# Sensitivity and Uncertainty Analysis on UAM Benchmark Exercise I-2 with MCS

Yunki Jo<sup>a</sup>, Peng Zhang<sup>a</sup>, Hyunsuk Lee<sup>a</sup>, and Deokjung Lee<sup>a\*</sup>

<sup>a</sup>Department of Nuclear Engineering, Ulsan National Institute of Science and Technology 50 UNIST-gil, Ulsan, 44919, Republic of Korea

\*Corresponding author: deokjung@unist.ac.kr

## 1. Introduction

Sensitivity and uncertainty (S/U) analysis is important for design and safety analysis of nuclear reactor. It can be used to introduce appropriate safety margins and define the top contributors to the uncertainties. The OECD/NEA Uncertainty Analysis in Modelling (UAM) benchmark for design, operation, and safety analysis of light water reactors (LWRs) has been developed to quantify the uncertainties in LWR systems [1].

Recently, a new generalized perturbation theory formulation has been developed by Sung Hoon Choi et al [2-3]. It can be used to calculate sensitivity coefficients not only for the ratio of two reaction rates, but a single reaction rate also. The new GPT formulation has been implemented in MCS, and verification has been performed for TMI-1 PWR pin-cell problem [4].

In this paper, S/U analyses have been performed with MCS on 2-D TMI-1 PWR fuel assembly and mini-core problems. There are two different states, which are Hot full power (HFP) unrodded and HFP rodded cases, for the fuel assembly problems. HFP state is considered for the mini-core problem.

# 2. Methods and Results

The GPT formulation and sandwich rule used in S/U analysis are briefly introduced in Section 2.1. The models used in this study are introduced in Section 2.2. The comparison of numerical results with different covariance data library is presented in Section 2.3.

### 2.1 Methodology

A general response Q with the form of single reaction rate is shown in Eq. (1). It can be defined as a response function for the new GPT formulation [3].

$$Q = \frac{\langle \mathbf{R}_1 S \rangle}{\langle S \rangle},\tag{1}$$

where  $\mathbf{R}_1$  is a response operator of the general response Q, and S is the fission source density. The brackets,  $\langle \rangle$ , indicate an inner product or an integral over the phase space. The sensitivity coefficient of Eq. (1) can be written as [3]

$$S_x^Q = \frac{\Delta Q}{Q} = \frac{\left\langle \Delta \mathbf{R}_1 S \right\rangle}{\left\langle \mathbf{R}_1 S \right\rangle} + \frac{\left\langle \mathbf{R}_1 \Delta S \right\rangle}{\left\langle \mathbf{R}_1 S \right\rangle} - \frac{\left\langle \Delta S \right\rangle}{\left\langle S \right\rangle}.$$
 (2)

where *x* is the perturbed input parameter such as neutron cross section.

A general response with the ratio of two reaction rates and its sensitivity coefficient are expressed as Eq. (3) and Eq. (4), respectively.

$$Q = \frac{\langle \mathbf{R}_1 S \rangle}{\langle \mathbf{R}_2 S \rangle},\tag{3}$$

$$S_{x}^{Q} = \frac{\langle \Delta \mathbf{R}_{1} S \rangle}{\langle \mathbf{R}_{1} S \rangle} - \frac{\langle \Delta \mathbf{R}_{2} S \rangle}{\langle \mathbf{R}_{2} S \rangle} + \frac{\langle \mathbf{R}_{1} \Delta S \rangle}{\langle \mathbf{R}_{1} S \rangle} - \frac{\langle \mathbf{R}_{2} \Delta S \rangle}{\langle \mathbf{R}_{2} S \rangle}, \qquad (4)$$

where  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are response operator in numerator and denominator of the general response Q, respectively.

The uncertainty of response Q can be calculated with a sandwich rule.

$$u_{Q}^{2} = S_{x_{1}}^{Q} C_{x_{1}, x_{2}} \left( S_{x_{2}}^{Q} \right)^{T}, \qquad (5)$$

where  $C_{x_1,x_2}$  is the relative covariance matrix between the input parameter  $x_1$  and  $x_2$ . The 44-group ENDF/B-VII.1 and 44-group SCALE6.1 covariance data are used in this study. The 44-group ENDF/B-VII.1 covariance data are produced by NJOY.

#### 2.2 Model Description

The 2-D  $15 \times 15$  TMI-1 PWR fuel assembly with two different states (HFP-unrodded and HFP-rodded) and 2-D  $3 \times 3$  mini-core problem at HFP [1] are used in this study. Fig. 1 shows the octant of TMI-1 PWR fuel assembly for the rodded case.

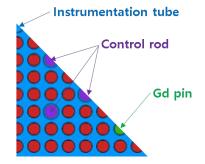


Fig. 1. Design of 1/8 of TMI-1 PWR fuel assembly for the rodded case.

The 2-D  $3 \times 3$  mini-core problem consists of one rodded assembly and eight unrodded assemblies. The rodded assembly is located at the center and eight unrodded assemblies are surrounding it. Fig. 2 shows the octant of TMI-1 PWR mini-core problem.

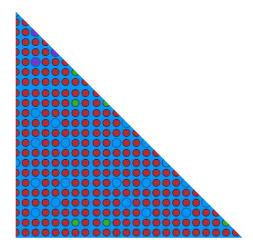


Fig 2. Design of 1/8 of TMI-1 PWR mini-core problem.

## 2.3 Numerical Results

S/U analysis has been performed for the TMI-1 assembly and mini-core problems with MCS. The ENDF/B-VII.1 nuclear data library is used as the continuous-energy neutron cross section. The 44-group ENDF/B-VII.1 covariance data and 44-group SCALE6.1 covariance data are used for uncertainty quantification. Table I and Table II show the uncertainty summary for TMI-1 HFP-unrodded and HFP-rodded assembly cases.

Table I: Uncertainty summary for TMI-1 PWR fuel assembly with HFP and unrodded case

	MCS/GPT	
Covariance	44G ENDF	44G
Library	/B-VII.1	SCALE6.1
Eigenvalue	0.70884	0.44503
$\Sigma_{a1}$	0.82065	0.86436
$\Sigma_{a2}$	0.28141	0.25160
$\Sigma_{f1}$	0.30238	0.32370
$\Sigma_{f2}$	0.29591	0.29300
$\nu \Sigma_{f1}$	0.62026	0.44131
$\nu \Sigma_{f2}$	0.72223	0.40999
$\phi_1$	4.25534	2.92006
$\phi_2$	1.33657	0.96549

Table II: Uncertainty summary for TMI-1 PWR fuel assembly with HFP and rodded case

	MCS/GPT	
Covariance	44G ENDF	44G
Library	/B-VII.1	SCALE6.1
Eigenvalue	0.69670	0.46031
$\Sigma_{a1}$	0.87629	0.90205

$\Sigma_{a2}$	0.24332	0.21865
$\Sigma_{f1}$	0.30141	0.32058
$\Sigma_{f2}$	0.29704	0.29183
$\nu \Sigma_{f1}$	0.60937	0.44011
$\nu \Sigma_{f2}$	0.70688	0.40368
$\phi_1$	4.90049	3.53055
$\phi_2$	2.32630	1.62360

Fig. 3 shows the eigenvalue uncertainty as a function of nuclide-reactions for TMI-1 HFP-unrodded assembly case. The top six contributors to the eigenvalue uncertainty are shown in Fig. 3. The <sup>235</sup>U nubar cross section has the largest effect on the eigenvalue uncertainty in both ENDF/B-VII.1 case and SCALE6.1 case.

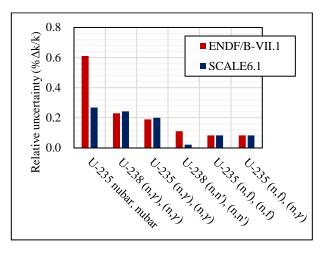


Fig. 3. Eigenvalue uncertainty for TMI-1 HFP-unrodded assembly case.

There are large discrepancies in the eigenvalue uncertainties between the cases using ENDF/B-VII.1 covariance data and SCALE6.1 covariance data. The discrepancies are mainly caused by the different source of <sup>235</sup>U nubar covariance data. SCALE6.1 covariance data for <sup>235</sup>U nubar are consistent with JENDL-3.3 data library [5], and it was decreased from 0.7% to 0.3% in energy range below 0.5 eV [5].

 Table III: Eigenvalue uncertainty for TMI-1 PWR mini-core with HFP

	MCS/GPT	
Covariance	44G ENDF	44G
Library	/B-VII.1	SCALE6.1
Eigenvalue	0.71159	0.44534

Table III shows the eigenvalue uncertainty for TMI-1 PWR mini-core at HFP state. The eigenvalue uncertainty with ENDF/B-VII.1 covariance data is 0.7%, and it is 0.45% with SCALE6.1 covariance data. It shows a similar tendency with the TMI-1 HFP assembly cases.

### 3. Conclusions

S/U analyses have been performed with MCS on the 2-D TMI-1 PWR fuel assembly with HFP-unrodded and HFP-rodded states, and 2-D mini-core at HFP state. ENDF/B-VII.1 nuclear data library is used as a continuous-energy neutron cross section. Two different covariance data, which are 44-group ENDF/B-VII.1 and 44-group SCALE6.1, are used for the uncertainty quantification. There are large discrepancies between the cases using ENDF/B-VII.1 covariance data and SCALE 6.1 covariance data. The eigenvalue uncertainty with ENDF/B-VII.1 covariance data is 0.7%, and it is 0.45% when SCALE6.1 covariance data is used. The discrepancies are caused by the <sup>235</sup>U nubar covariance data in SCALE6.1 which are consistent with JENDL-3.3 library. ENDF/B-VII.1 covariance data for <sup>235</sup>U nubar is overestimated. The uncertainty results with MCS are reasonable for TMI-1 PWR fuel assembly and mini-core problems.

### ACKNOWLEDGEMENTS

This work was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIT). (No. NRF-2017M2B2A9A02049916)

### REFERENCES

[1] K. Ivanov, M. Avromova, S. Kamerow, I. Kodel, E. Sartori, E. Ivanov and O. Cabellos, Benchmarks for Uncertainty Analysis in Modelling (UAM) for the Design, Operation and Safety Analysis of LWRs, NEA/NSC/DOC, 2012

[2] S. H. Choi and H. J. Shim, Memory-efficient calculations of adjoint-weighted tallies by the Monte Carlo Wielandt method, *Ann. Nucl. Energy*, Vol. 96, p. 287-294, 2016.

[3] S. H. Choi, H. J. Shim and C. H. Kim, Development of Generalized Perturbation Theory Algorithms for Monte Carlo Eigenvalue Calculations, *Nucl. Sci. Eng.*, Vol. 189, p. 171-187, 2018.

[4] Y. Jo, J. Yu, H. Lee and D. Lee, Sensitivity and Uncertainty Analysis of UAM TMI-1 Pin-cell with UNIST Monte Carlo Code MCS, *Trans. of Korean Nuclear Soc. Spring Meeting*, Jeju, Korea, May 17-18, 2018.

[5] B. T. Rearden and M. A. Jessee, SCALE Code System, ORNL/TM-2005/39, Oak Ridge National Laboratory, 2016.