Mathematical Expressions for the Fractional Release of a Fission Product from an HTR Nuclear Fuel Element

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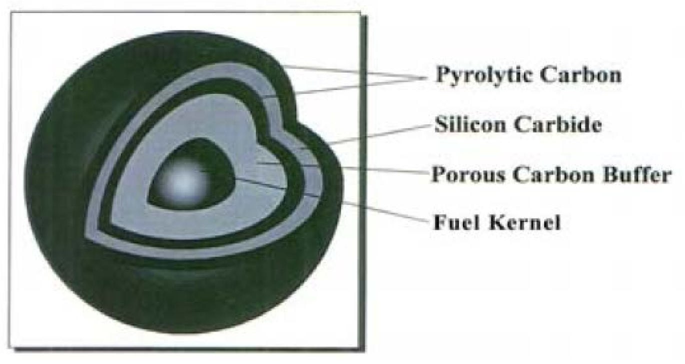
Objectives

- The quantity called fractional release (FR) (or release fraction) is used as a measure quantifying the fission product (FP) release in many high temperature reactor (HTR) fuel performance analysis code.
- This study describes the mathematical expressions of the FP releases from an HTR fuel element, and the analytical forms of FRs for the radioactive and stable isotopes diffusing out of a spherical particle.



Fractional Release Formulae

Radioactive fission product

C = concentration (atoms/m^3), $\dot{B}$ = volumetric birth rate (atoms/(m^3 s)), λ = decay constant (s^{-1}), D = diffusion coefficient (m^2/s), r = radial coordinate (m), t = time (s), z = 0 for a slab, 1 for a cylinder, 2 for a sphere, J = mass current, (atoms/(m^2 s)), $C^{(0)}$ = initial concentration (atoms/m^3), $C_V = \int_V C dV$ = amount in a volume (atoms), $\dot{R}_V$ = release rate (atoms/s), $\dot{B}_V = \int_V \dot{B} dV$ = birth rate (atoms/s), V = volume (m^3), R_V = amount released (atoms), B_V = amount generated (atoms), A_s = surface area (m^2), $\mu = \lambda a^2/D$, $\tau(t) = \int_0^t [D(r,x)/a^2]dx$, a = radial coordinate of the outermost location (m), $F_{irr}^{(e)}$ = fractional release at the end of irradiation.

Phases	Quantities	Fractional release formulae for a sphere with a constant D and a constant $\dot{B}$
Irradiation phase: $\frac{\partial C}{\partial t} = \dot{B} - \lambda C + \frac{1}{r^z}\frac{\partial}{\partial r}\left(r^z D \frac{\partial C}{\partial r}\right)$ $C(a,t) = C(r,0) = 0$ $J(0,t) = 0$ $\frac{dC_V}{dt} = \dot{B}_V - \lambda C_V - \dot{R}_V$ $B_V = C_V + R_V$	$\dot{R}_V = A_s J(a,t)$ $R_V = e^{-\lambda t}\int_0^t e^{\lambda x}\dot{R}_V dx$ $B_V = e^{-\lambda t}\int_0^t e^{\lambda x}\dot{B}_V dx$	$J(\mathrm{a,t}) = \frac{a}{3}\dot{B}\left[\frac{3}{\sqrt{\mu}}\left(coth\sqrt{\mu} - \frac{1}{\sqrt{\mu}}\right) - 6e^{-\mu\tau}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2+\mu}\right]$ $H = \dot{R}_V/\dot{B}_V = \frac{\text{total rate at which atoms are being released}}{\text{total rate at which atoms are being produced}}$ $= \frac{3}{\sqrt{\mu}}\left(coth\sqrt{\mu} - \frac{1}{\sqrt{\mu}}\right) - 6e^{-\mu\tau}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2+\mu}$ $\approx \begin{cases} \frac{3}{\sqrt{\mu}} erf\sqrt{\mu\tau} - \frac{3}{\mu}(1-e^{-\mu\tau}) & ,\tau \le 1/\pi^2 \\ \frac{3}{\sqrt{\mu}}\left(coth\sqrt{\mu} - \frac{1}{\sqrt{\mu}}\right) - \frac{6e^{-(\pi^2+\mu)\tau}}{\pi^2+\mu} & ,\tau > 1/\pi^2 \end{cases}$ $F = R_V/B_V = \frac{\text{amout of external nondecayed atoms}}{\text{amount of nondecayed atoms in the system}}$ $= \frac{1}{1-e^{-\mu\tau}}\left[\frac{3}{\sqrt{\mu}}\left(coth\sqrt{\mu} - \frac{1}{\sqrt{\mu}}\right) - e^{-\mu\tau} + 6\mu e^{-\mu\tau}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2(n^2\pi^2+\mu)}\right]$ $= \frac{1}{1-e^{-\mu\tau}}\begin{cases} \frac{3}{\sqrt{\mu}}\left(erf\sqrt{\mu\tau} - 2\sqrt{\frac{\mu\tau}{\pi}}e^{-\mu\tau}\right) - \frac{3}{\mu}[1-(1+\mu\tau)e^{-\mu\tau}] & ,\tau \le 1/\pi^2 \\ \frac{3}{\sqrt{\mu}}\left(coth\sqrt{\mu} - \frac{1}{\sqrt{\mu}}\right) - e^{-\mu\tau} + \frac{6\mu e^{-(\pi^2+\mu)\tau}}{\pi^2(\pi^2+\mu)} & ,\tau > 1/\pi^2 \end{cases}$
Heating phase: $\frac{\partial C}{\partial t} = -\lambda C + \frac{1}{r^z}\frac{\partial}{\partial r}\left(r^z D \frac{\partial C}{\partial r}\right)$ $C(a,t) = 0, C(r,0) = C^{(0)}$ $J(0,t) = 0$ $\frac{dC_V}{dt} = -\lambda C_V - \dot{R}_V$ $0 = C_V - C_V^{(0)}e^{-\lambda t} + R_V$	$\dot{R}_V = A_s J(a,t)$ $R_V = e^{-\lambda t}\int_0^t e^{\lambda x}\dot{R}_V dx$ $C_V^{(0)} = \int_V C^{(0)} dV$	$J(\mathrm{a,t}) = \frac{2C^{(0)}D}{a}\sum_{n=1}^{\infty} e^{-(n^2\pi^2+\mu)\tau}$ $F^{INT} = R_V/C_V^{(0)} = \frac{\text{amout of external nondecayed atoms}}{\text{amount of atoms at the initial time of heating}}$ $= e^{-\mu\tau}\left(1 - 6\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2}\right) \approx e^{-\mu\tau}\begin{cases} 6\sqrt{\tau/\pi} - 3\tau & ,\tau \le 1/\pi^2 \\ 1 - \frac{6}{\pi^2}e^{-\pi^2\tau} & ,\tau > 1/\pi^2 \end{cases}$ $F = [1 - F_{irr}^{(e)}]F^{INT} = \frac{\text{amout of external nondecayed atoms}}{\text{amount of nondecayed atoms in the system at the end of irradiation}}$

Stable fission product

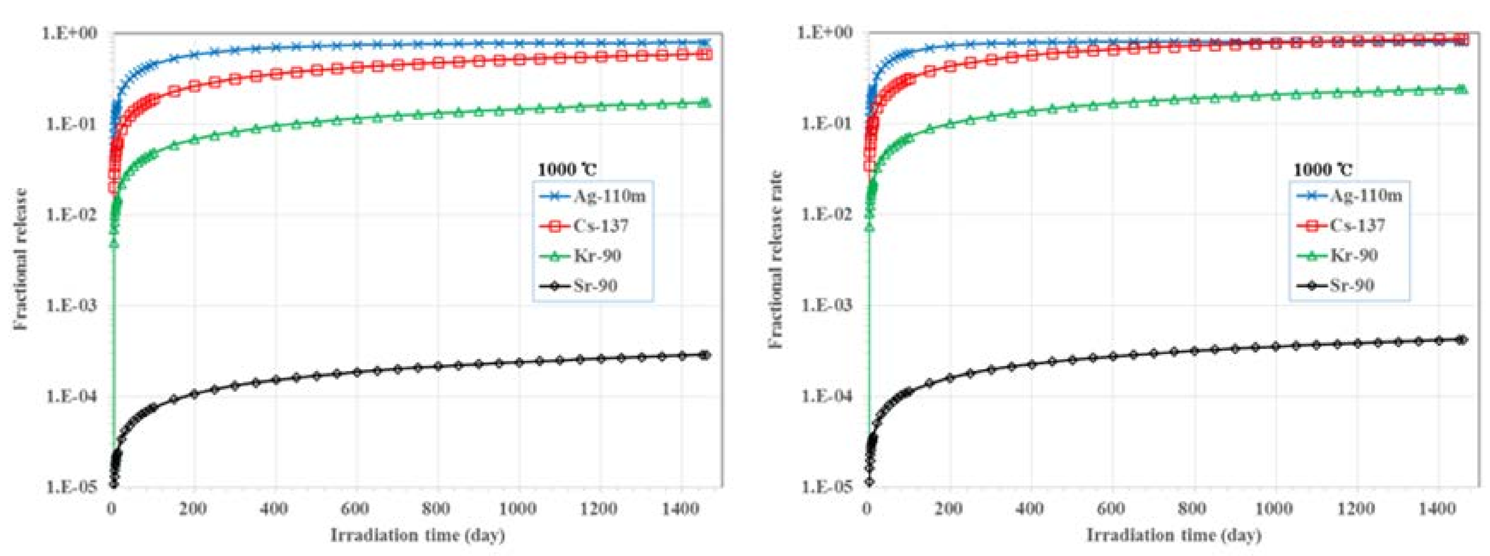
C = concentration (atoms/m^3), $\dot{B}$ = volumetric birth rate (atoms/(m^3 s)), λ = decay constant (s^{-1}), D = diffusion coefficient (m^2/s), r = radial coordinate (m), t = time (s), z = 0 for a slab, 1 for a cylinder, 2 for a sphere, J = mass current, (atoms/(m^2 s)), $C^{(0)}$ = initial concentration (atoms/m^3), $C_V = \int_V C dV$ = amount in a volume (atoms), $\dot{R}_V$ = release rate (atoms/s), $\dot{B}_V = \int_V \dot{B} dV$ = birth rate (atoms/s), V = volume (m^3), R_V = amount released (atoms), B_V = amount generated (atoms), A_s = surface area (m^2), $\mu = \lambda a^2/D$, $\tau(t) = \int_0^t [D(r,x)/a^2]dx$, a = radial coordinate of the outermost location (m), $F_{irr}^{(e)}$ = fractional release at the end of irradiation.

Phases	Quantities	Fractional release formulae for a sphere with a constant D and a constant $\dot{B}$
Irradiation phase: $\frac{\partial C}{\partial t} = \dot{B} + \frac{1}{r^z}\frac{\partial}{\partial r}\left(r^z D \frac{\partial C}{\partial r}\right)$ $C(a,t) = C(r,0) = 0$ $J(0,t) = 0$ $\frac{dC_V}{dt} = \dot{B}_V - \dot{R}_V$ $B_V = C_V + R_V$	$\dot{R}_V = A_s J(a,t)$ $R_V = \int_0^t \dot{R}_V dx$ $B_V = \int_0^t \dot{B}_V dx$	$J(\mathrm{a,t}) = \frac{a}{3}\dot{B}\left(1 - 6\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2}\right)$ $H = \dot{R}_V/\dot{B}_V = \frac{\text{total rate at which atoms are being released}}{\text{total rate at which atoms are being produced}}$ $= 1 - 6\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2} \approx \begin{cases} 6\sqrt{\tau/\pi} - 3\tau & ,\tau \le 1/\pi^2 \\ 1 - \frac{6}{\pi^2}e^{-\pi^2\tau} & ,\tau > 1/\pi^2 \end{cases}$ $F = R_V/B_V = \frac{\text{amout of atoms released}}{\text{amount of atoms produced in the system}}$ $= 1 - \frac{1}{15\tau} + \frac{6}{\tau}\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^4\pi^4} \approx \begin{cases} 4\sqrt{\frac{\tau}{\pi}} - \frac{3}{2}\tau & ,\tau \le 1/\pi^2 \\ 1 - \frac{1}{15\tau} + \frac{6}{\pi^2\tau}e^{-\pi^2\tau} & ,\tau > 1/\pi^2 \end{cases}$
Heating phase: $\frac{\partial C}{\partial t} = \frac{1}{r^z}\frac{\partial}{\partial r}\left(r^z D \frac{\partial C}{\partial r}\right)$ $C(a,t) = 0, C(r,0) = C^{(0)}$ $J(0,t) = 0$ $\frac{dC_V}{dt} = -\dot{R}_V$ $0 = C_V - C_V^{(0)} + R_V$	$\dot{R}_V = A_s J(a,t)$ $R_V = \int_0^t \dot{R}_V dx$ $C_V^{(0)} = \int_V C^{(0)} dV$	$J(\mathrm{a,t}) = \frac{2C^{(0)}D}{a}\sum_{n=1}^{\infty} e^{-n^2\pi^2\tau}$ $F^{INT} = R_V/C_V^{(0)} = \frac{\text{amout of atoms released}}{\text{amount of atoms at the initial time of heating}}$ $= 1 - 6\sum_{n=1}^{\infty}\frac{e^{-n^2\pi^2\tau}}{n^2\pi^2} \approx \begin{cases} 6\sqrt{\tau/\pi} - 3\tau & ,\tau \le 1/\pi^2 \\ 1 - \frac{6}{\pi^2}e^{-\pi^2\tau} & ,\tau > 1/\pi^2 \end{cases}$ $F = [1 - F_{irr}^{(e)}]F^{INT} = \frac{\text{amout of atoms released}}{\text{amount of atoms produced in the system until the end of irradiation}}$

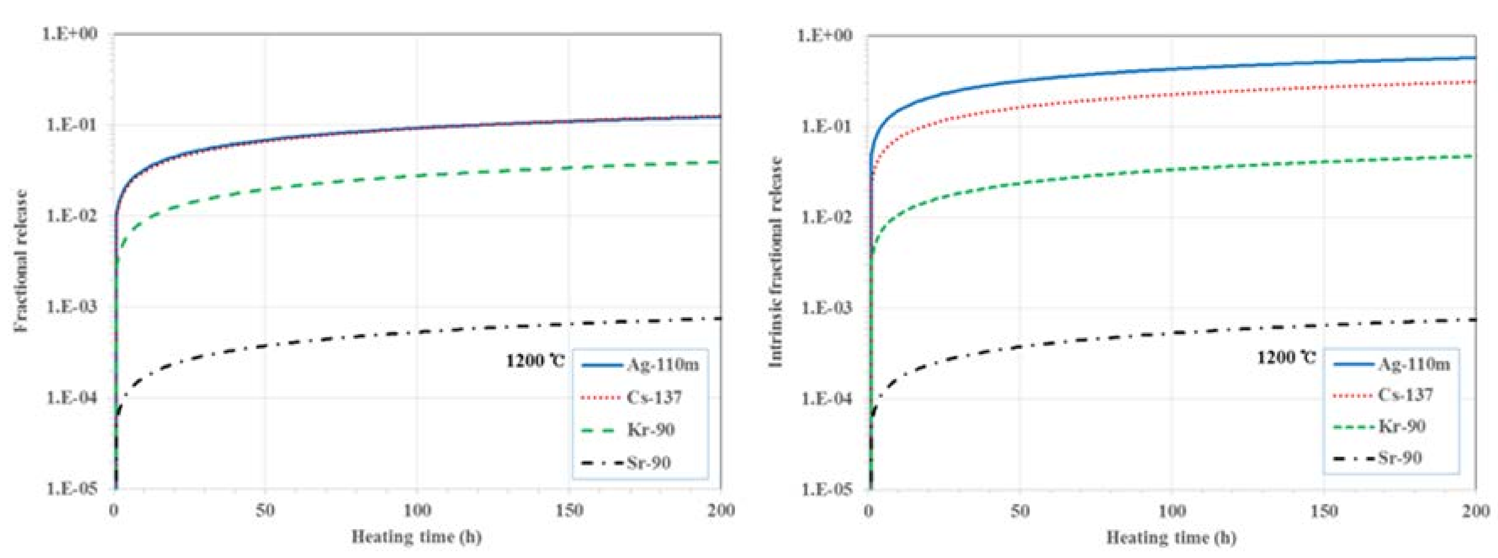
Calculation Results and Summary

- The mathematical fractional releases were classified systematically for radioactive and stable isotopes diffusing out of a fuel element under irradiation and accident conditions.
- Only when the diffusing medium is a single layer, it is possible to get an analytical form of fractional release. In other cases, the fractional releases must be calculated numerically.
- The choice of either the fractional release or the fractional release rate depends on whether the amount generated or the birth rate on which they are based on is more easily identifiable.
- According to the fractional releases calculated, the radioactive isotopes ^{110m}Ag, ^{137}Cs, ^{85}Kr and ^{90}Sr release from a spherical UO_2 kernel more in that order. The strontium release is negligible.

FR and its rate in a UO_2 kernel during irradiation



FR and its rate in a UO_2 kernel during heating



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