Application of Nodal Equivalence Theory in CANDU analysis with APEC Method

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1. Introduction

Coarse-mesh nodal method is very successful to the light water reactors (LWR) due to the nodal equivalence theory [1]. In the generalized equivalence theory, neutron flux is considered discontinuous at the interfaces between the fuel assemblies to preserve the net current. In spite of the success of the nodal methods in LWRS, a simple coarse-mesh finite difference method (FDM) is still used for the CANDU reactor analysis [2]. It is well known that coarse-mesh FDM provides very nice results but it is reported that a fine-mesh FDM may results in a worse accuracy [2]. Several attempts have been made to apply the simplified equivalence theory commonly used in PWR core analysis to CANDU core, but not very successful. In this study, the albedo-corrected parametrized equivalence constants (APEC) method [3] is introduced to consider the nodal equivalence theory in the CANDU analysis. This method is described herein, and representative results of the method calculations are presented.

2. Application of APEC method to CANDU core analysis

In the APEC method, two-group cross section (XS) and discontinuity factor (DF) of the standard CANDU fuel lattice are functionalized in terms of node-wise current-to-flux ratio (CFR) and surface CFRs simultaneously. The XS and DF are updated during nodal calculations using the APEC functions and CFR values of each fuel lattice inside the core. The corrected group constants will provide the nodal equivalence of homogenized fuel assembly, which will leads to an accurate core analysis.

2.1 Functionalization of fuel assembly XS

It was demonstrated that, if a fuel assembly is symmetric, then two-group homogenized XS have strong relationship with assembly-wise CFR at assembly m, defined as below:

$$CFR_g^m = \frac{\sum_s J_g^{m,s}}{\sum_s \phi_g^{m,s}} \tag{1}$$

where the numerator is the summation of the g^{th} group outward net currents of assembly m and the denominator is the summation of the g^{th} group surface fluxes of assembly m. An assembly-wise CFR is a more explicit definition of neutron leakage of the fuel assembly and is normalized parameter to represent surface integrated leakage of the assembly. The CFR could be positive or negative or even zero. Positive CFR refers to the net outgoing neutron leakage and negative CFRs refers to the net incoming neutron from the neighboring fuel assembly. Therefore, spectrum will be change at the interface of fuel assembly due to these positive or negative CFRs and homogenized few-group constants should be different from the single assembly calculations.

In the standard APEC method, the XS changes due to the non-zero leakage are functionalized with CFR as follows for node m reaction x and group g :

$$\Sigma_{x,g}^m = \Sigma_{x,g}^0 + \Delta\Sigma_{x,g}^m \tag{2}$$

$$\Delta\Sigma_{x,g}^m = a_{x,g} CFR_g^m + C_{x,g} \tag{3}$$

where, $\sum_{x,g}^m$ is the corrected XS, $\sum_{x,g}^0$ is the XS from single lattice calculations, and $\Delta\sum_{x,g}^m$ is the XS change in Eq. (2). To obtain the coefficients, $a_{x,g}$ and constants $C_{x,g}$ the change in XS data as function of different CFR values are required. Various CFR values can be obtained from a color-set model where different fuel assemblies surround fuel assembly. Therefore, XS change due to CFR can be functionalized by fitting APEC functions to the color-set calculations results. The function fitting is done by using the least-square method.

2.2 Functionalization of fuel assembly DF

According to the nodal equivalence theory, the accurate nodal solution is guaranteed only when both XS and DF are corrected simultaneously [1]. Therefore, functionalization of DF is also important to improve nodal accuracy. In the previous study, the DF of specific surface has strong relation with the surface-wise CFR [4], such as:

$$CFR_g^s = \frac{J_g^s}{\phi_g^s} \tag{4}$$

Therefore, ΔDF_g^s is defined as a linear function of surface CFR:

$$DF_g^s = ADF_g + \Delta DF_g \tag{5}$$

$$\Delta DF_g^s = a_g^s \times CFR_g^s + C_g^s \tag{6}$$

where, ADF_g is the assembly discontinuity factor (ADF). ADF can be evaluated as the ratio of

heterogeneous surface flux to the node-average flux in the single lattice calculation results.

As DFs are strongly dependent on the position, inner core, outer core and the surface relative to the reflector or the absorbing materials like liquid zone controller (LZC) should be considered during functionalization.

2.3 Lattice and Color-set calculation for APEC function

Among the several reactivity devices in CANDU, only LZC was considered in this study. To represent the vertically located LZC in a 2-D core, LZC was modeled as a 1.0827 cm thick plate. The thickness of LZC plate was evaluated to be equivalent to the original LZC in terms of reactivity. Figures 1 and 2 show the standard CANDU lattice and LZC loaded CANDU lattice. Two color-set models with different LZC locations were considered as shown in Figs. 3 and 4. In order to determine the burnup dependent APEC functions, fuel lattice of various burnup was loaded in the color-set model. A standard CANDU lattice was depleted from fresh to 12 GWd/tU for generating burnup dependent XSs and DFs with reflective boundary condition. Monte Carlo code Serpent2 [~~6~~5] performed depletion calculation and color-set calculation.

It is important to note that the XS of LZC loaded lattice is strongly dependent to the position of LZC. Therefore, the APEC function for XS correction of LZC loaded lattice was considered as both burnup and position dependent. There are three categories of lattice position such as inner side, outer side and central LZC depending on the relative position of LZC as shown in Figs. 3 and 4. For the LZC loaded lattice, DF APEC functions were prepared on three surfaces such as LZC side, LZC opposite side and perpendicular to LZC as shown in Fig. 2.

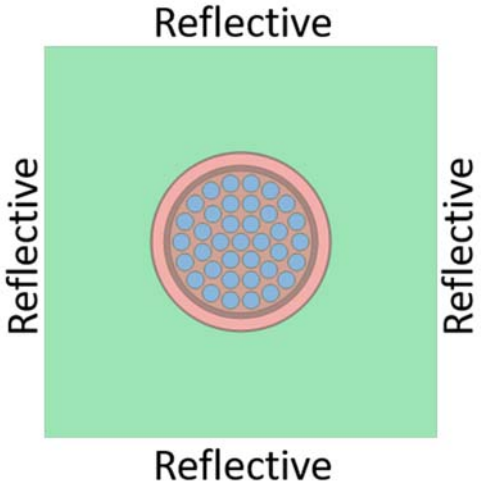

Fig. 1. Standard lattice model in CANDU reactor.

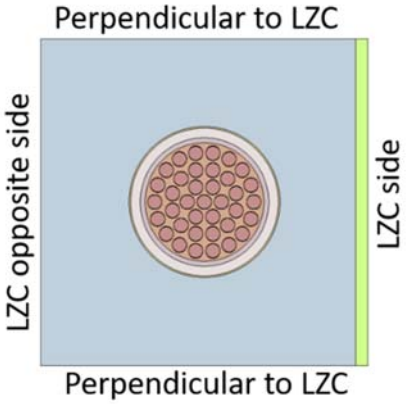

Fig. 2. LZC loaded CANDU lattice.

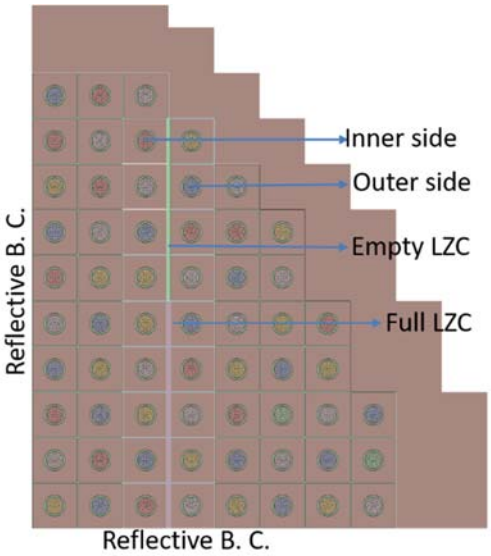

Fig. 3. Color-set for APEC function.

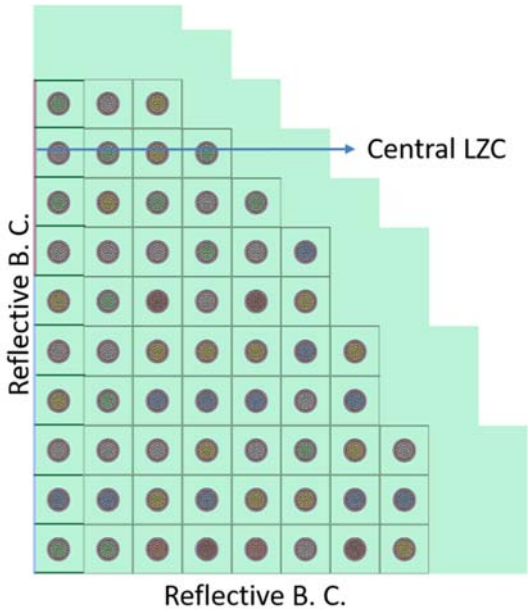

Fig. 4. Color-set for APEC function (LZC at center).

Serpent provides the net current boundary condition for every lattice in the color-set. Then 1x1 NEM equivalent DF is calculated by solving the fixed source problem. These 1x1 NEM equivalent DF is functionalized to determine the APEC function for DF correction. As DFs are strongly dependent on positions, DF APEC function is generated on both burnup and position dependent. Figures 5 and 6 show the APEC function for XS and DF respectively.

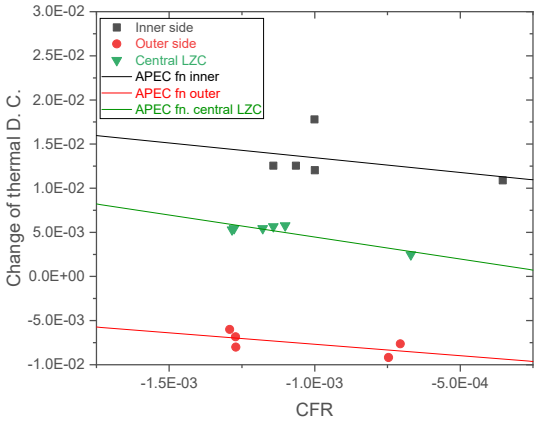

Fig. 5. APEC function for the thermal diffusion coefficient of the LZC loaded lattice.

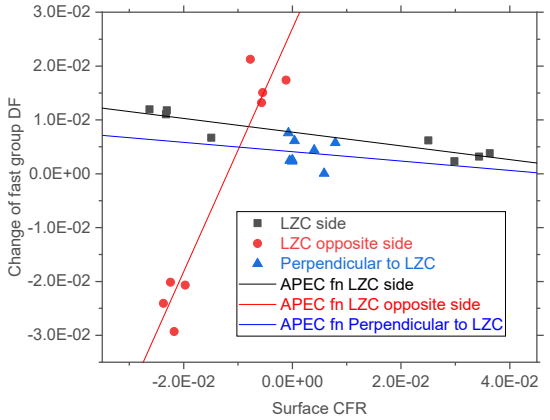

Fig. 6. APEC function for the fast group DF of the LZC loaded lattice.

3. Numerical Results

A 2-D CANDU core benchmark problem and its 3 variants are considered to investigate the impacts of the nodal equivalence theory. Four variants are loaded with different burnup distribution of fuel from zero to 12 GWd/tU and is surrounded by a heavy water reflector. In these problems, only LZC reactivity control devices are considered. The APEC method is implemented in an in-house nodal expansion method (NEM) code [3]. For the comparison, nodal calculation were done in the following way: 1) the conventional FWC with no DF, 2) the conventional FWC with ADF, 3) B1 critical spectrum corrected XS, 4) XS correction by APEC function only, 5) Simultaneous correction of XSs and DFs by APEC function. The nodal analysis results were summarized in Table I for both 2x2 and 1x1 node per lattice.

Table I: Results of the Nodal analysis

XS	DF	k-eff	$\Delta\rho$ (pcm)	Power error, (%)	
				RMS	Max.
2-D CANDU benchmark					
Reference		1.012023	-	-	-
FWC*	no DF	1.011972	5.06	3.89	12.09
FWC*	ADF	1.012024	0.08	1.64	5.36
FWC*	ADF	1.012631	38.36	1.53	5.12
FWC	no DF	1.012136	10.97	4.84	15.33
FWC	ADF	1.012158	13.19	1.44	5.45
B_1	ADF	1.011766	25.09	1.37	5.07
APEC	ADF	1.012199	17.17	1.74	5.21
APEC	APEC	1.012079	5.51	0.58	1.33
Variant 1					
Reference		1.016638	-	-	-
FWC*	no DF	1.016559	7.64	3.84	11.77
FWC*	ADF	1.016614	2.34	1.66	5.33
FWC*	ADF	1.016267	35.86	1.55	4.83
FWC	no DF	1.016713	7.26	4.79	14.90
FWC	ADF	1.016740	9.84	1.39	5.12
B_1	ADF	1.016395	23.51	1.36	4.56
APEC	ADF	1.016808	16.45	1.76	4.20
APEC	APEC	1.016632	0.56	0.57	1.62
Variant 2					
Reference		1.014296	-	-	-
FWC*	no DF	1.014309	1.32	3.65	12.13
FWC*	ADF	1.014361	6.37	1.79	4.82
FWC*	ADF	1.014037	25.15	1.62	4.35
FWC	no DF	1.014451	15.07	4.57	14.27
FWC	ADF	1.014477	17.59	1.42	4.60
B_1	ADF	1.014154	13.78	1.31	4.13
APEC	ADF	1.014574	27.05	1.80	4.81
APEC	APEC	1.014429	12.98	0.51	1.34
Variant 3					
Reference		1.013770	-	-	-
FWC*	no DF	1.013710	5.93	3.94	12.13
FWC*	ADF	1.013702	6.68	1.62	5.10
FWC*	ADF	1.013414	34.65	1.54	4.92
FWC	no DF	1.013872	9.86	4.89	15.05
FWC	ADF	1.013835	6.26	1.37	4.94
B_1	ADF	1.013549	21.57	1.34	4.69
APEC	ADF	1.013981	20.52	1.66	4.60
APEC	APEC	1.013825	5.31	0.60	1.80

*2x2 node per assembly

It is observed that the reactivity error is not a big issue in these four cores but the power distribution error is very large. The root mean square (RMS) values of the relative lattice power distribution error are over 3.50% with maximum error over 12% for all four variants. Simultaneous correction of XS and DF using the APEC method reduces relative error of lattice power

distribution that RMS value is less than 1% and the maximum error is less than 2%. It is also observed from the summary table that the simplified equivalence theory (SET) using FWC and ADF can significantly reduce error in terms of both reactivity and power for all four variants.

Relative errors of both FWC and APEC corrected XSs of 2-D benchmark are presented in Table II. Simultaneous application of the APEC XS correction and APEC DF correction eliminated almost all errors from the XSs. Figures 7, 8 and 9 show 1/4th part of whole core the power distribution error before and after the APEC-based SET application for 2-D benchmark.

Table II: RMS errors of XSs for 2-D benchmark.

	Relative error (%)					
	FWC		APEC XS		APEC XS & DF	
	RMS	Max.	RMS	Max.	RMS	Max.
D_1	0.26	0.90	0.05	0.19	0.02	0.10
D_2	0.17	0.58	0.03	0.13	0.01	0.06
Σ_{a1}	0.55	1.94	0.09	0.36	0.05	0.18
Σ_{a2}	0.35	1.27	0.17	1.09	0.13	0.83
$\nu\Sigma_{f1}$	2.76	2.24	0.47	1.79	0.17	0.81
$\nu\Sigma_{f2}$	0.42	1.58	0.17	1.08	0.12	0.79
$\Sigma_{s1\rightarrow2}$	3.47	12.45	0.77	2.79	0.51	1.53
$\Sigma_{s2\rightarrow1}$	8.42	30.30	1.74	6.18	0.68	2.80

Fig. 7. Power error (%) from the reference and FWC with no DF for 2-D benchmark.

Fig. 8. Power error (%) from reference and FWC with ADF for 2-D benchmark.

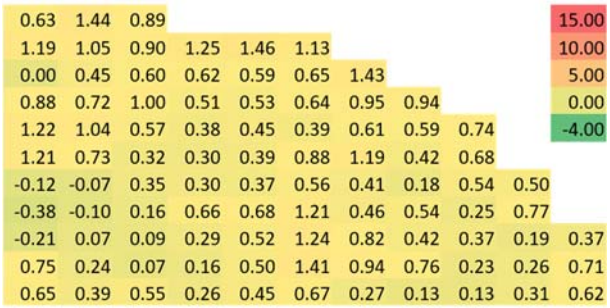

Fig. 9. Power error (%) from reference and APEC corrected XSs and DFs for 2-D benchmark.

4. Conclusions

The nodal equivalence theory has been used for CANDU reactor core analysis. In order to satisfy the nodal equivalence and obtain the accurate solution, APEC method applied for correcting the XSs and DFs. In the APEC method both burnup and position dependent functions were determined. A realistic CANDU core with reactivity device LZC has been considered in this analysis. The 2-D CADNU analysis indicates that the standard nodal equivalence theory improves the accuracy of the nodal solution with the APEC application.

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