1. Introduction

A large number of coated fuel particles (CFPs) are contained in a fuel element of a high temperature reactor (HTR). The integrity of CFPs greatly affects the characteristics of the fission product migration in a fuel element. It should be evaluated how various mechanical and chemical hazards deteriorate the integrity of the CFPs during the operation of an HTR. The CFP usually consists of a fuel kernel in its innermost center and four surrounding coating layers such as a low-density pyrocarbon called buffer, an inner high-density pyrocarbon (IPyC), a silicon carbide (SiC), and an outer high-density pyrocarbon (OPyC) from its inside part. The sizes and material properties of the CFP components are statistically variable. Those statistical variations must be considered when doing a performance analysis on a batch of CFPs.

This study describes how to generate random CFPs for a fuel performance analysis, and shows an example of the random particle calculations.

2. Random Particle Generation

The CFP quantities with normal distribution are the diameter of a kernel, the thickness and density of a buffer, the thicknesses, densities, and bacon anisotropy factors (BAFs) of coating layers. The CFP quantities with Weibull distribution are the strengths of coating layers. They are stochastically independent of each other. They can be generated using uniform deviates whose range is from 0 to 1, where uniform deviates are just random numbers that come from a uniform distribution.

2.1. Generating normal deviates from uniform deviates

Two independent standard normal distribution deviates are produced from two independent uniform distribution deviates using the Box-Muller method [1]:

\[ y_1 = \sqrt{-2 \ln R^2/R^2} \left( 2x_1 - 1 \right), \]

\[ y_2 = \sqrt{-2 \ln R^2/R^2} \left( 2x_2 - 1 \right), \]

where \( R^2 = (2x_1 - 1)^2 + (2x_2 - 1)^2 \), \( y = \) the standard normal distribution deviate \( \in (-\infty, \infty) \), and \( x = \) the uniform distribution deviate \( \in (0, 1) \). A usual random number generator (RNG) produces \( x_1 \) and \( x_2 \).

The CFP quantities with normal distribution can be calculated using a standard normal distribution deviate \( y \):

\[ t = \bar{t} + y \cdot \text{STD}_t, \]

where \( \bar{t} \) and \( \text{STD}_t \) = the mean value and standard deviation of \( t \), respectively, that are experimentally given.

2.2. Generating Weibull deviates from uniform deviates

The random variable \( y \) with a probability distribution \( f(y) \) is related to the uniform random variable \( x \) on (0, 1) by the fundamental transformation law of probabilities [2]:

\[ f(y) = \frac{dx}{dy}, \]

where \( f \) = the probability density function of \( y \). Solving Eq. (4) gives the random variables \( x \) and \( y \):

\[ x = F(y) = \int_{-\infty}^{y} f(z)dz, \]

\[ y = F^{-1}(x), \]

where \( F \) = the cumulative distribution function of \( y \), and \( F^{-1} \) = the inverse function of \( F \). The random variable \( y \) is expressed by Eq. (7) if it is in a Weibull distribution of Table I [3]:

\[ y = \lambda [-\ln(1-F)]^{1/m}. \]

A usual RNG produces \( F \) on (0, 1).

The probability distribution for the strength of a CFP coating layer is defined by a Weibull distribution. Table II shows the strength distributions according to the scale parameter of a Weibull distribution. Fig. 1 shows the variation of random Weibull strength deviate over uniform deviate.

<table>
<thead>
<tr>
<th>Table I: A standard Weibull distribution</th>
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</table>
| \( f(y) = \begin{cases} 
\frac{m \cdot \lambda^m y^{m-1} e^{-(y/\lambda)^m}}{\Gamma(m)}, & y \geq 0 \\
0, & y < 0
\end{cases} \) |
| \( F(y) = \begin{cases} 
1 - e^{-(y/\lambda)^m}, & y \geq 0 \\
0, & y < 0
\end{cases} \) |
| \( \gamma_{\text{mean}} = \lambda \Gamma(1+1/m) \) |
| \( \gamma_{\text{med}} = \lambda (\ln 2)^{1/m} \) |
| \( \sigma_y = \lambda^2 \left[ \Gamma(1+2/m) - [\Gamma(1+1/m)]^2 \right] \) |
where \( f \) = the probability density function, 
\( F \) = the cumulative distribution function, 
\( y \) = the Weibull distributed random variable, 
\( m \) = the shape parameter (\( > 0 \)), 
\( \lambda \) = the scale parameter (\( > 0 \)), 
\( y_{\text{mean}} \) = the mean value of \( y \), 
\( y_{\text{med}} \) = the median value of \( y \), 
\( V_y \) = the variance of \( y \), 
\( \Gamma \) = the gamma function.

A method generating random CFPs has been developed that use only a random number generator during a fuel performance analysis. The method produces statistical properties of a batch of CFPs using a uniform random number produced by a usual random number generator. The method is utilized in a statistical fuel performance analysis on a batch of CFPs.

ACKNOWLEDGEMENTS

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REFERENCES

Table II: Weibull distribution for the strength of a CFP coating layer

<table>
<thead>
<tr>
<th>Scale parameter $\lambda$</th>
<th>$f(s)$</th>
<th>$F(s)$</th>
<th>$s_{\text{mean}}$ or $s_{\text{med}}$</th>
<th>$V_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{\text{mean}} = \frac{s_{\text{med}}}{(n2)^{1/m}} \Gamma(1 + 1/m)$</td>
<td>$\frac{m \ln 2}{s_{\text{med}}} \left( \frac{s}{s_{\text{med}}} \right)^{m-1} e^{-\ln 2 (s/s_{\text{med}})^m}$, $s \geq 0$</td>
<td>$1 - e^{-\ln 2 (s/s_{\text{med}})^m}$, $s \geq 0$</td>
<td>$\frac{s_{\text{med}}}{s_{\text{mean}}} = \frac{(n2)^{1/m}}{\Gamma(1 + 1/m)}$</td>
<td>$\frac{s_{\text{med}}^2}{s_{\text{mean}}} = \frac{(n2)^{2/m}}{\Gamma(1 + 2/m) - \left( \frac{\Gamma(1 + 1/m)}{\Gamma(1 + 1/m)} \right)^2}$</td>
</tr>
<tr>
<td>$s_{\text{med}} = \frac{\ln(1-F)}{\ln 2}$</td>
<td>$\frac{m \Gamma(1 + 1/m)^m}{s_{\text{mean}}} \left( \frac{s}{s_{\text{med}}} \right)^{m-1} e^{-\left[ \frac{\Gamma(1 + 1/m)}{\Gamma(1 + 1/m)} \right] s/s_{\text{mean}}}$, $s \geq 0$</td>
<td>$1 - e^{-\left[ \frac{\Gamma(1 + 1/m)}{\Gamma(1 + 1/m)} \right] s/s_{\text{mean}}}$, $s \geq 0$</td>
<td>$s_{\text{med}} = \frac{s_{\text{mean}}}{(n2)^{1/m}} \Gamma(1 + 1/m)$</td>
<td>$\frac{s_{\text{med}}^2}{s_{\text{mean}}} = \frac{(n2)^{2/m}}{\Gamma(1 + 2/m) - \left( \frac{\Gamma(1 + 1/m)}{\Gamma(1 + 1/m)} \right)^2}$</td>
</tr>
</tbody>
</table>

$s = \text{the strength (MPa)}, s_{\text{med}} = \text{the median strength (MPa)} > 0, s_{\text{mean}} = \text{the mean strength (MPa)} > 0, m = \text{the shape parameter} > 0,$

$f = \text{the Weibull probability density function of } s, F = \text{the Weibull cumulative distribution function of } s, V_s = \text{the variance of } s \text{ (MPa)}^2 > 0, \Gamma = \text{the gamma function.}$