Transactions of the Korean Nuclear Society Spring Meeting
Jeju, Korea, May 23-24, 2019

Effect of the Unbalanced Ray Distribution in the the Method of Characteristics

Gil Soo Lee [a*], Jaejun Lee [a], Adriaan Buijs [b]
[a] *Korea Institute of Nuclear Safety,62 Gwahak-ro, Yuseong-gu, Daejeon, 34112, Korea*
[b] *McMaster University, Department of Engineering Physics, 1280 Main Street West, Hamilton, ON, L8S 4L8, Canada*
[*] *Corresponding author: gslee@kins.re.kr*

1. Introduction

The method of characteristics (MOC) is widely used in many nuclear design codes and can treat complex geometries[1,2]. These complex geometries are approximated by rectangles, which are results of ray tracing. Since ray tracing is usually performed on the cell or fuel assembly, the ray distribution in certain computational mesh in a unit domain of ray tracing can be unbalanced and the results can be biased.

This paper explains how unbalanced ray distribution happens and provides numerical results for the effect of unbalanced ray distribution. The weighted MOC (WMOC)[3] is briefly introduced and numerical results are presented. The numerical results show that the effect of ray distribution need to be considered carefully and the WMOC seems to be less affected by unbalanced ray distribution.

2. Methods and Results

2.1 The Method of Characteristics (MOC)

MOC performs 1-D transport calculations along the characteristic lines for approximating a computational mesh geometry. Ray tracing is required for MOC as shown in an example for the triangular mesh in Fig. 1. The triangular geometry is approximated by the shaded rectangles.

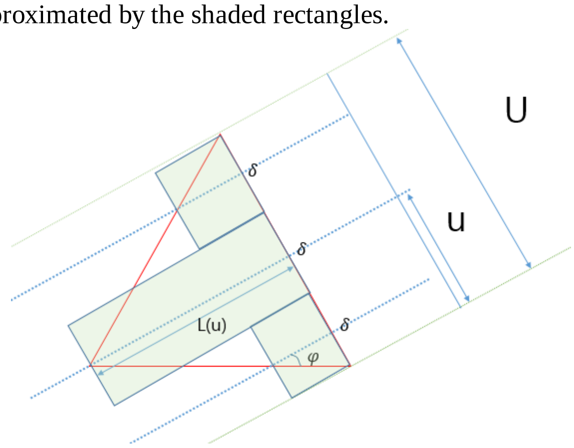


Fig.1. Example ray tracing for geometrical treatment

The discrete ordinate 1-D within-group neutron transport equation is well known and shown in the following :

$$\mu\frac{d\psi(x)}{dx} + \Sigma(x)\psi(x) = q(x). \qquad (1)$$

Assuming uniform cross sections and a flat source on a mesh, the solution of Eq.(1) for the characteristic line $L(u)$ in Fig.1 is obtained as follows :

$$\psi_{out}(u) = \psi_{in}(u)\exp\left(-\frac{\Sigma L(u)}{\mu}\right) + \frac{q}{\Sigma}\left(1-\exp\left(-\frac{\Sigma L(u)}{\mu}\right)\right). \qquad (2)$$

$$\bar{\psi}(u) = \frac{\mu\left(\psi_{in}(u) - \psi_{out}(u)\right) + qL(u)}{\Sigma L(u)}. \qquad (3)$$

The mesh-integrated angular flux can be calculated and approximated in conventional MOC calculations as follows :

$$\bar{\psi}V = \int_0^U \bar{\psi}(u)L(u)\,du \approx \sum_i \bar{\psi}(u_i)L(u_i)\delta. \qquad (4)$$

2.2 Unbalanced Ray Distribution

Ray tracing usually performed on a cell, fuel assembly, or whole problem, not on each computational mesh. Thus each computational mesh can have different ray distribution depending on its position. Fig.2 shows the sample ray distribution in a computational mesh.

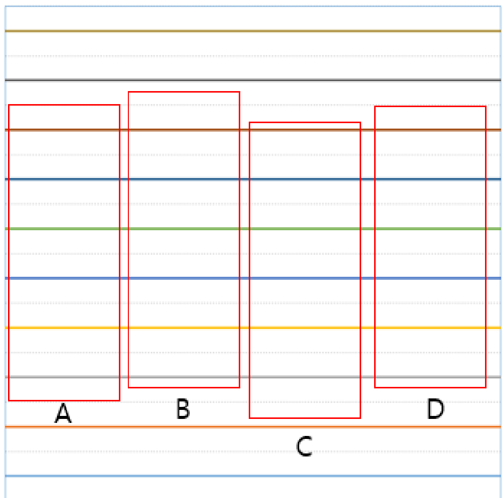


Fig.2 Sample ray distribution in a computational mesh

Each mesh in Fig.2 has 6 rays, and position of each ray of mesh A is center of each equally divided region. While the ray distribution in computational mesh A is balanced as explained, other meshes B, C, D are not balanced as shown in Fig.2. This unbalanced ray distribution can lead MOC calculation with Eq.(4) to biased result as in Fig.3.

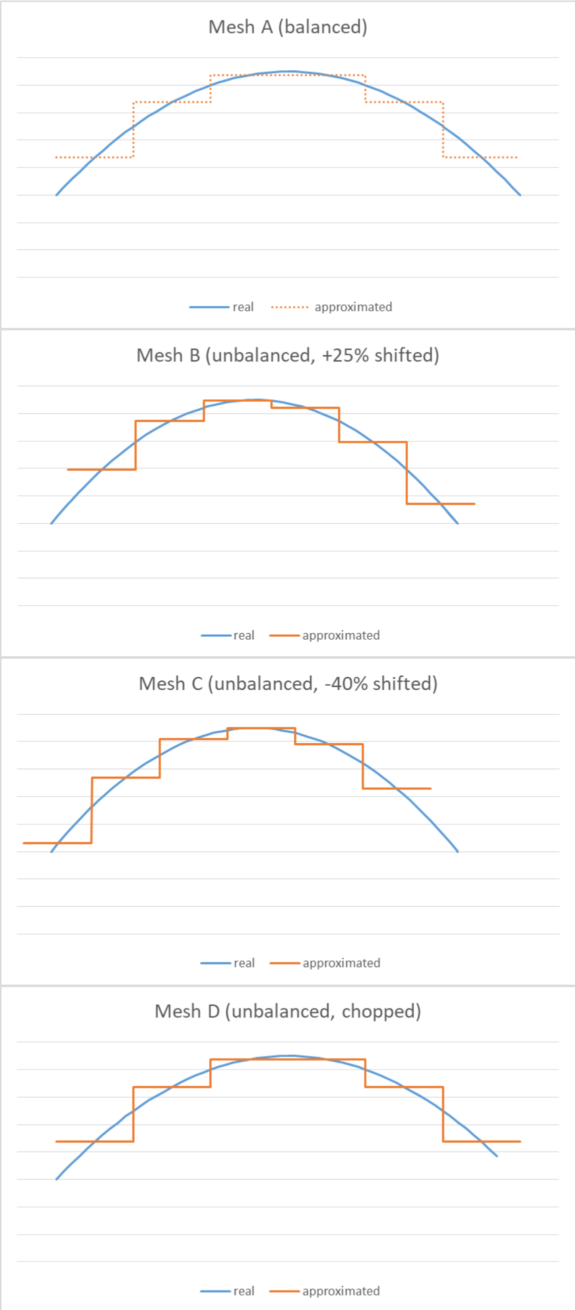


Fig.3 Expected results with unbalanced ray distribution

2.3 Weighted MOC

Eq.(4) using in the conventional MOC(CMOC) is the numerical integration form with weights of ray spacing. Since weights are determined without considering ray distribution, The effect of unbalanced ray distribution might be large. The weighted MOC(WMOC) calculates the weights considering geometry and ray distribution, which means that the effect of unbalanced ray distribution is expected to be smaller in WMOC.

In the WMOC schemes, line-averaged angular flux is approximated in the polynomial with spline interpolation. Starting from Eq.(4),

$$\int_0^U \bar{\psi}(u)L(u)\,du = \sum_j \int_{U_j}^{U_{j+1}} \bar{\psi}(u)L(u)\,du. \quad (5)$$

When the line-averaged angular flux is approximated with interpolation polynomial using Lagrange basis polynomial, Eq.(5) becomes

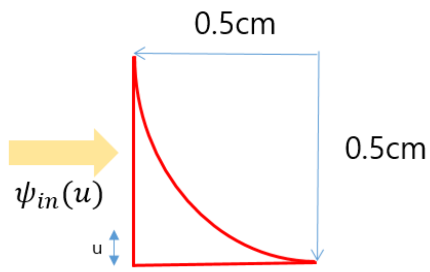
$$\sum_j \int_{U_j}^{U_{j+1}} \bar{\psi}(u)L(u)\,du \approx \sum_j \sum_i^{U_j \le u_i \le U_{j+1}} w_{i,j}\bar{\psi}(u_i) \quad (6)$$

$$w_{i,j} = \int_{U_j}^{U_j+\Delta_j} L(u)\,\ell_i(u)\,du, \quad (7)$$

and $\ell_i(u)$ is Lagrange basis polynomial.

2.4 Numerical Results

Let us consider test problem in Fig.4, which can be used as a computational mesh between square and ring geometries.



Total Cross Section	1.0
Scattering Cross Section	0.0
Internal neutron source	0.0
Incoming angular flux	1.0 for case 1 2.0-u/0.5 for case 2

Fig.4 Description of the test problem

The mesh integrated angular fluxes whose directional cosines to x, y, and z axis are 1.0, 0.0 and 0.0, respectively, were calculated with different ray distributions considering ray shifting. Two incoming angular flux assumptions were used and the results of the mesh integrated angular flux are shown in Figs.5 and 6. The numerical errors from comparison results with reference solution with very fine calculation(0.00002cm subinterval and 6 points Gaussian quadrature within subinterval in Eq.(5)) are presented as functions of ray distribution shifting (r), where ray distribution is as follows :

$$u_i = \frac{(i-0.5+r)U}{N}. \quad (8)$$

In Eq.(8), U represents the length of the normal direction to the neutron flight as in Fig. 1, N is the number of rays in a mesh (i.e., δ=U/N) and r represents how much ray distribution is shifted. For example, r for meshes B and C in Figs. 2 and 3 are 0.25 and -0.40, respectively. The ray distribution is balanced when r=0. For the case 1, the results using track length renomalization which is usually used in the MOC calculation were not affected unbalanced ray distribution significantly, while CMOC without effort for preserving volume of computational mesh affected significantly.

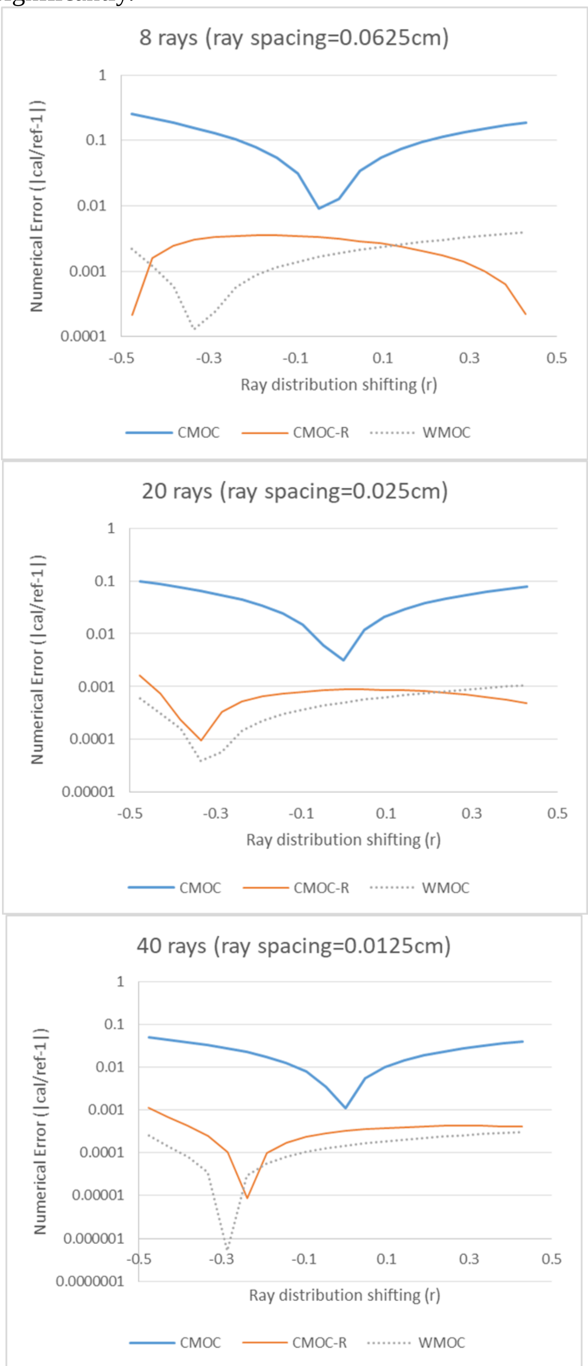


Fig.5 Numerical Errors (case1) for various ray distributions

The track length renormalization preserves the volume of the computational mesh, but it uses same weight (ray spacing) for every ray in a mesh, which can cause inaccurate solution when there is a peak in mesh boundary. The incoming angular flux distribution in case 2 is one of the examples.

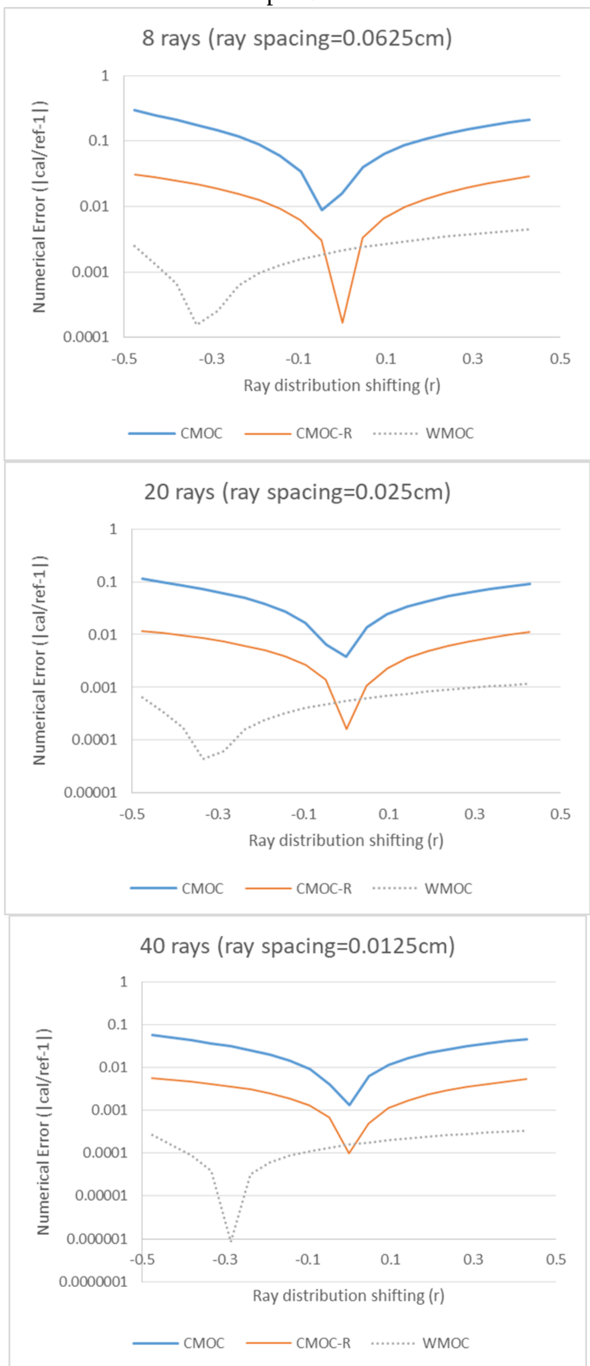


Fig.6 Numerical Errors (case2) for various ray distributions

Fig.6 shows that CMOC with track length renormalization can be affected by unbalanced ray distribution, as expected. In a real applications, computational mesh is determined to be little angular flux change within a mesh. Thus, it is expected that these effect is not significant in global, while there is a possibility that unbalanced ray distribution affects the solution locally. Also sensitivity studies on the ray spacing should be performed carefully. The numerical errors may seem to be small enough when only balanced ray distribution is assumed. Unlike CMOC results, the WMOC results which use pre-calculated weights for ray position seem to be less affected as shown in Figures 5 and 6.

To quantify the effect of unbalanced ray distribution in CMOC and WMOC, lots of case studies using different shapes of mesh with different angles need to be followed.

3. Conclusions

The computational mesh can have unbalanced ray distribution in the MOC calculation, and the unbalanced mesh can distort the calculation result. Numerical results of test problem show that the calculated results can be affected by the ray distribution.

The weighted MOC calculation provided reliable results less affected by ray distribution for the test problem. To ensure the effect of unbalanced ray distribution, lots of case studies need to be followed.

REFERENCES

[1] M. A. Jessee et al, "POLARIS -2D LIGHT WATER REACTOR LATTICE PHYSICS MODULE," SCALE 6.2 Manual, 2018.

[2] K. S. Kang et al., "Transport Lattice Code KARMA 1.1," *Trans. of the KNS autumn meeting,* 2009.

[3] G. S. Lee et al., "Weighted Average Solution of the Method of Characteristics," *Trans. of the ANS annual meeting,* 2019. (will be presented)