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The Rationale of Leakage Parameters adopted in Leakage Feedback Method

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1. Introduction

A conventional two-step core calculation procedure [1] employing B_1 critical leakage correction has been adopted as the standard procedure for reactor design analyses for a long time because of its indispensable merits due to far lower cost than Direct Whole Core (DWC) calculations and acceptable accuracy. However, since the neighbor-irrelevant correction of B_1 induces biased reactivity errors and checkerboard-like power distribution errors, Smith mentioned that "B_1 spectrum calculation should not be used in commercial LWR analysis" [2]. Thus, alternatives have been developed.

One of the method, Leakage Feedback Method (LFM) [3], employs an iterative leakage feedback correction on few-group constants (GC). The GCs of fuel assemblies (FA) are functionalized with respect to both two group leakage-to-removal fractions (LF) and spectral index shift (SIS) as an additional parameter for peripheral assemblies (PA). The method proved its validity and its performance for various LWR cores [4] but a proper rationale of the functionalization formula is not clearly verified yet. In this study, the fitting formula of GCs for LFM is introduced with the explanation of how each leakage parameter impacts on GCs. Moreover, the fitting formula is compared with a similar method, Albedo-corrected Parameterized Equivalence Constant (APEC) [5] which also uses leakage-dependent GCs. For the assessment, the errors of evaluated GCs from fitting formulas for 2D APR1400 core problem are analyzed.

2. Functionalization of Few-group Constants

2.1 Advanced Leakage Feedback Method

In LFM, there are two key ideas: 1) consideration of the effect of the cross-group leakage and 2) a special treatment on PAs (PAT). For the functionalization of few-GCs, the leakage-to-removal fractions and the spectral index (fast-to-thermal flux ratio) shift defined as following equations are used as fitting parameters:

$$l_G = \left(\sum_{surf} J_G^{surf} A_{surf} \right) / \left(\int_{Asy} \Sigma_{r,G} \phi_G dV \right), \quad (1)$$

$$\Delta\Gamma = \frac{\phi_1}{\phi_2} - \frac{\phi_1^{SA}}{\phi_2^{SA}}. \quad (2)$$

Note that G stands for the few-group index (two-group in general), superscript SA for the GCs generated from a single assembly infinite medium condition (SA), and subscript 1 and 2 stand for fast and thermal group index to differentiate with three-group in Section 2.4. For FAs, GCs are functionalized using the formula written in Eq. (3) while setting γ_G as zero for inner assemblies (IA) and non-zero for PAs. The difference in formula is intended to properly reflect the different leakage characteristic of PAs due to the presence of reflectors in neighbor.

$$\frac{\bar{\Sigma}_G - \bar{\Sigma}_G^{SA}}{\bar{\Sigma}_G^{SA}} = \alpha_G l_1 + \beta_G l_2 + \gamma_G \Delta\Gamma. \quad (3)$$

2.2 APEC Method

APEC is another alternative method to B_1 that uses leakage-dependent GCs. As it can be noticed from its name, albedo-like leakage parameter, current-to-flux ratios (CFR) defined as Eq. (4), is used.

$$\tilde{l}_G = \sum_{surf} J_G^{surf} / \sum_{surf} \phi_G^{surf}. \quad (4)$$

And the fitting formulas used in APEC are as follows:

$$\begin{cases} \bar{\Sigma}_1 - \bar{\Sigma}_1^{SA} = \alpha_1 \tilde{l}_1 + \beta_1 \tilde{l}_2 + \gamma_1 \Delta\Gamma + \delta_1 \\ \bar{\Sigma}_2 - \bar{\Sigma}_2^{SA} = \beta_2 \tilde{l}_2 + \beta_2' \tilde{l}_2^2 + \gamma_2 \Delta\Gamma + \delta_2 \end{cases}. \quad (5)$$

Note that in APEC, δ_G is zero for IAs and γ_G is zero for PAs.

Through several updates of APEC, the method adopts similar strategies with LFM but there are many differences: 1) target of correction (LFM: relative GC difference, APEC: absolute GC difference), 2) leakage parameters (LFM: reaction rate based LFs, APEC: albedo-like CFRs), 3) different formulas for thermal GCs (note that β_2 is decisive term for both formulas), 4) totally opposite use of SIS for PA treatment (LFM: γ_G is non-zero for PAs, APEC: γ_G is zero for PAs). Also, there are minor differences such as the configuration of colorset problems for obtaining values for the fitting.

2.3 Spatial and Spectral Errors

The error of few-GCs can be separated into two parts, spatial and spectral errors. They can be derived from the condensation equation of homogenized XS:

$$\bar{\Sigma}_G^* = \sum_{g \in G} \bar{\Sigma}_g^* \tilde{\phi}_g^* \quad (6)$$

where g for multi-group and G for the few-group index, asterisk for true (reference) values which can be obtained

from core calculations, the bar for homogenized values, and the tilde for normalized values in unity which means:

$$\sum_{g\in G}\tilde{\phi}_g = 1. \tag{7}$$

The difference of the homogenized XS and the spectrum between SA and core condition can be written as:

$$\Delta\bar{\Sigma}_g = \bar{\Sigma}_g^{*} - \bar{\Sigma}_g^{SA}, \tag{8}$$

$$\Delta\tilde{\phi}_g = \phi_g^{*} - \phi_g^{SA}. \tag{9}$$

Inserting Eq. (8) and (9) in Eq. (6) results in:

$$\begin{aligned}\bar{\Sigma}_G^{*} &= \sum_{g\in G}\left(\bar{\Sigma}_g^{SA} + \Delta\bar{\Sigma}_g\right)\left(\tilde{\phi}_g^{SA} + \Delta\tilde{\phi}_g\right)\\ &= \sum_{g\in G}\begin{pmatrix}\bar{\Sigma}_g^{SA}\tilde{\phi}_g^{SA}\\ +\tilde{\phi}_g^{SA}\Delta\bar{\Sigma}_g + \bar{\Sigma}_g^{SA}\Delta\tilde{\phi}_g + \Delta\bar{\Sigma}_g\Delta\tilde{\phi}_g\end{pmatrix}\\ &= \bar{\Sigma}_G^{SA} + E_G^{Hom} + E_G^{Con} + O(E^2).\end{aligned} \tag{10}$$

In traditional methods, leakage corrections on GCs are commonly done in two steps: 1) rehomogenization technique [6] before the energy condensation and 2) spectral correction [7] in a homogenized medium to catch spatial and spectral effects. However, in recent methods, these two effects are considered at once. In LFM, the target of functionalization as shown in Eq. (3) is the sum of spatial, spectral and, the cross errors:

$$\frac{\bar{\Sigma}_G^{*} - \bar{\Sigma}_G^{SA}}{\bar{\Sigma}_G^{SA}} = \tilde{E}_G^{Hom} + \tilde{E}_G^{Con} + \tilde{O}(E^2). \tag{11}$$

Since the whole error can be obtained from colorset results, even the cross effects are counted in the recent methods.

2.4 Rationale of cross-group leakage dependency

Because of the outstanding performance of the advanced leakage correction method LFM, the rationale of the functionalization has not drawn a sufficient attention so far. In principle, thermal flux nor thermal leakage cannot give any impact on the fast group XS as they do not appear in the homogenization nor group condensation equation as introduced in Eq. (6). However, the studies [3][4] have clearly shown the existence of the thermal leakage dependency of fast group XSs. Author's understanding of the phenomenon is described.

Consider a two-node domain which already has been homogenized as represented in Fig. 1. In this explanation, the group is subdivided into three: fast, resonance (or a.k.a. intermediate) and thermal. Note that the fast group in the two-group approach (named as a whole-fast group) includes fast and resonance groups in the three-group.

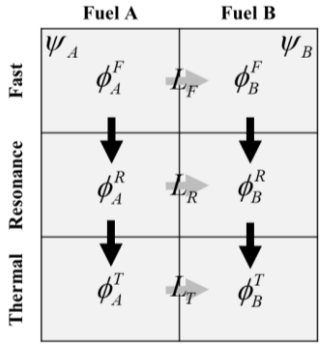

Fig. 1. Schematic diagram of the two-node domain.

Life of neutrons starts from fission reactions. Source of the fast neutron is the fission and there is no other factor that perturbs the behavior of the fast neutrons. Thus, the leakage in the fast group solely depends on the gradient of the fission source as follows:

$$L_F \propto \Delta\psi_{A,B}. \tag{12}$$

Then neutrons are slow-downed. The probability that the fast neutrons down-scatter to resonance group without getting absorbed remains in a similar level almost regardless of neighbors or composition:

$$p_F = \frac{\Sigma_{F\to R}}{\Sigma_{r,F}}. \tag{13}$$

A typical value for LWR FAs is 0.92 (and 0.99 for reflectors which purely down-scatters).

In the resonance range, the source is the down-scattered neutrons from the fast group. Thus, the source is determined by the fast neutrons which are affected by the gradient of fission source. In the same manner, the intermediate range leakage affects the scattered source toward the thermal group, and the thermal source determines the thermal leakage. Thus, the intermediate range leakage indirectly affects the thermal leakage.

On the other hand, the spectral shift inside the whole-fast group (fast + resonance) is determined by fast and intermediate leakage. Thus, the fast and the intermediate range leakage affect the GCs of the whole-fast group. Taking into account the things mentioned so far, the rationale of the thermal leakage dependency on the whole-fast group XS is to *indirectly represent the effect of intermediate leakage through its consequence in thermal leakage*.

2.5 Spectral Index Shift

To catch the effect of the drastic spectral shift due to reflectors, LFM adopts SIS as an additional fitting parameter while the APEC method utilizes SIS for the fitting of IAs and excludes it for PAs. Since the two

methods completely contradict each other, it is required to state the rationale of each method.

From the thermal group balance equation,

$$\Sigma_{r,2}\phi_2 + L_2 = \Sigma_{1\to 2}\phi_1, \tag{14}$$

the SI which is a fast-to-thermal flux ratio can be derived as:

$$\Gamma = \frac{\phi_1}{\phi_2} = \frac{\Sigma_{r,2} + l_2}{\Sigma_{1\to 2}} \tag{15}$$

where

$$l_2 = L_2 / \phi_2 . \tag{16}$$

From Eq. (15), since GCs of a thermal group solely depend on thermal LF, it is clear that the numerator of SI is a function of thermal LF. Excluding the thermal LF dependent terms in the numerator of SI, the leftover term that determines SI is a down-scattering XS and this can be rewritten in three-group as follows:

$$\Sigma_{1\to 2} = \Sigma_{R\to T}\frac{\phi_R}{\phi_F + \phi_R} . \tag{17}$$

From the expression above, it can be noticed that SI is the representative of the spectrum inside the whole-fast group and that is how SI works as a fitting parameter.

For IAs, the down-scattering probability from Eq. (13) which is the decisive factor of the spectrum inside the whole-fast group is almost constant for all FA types. And noting that the probability for the resonance range also remains in a similar level (0.70 to 0.75), neutrons will experience the similar probability of down-scattering whether it migrates to neighbor or not. Therefore, the tendency of SIS remains constant for FA checkerboards (CB). From Fig. 2, it can be clearly noticed that SIS is a function of two group LFs.

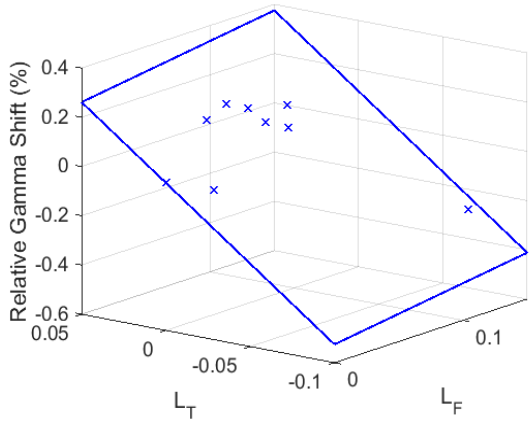

Fig. 2 Spectral Index Shift with respect to the leakage fractions from APR1400 C0 checkerboards. Goodness of Fitting:0.999.

However, since the reflectors have a bit different probability to down-scatter (0.94 for intermediate range), neutrons will experience a different probability of down-scattering. And the net probability depends on how many neutrons go out to the reflector sides. This results in the difference of the SI shift tendencies among the PAs at the edge (facing 1 side toward reflector) and at the corner (facing 2 sides toward reflector) as clearly represented in Fig. 5.

3. Evaluation of Fitting Formulas

For the validation of fitting formulas adopted in LFM, we evaluated the goodness of fitting and the RMS error of estimated GCs compared to the core values. The target core of evaluation is 2D APR1400 core [8] with a loading pattern as Fig. 3, at BOC, hot temperature without thermal feedback. To obtain the points for GC fittings, colorset problems with configurations in Fig. 4 are solved by nTRACER using transport corrected P_0 XS. The number of different assembly type combination per one assembly is 9 for the APR1400 fresh core. From CBs, 9 points can be obtained and from 3×3 problems, 5×9 points can be obtained considering 5 different positions.

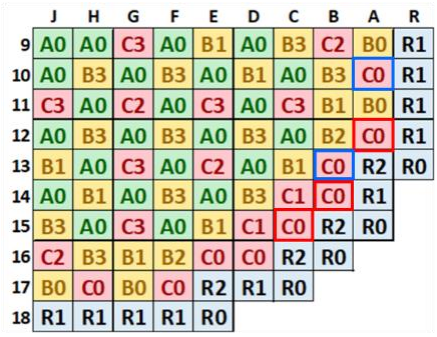

Fig. 3 Configuration of the APR1400 Fresh Core.

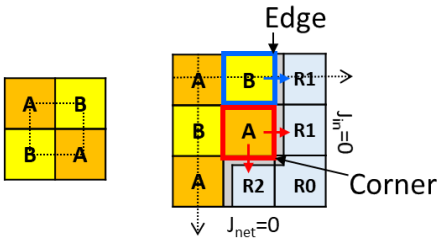

Fig. 4. Configurations of the checkerboard (left) and the 3×3 local problem (right).

3.1 Spectral Index Shift Tendency

Fig. 5 shows the tendency of SIS of PAs at the edge and the corner from 3×3 problems in each red and blue, and the actual SIS values from the marked assemblies in Fig. 3 in black dots. Since two kinds of PAs (edge and corner) show different tendency of SIS, the consideration of SIS in the functionalization of PAs is necessary to properly count the effect from reflectors.

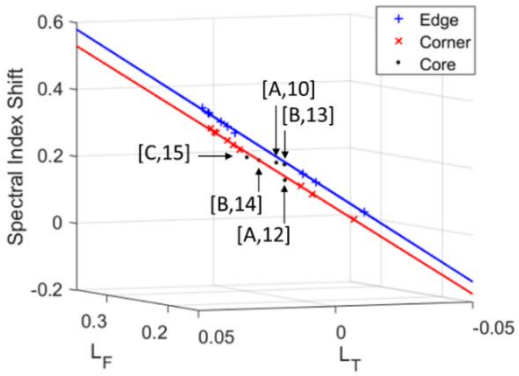

Fig. 5 Spectral Index Shift with respect to the leakage fractions from the edge and the corner assemblies of 3×3 local problems for APR1400 C0.

3.2 Errors of Evaluated GCs

The effectiveness of each formula is assessed by the RMS error of evaluated GCs with a reference leakage parameter defined as Eq. (18) for B3 and C0 assemblies in APR1400 as a representative of each IA and PA. For the fitting, fuel-only CBs are used for IA, and 3×3 local problems are additionally used for PA.

$$E = \sqrt{\sum_{i}^{N} \left(\frac{\Sigma_{Core}^{i} - \Sigma(L_{Core}^{i})}{\Sigma_{Core}^{i}} \right)^{2} / N} \qquad (18)$$

Table I. RMS error (in pcm) of evaluated fast GCs with different fitting formulas for APR1400 B3 (IA).

Fit. Para.	D_1	Σ_{a1}	Σ_{f1}	Σ_{12}
l_1, l_2	13	30	42	**59**
l_1, $\Delta\Gamma$	13	25	39	**49**
l_F, l_R	18	11	17	**21**
l_1, l_2, $\Delta\Gamma$	17	10	**26**	16

Table II. RMS error (in pcm) of evaluated fast GCs with different fitting for APR1400 C0 (PA).

Fit. Para.	D_1	Σ_{a1}	Σ_{f1}	Σ_{12}
l_1, l_2	122	418	219	**609**
l_1, $\Delta\Gamma$	120	390	179	**519**
l_F, l_R	45	**100**	15	35
l_1, l_2, $\Delta\Gamma$	24	21	18	**28**

Bold numbers in tables are the largest error for each fitting. Remarks from two tables are as follows:

- IA GCs are well fitted even with l_1, l_2.
- Consideration of $\Delta\Gamma$ as an additional fitting parameter to l_1, l_2 improves the accuracy of fitted GCs significantly for both IAs and PAs.
- Although l_F, l_R cannot be used in a practical case, they best-estimate the GCs for both IAs and PAs among two variable fittings.

4. Conclusions

In this study, the rationales of thermal LF and SIS in the fitting formula of fast GCs are demonstrated. Thermal LF works as an indirect parameter of the intermediate leakage. From the evaluation of the fitting formulas for APR1400, the statement is verified by proving that GCs can be functionalized with respect to the fast and the intermediate leakages.

SI shift is a representative of spectrum inside the whole-fast group. Since the tendencies of SIS are different for the edge and the corner PAs, the consideration of SIS for PAs results in the accuracy enhancement of fitted GCs. Among four different fitting formulas for leakage dependent GCs, using three variables, fast and thermal LFs and SIS, showed the best accuracy for both IA and PA. However, the author strongly insists to reconsider the use of SIS for IA despite its accuracy enhancement because SIS in IAs are quite dependent to two group LFs while noting that fitting without SIS is still not bad.

The estimated GC error of LFM is less than 30 pcm in RMS. With a help of advance in a leakage correction method, the two-step method employing LFM achieved less than 36 pcm in reactivity and 0.7 % in assembly-wise power error in RMS for the 2D APR1400 core compared to the DWC reference. Assessment of LFM on various core problems with thermal feedback and rod insertion will be presented in the upcoming paper.

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