explicit and implicit uncertainty by MUSAD on Ex.I-1b CZP and HZP problems. The explicit uncertainty caused by ^{238}U absorption-absorption in MUSAD result was overestimated to about 40% comparing with McCARD result. On the other hand, the differences with the reference decrease to about 4% in the CZP case and 11% in the HFP case, respectively, when considering the implicit effect by the resonance self-shielding effect. Moreover, it reveals that if applying the implicit effect correction by the DH, the difference in the HFP case decreases to 2.3%. When considering the simple calculation for the correction factor as shown in Eq.(10), it seems that the proposed method is considerably effective.

3. Conclusions

For considering the implicit effect by the DH, the correction method of the implicit uncertainty has been implemented into DeCART/MUSAD code system. The additional correction factor for the implicit effect was defined and approximately calculated using the self-shielding factor of the DH based on Sanchez method. The verification calculation was performed on MHTGR 350 Ex.I-1b and the differences with McCARD result decrease from 40% to 2.3% in HFP case.

From this study, it is expected that DeCART/MUSAD code can reasonably produce the uncertainty considering the implicit effect on VHTR problem.

Table I: k_{inf} uncertainty for Ex.I-1b

Problem	Ex.I-1b CZP				Ex.I-1b HFP			
Code	McCARD	MUSAD			McCARD	MUSAD		
Contributor \ Type	Explicit+ Implicit	Explicit	Explicit+ Implicit 1*	Explicit+ Implicit 2**	Explicit+ Implicit	Explicit	Explicit+ Implicit 1*	Explicit+ Implicit 2**
^{235}U ν-ν	0.617	0.617	0.617	0.617	0.611	0.612	0.612	0.612
^{235}U abs-abs	0.239	0.240	0.239	0.239	0.237	0.238	0.238	0.237
^{235}U fis-fis	0.065	0.065	0.065	0.065	0.071	0.071	0.071	0.071
^{238}U abs-abs	**0.316**	**0.437**	**0.328**	**0.312**	**0.388**	**0.555**	**0.432**	**0.397**
Total	0.753	0.815	0.762	0.755	0.784	0.883	0.811	0.793

* : Implicit uncertainty by resonance self-shielding effect
** : Implicit uncertainty by resonance and double heterogeneity

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