A Leakage Correction with SPH Factors for Two-group Cross-section in MOX-loaded PWR Core

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1. Introduction

Recently, a high-fidelity multi-physics and multidimensional reactor analysis is gaining more attention because of its accurate predictions of local parameters. For advanced reactor design and analysis, it is essential to provides the accurate and detail power profile. In terms of accuracy, the direct whole-core transport calculation is quite promising for the high-fidelity reactor analysis. However, it is very costly in terms of the computational burden and memory. Another possible solution is the pin-by-pin core calculation with a low-order operator such as diffusion.

The two-step procedure is the most widely used method for both nodal and pin-by-pin analysis. In the two-step procedure, there are two well-known techniques to reduce the spatial homogenization error of group constants; generalized equivalence theory (GET) [1,2] and super-homogenization method (SPH) [3,4]. The two-step pin-wise reactor analysis is performed with pin-wise spatial homogenization in the infinite lattice transport analysis. Consequently, the resulting pin-wise group constants always subject to unavoidable and noticeable errors due to the unphysical boundary condition. It causes non-negligible inaccuracy in the two-step procedure.

In our previous study, a new leakage correction method, named GET Plus SPH (GPS) method [5,6], was introduced to improve accuracy of the standard (GET-based) two-step procedure. In the GPS method, the pin-wise XS-dependent SPH factor are introduced and parameterized as function of the pin-wise currentto-flux ratio (CFR). Also, the pin-wise reaction rates are corrected by the updated XS-dependent SPH factor. The feasibility of the GPS function are investigated in the several UOX variant cores. It is demonstrated that GPS method can improve substantially the accuracy of the pin-wise reactor analysis [5,6].

In this study, the GPS method is validated against various partially MOX-loaded benchmark problems. A 2-D method of characteristics (MOC) based lattice code DeCART2D [7] is used for all transport calculations. The pin-by-pin core analyses are performed using the HCMFD algorithm [8].

2. GPS method

In the standard two-step reactor analysis, the fluxweighted constants (FWCs) are the cornerstone of generating equivalent group constants. However, they have non-negligible discrepancy from the real reference values, since they are determined in the all-reflective boundary condition.

The key idea of the GPS method is correcting the pinwise group constants by taking into account pin-wise neutron leakage. In the GPS method, the modified SPH concept, XS-dependent SPH factor is functionalized to reduce the discrepancy of the FWCs.

2.1 XS-dependent SPH factor

In the SPH method, the average reaction rate is preserved by introducing the group-dependent SPH factor μ_{α} :

$$\tau_g = \mu_g \Sigma_g^{ref} \phi_g = \Sigma_g^{ref} \phi_g^{ref}, \tag{1}$$

where Σ_{g}^{ref} is reference cross-section, ϕ_{g}^{ref} is reference heterogeneous flux, and ϕ_{g} is homogenous flux.

From the definition of the SPH factor, the reference reaction rates can be preserved only if reference crosssections and corresponding SPH factors are available. However, the available information in the standard twostep procedure is only pin-wise group constants from the single fuel assembly calculation. To correct these pin-wise group constants, a XS-dependent SPH factor is introduced as follows:

$$SPH_{\Sigma_{g\alpha}} = SPH_g \times \frac{\Sigma_{\alpha g}^{ref}}{\Sigma_{\alpha g}^{SA}},$$
(2)

where $\sum_{\alpha g}^{SA}$ is cross-sections from the single fuel assembly calculation and *SPH_g* is the SPH factor.

2.2 GPS functionalization

Unlike the assembly-wise group constant, the pinwise group constants are sensitive to the pin location; pins near the baffle, GT and BA pin. In this study, the position-wise functionalization, such as 289 different pin positions in the 17x17 FA, is adopted to consider the different characteristics of the pins according to their location.

For the functionalization of the GPS functions, three different FA-wise and regions-wise functionalization are considered since they have different characteristics and position-dependency; 1) fuel pin inside FA of inner core region, 2) fuel pin inside FA neighboring the baffle-reflector region, 3) baffle-reflector regions.

In our previous study [5,6], the two-group pin-wise XS-dependent SPH factors have a strong relationship with the node-average CFR, defined as below;

$$CFR_g^m = \frac{\sum_s J_g^s}{\phi_s^m},\tag{3}$$

where J_g^s is surface net outward current ϕ_g^m is the nodeaverage flux.

The pin-wise changes in the XS-dependent SPH factor from their initial values are functionalized by change of node-average CFR as follows:

$$\Delta SPH_{1st_{\Sigma\alpha}} = a_{1,F} \Delta CFR_{1st} + a_{2,F} \Delta CFR_{2nd} + a_{3,F}, \quad (4)$$

$$\Delta SPH_{2nd_{\Sigma}\alpha} = a_{1,T} \Delta CFR_{1st} + a_{2,T} \Delta CFR_{2nd} + a_{3,T}, \quad (5)$$

where $a_{1,g}$, $a_{2,g}$, and $a_{3,g}$ are the fitting constants.

It is noted that the constant term, $a_{3,g}$, of the 1) 'fuel pin inside FA of inner core region' is always zero in the previous UOX-based core analyses [5]. In this partially MOX-loaded core analyses, however, the non-zero constant term, $a_{3,g}$, is used to consider the huge spectrum change.

The change of both XS-dependent SPH and nodeaverage CFR are defined as;

$$\Delta SPH_{g_{\Sigma}\alpha} = SPH_{g_{\Sigma}\alpha} - SPH_{g_{\Sigma}\alpha}^{SA}, \qquad (6)$$

$$\Delta CFR_g = CFR_g - CFR_g^{SA},\tag{7}$$

where $SPH_{g_{-}\Sigma\alpha}^{SA}$ and CFR_{g}^{SA} are the XS-dependent SPH (unity) and node-average CFR from single fuel assembly calculation.

2.3 Fitting data sets of GPS function

In order to determine proper coefficients of GPS functions, least square method is used and two different kinds of color-set calculations are considered; 1) 3x3 color-set and 2) L-shape baffle color-set as shown in Figs. 1 and 2. The fitting data sets from the '3x3 color-set' are used for GPS functions of the FA in inner core region. In the '3x3 color-set', the loading pattern are determined randomly to get properly perturbed and asymmetrical fitting data sets. The fitting data sets from the 'L-shape baffle color-set' are used for GPS functions of the FA neighboring the baffle-reflector and baffle-reflector itself.



Fig. 1. Color-set model for the FA in the inner core



Fig. 2. Color-set models for the baffle-reflector related regions.

In the GPS method, one-point correction of group constants is considered since the neutron leakage and flux are not preserved after the SPH correction. The node-average CFR from the GET-based two-step colorset is used as the input of the fitting data set and XSdependent SPH factors from the SPH fixed-source iteration with corresponding pin-wise DFs are used as the output of the fitting data set for the GPS functionalization.

3. Results and Discussion

In this study, the GPS method was applied to partially MOX-loaded KAIST 1A (ARO) benchmark problem [9] as shown in Fig. 3. There are three typical 17x17 UOX fuel assemblies (UOX-1: 2.0 w/o, UOX-2: 3.3 w/o UOX-2 with 16 BA fuel pins) and two 17x17 MOX FAs (MOX-1 and MOX-1 with 8 BA fuel pins). In the MOX FAs, three types of fuel pins are loaded with different Pu loading (4.3 w/o, 7.0 w/o, and 8.7 w/o). For the consistency, the baffle-reflector regions are also treated with the pin-wise group constants.



Fig. 3. Core configuration of KAIST 1A benchmark problem.

In the GPS method, the pin-by-pin core calculation initially starts with pin-wise XSs and DFs from the single FA calculation. The XS-dependent SPH factors are updated only once after the problem is converged. One notes that the XS-dependent SPH factors in the baffle-reflector regions are also updated since they are closely coupled with the fuel pins near baffle.

In this study, a complete set of GPS functions such as FA-type and baffle-type dependent GPS functions are

functionalized with fitting data sets from that are listed in Fig. 4 and Table 1. The U1, U2, U3, M1, and M2 indicate UOX-1, UOX-2, UOX-2 (BA16), MOX-1, and MOX-1 (BA8) type FA respectively.

U3	U2	M 1	U2	М1	Ul		Ul	мı	U2	Ul	M 1	U2
U2	M2	U2	U2	M1	U1		М1	М1	U2	Ul	M1	U2
М1	U2	U1	M2	U2	Ul		U2	U2	UI	мı	M 1	U2
U1	M 1	U3	U1	M1	U3		MI	U3	Ul	M2	M2	U3
M1	M2	U2	U1	M1	U3		U3	M2	U2	M2	M2	U3
U3	U2	U2	M1	M2	U2		Ul	U2	Ul	U3	U3	M1
						-						
U2	M2	M 1	U2	M2	MI		M2	М1	UI			
M2	M 1	U2	U2	M2	MI		M2	М1	U1			
M1	U2	U1	M2	M1	M2		мı	U2	Ul			
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Fig. 4. List of the '3x3 color-set' loading patters.

Table 1: List of color-set for the GPS functionalization

Model	Set of FAs in color-set models				
L-shape baffle Type-1	(U1,U2,M1), (M1,U2,U1)				
L-shape	(U1,U1,U1), (U1,U2,M1), (U1,M1,U2)				
baffle Type-2	(U2,M1,U1), (M1,U2,U1)				

Table 2 shows a summary of the pin-by-pin core analyses for the KAIST 1A benchmark problem. Even if the XS-dependent SPH factors are updated only once, it is clearly shown that the GPS method provides more accurate eigenvalue and pin-power than the standard GET-based two-step pin-by-pin core analysis as shown in Table 2 and Fig. 5.

Table 2: Summaries of KAIST 1A benchmark problem

Condition	k-eff	$\Delta \mathbf{k}_{eff}$ [pcm]	Pin-power % error Max (RMS)		
Ref. DeCART2D	1.116271	Ref	Ref		
Standard two-step	1.118000	172.93	3.94 (1.15)		
GPS	1.116599	32.78	-1.78 (0.34)		



Fig. 5. FA-wise Max and RMS pin power %error distribution of KAIST 1A benchmark problem.

To demonstrate that GPS method works for any other core configuration, four large MOX-loaded variants were considered as shown in Fig 6. Since the loading patterns of each variant are determined randomly, the variant cores have different power distributions. For large variants, the same GPS functions from the KAIST 1A benchmark problem were used without any modification.



For the MOX-loaded large variant cores, the pin-bypin core calculations are analyzed by using standard two-step method and GPS method. As shown in Table 3, the GPS correction effectively improves the accuracy of the pin-by-pin calculation. Although the maximum error in the pin-power is still significant, the RMS value of the pin-power is reduced by almost half.

Table 3: Summary of MOX-loaded large variants

Condition	k _{eff}	∆ k _{eff} [pcm]	Pin-power % error Max (RMS)						
Large Variant 1									
Ref. DeCART2D	1.136471	Ref	Ref						
Standard two-step	1.137403	93.25	4.84 (1.07)						
GPS	1.136639	16.76	-2.37 (0.41)						
Large Variant 2									
Ref. DeCART2D	1.127308	Ref	Ref						
Standard two-step	1.128433	112.47	4.40 (1.02)						
GPS	1.127239	-6.92	-2.11 (0.35)						
Large Variant 3									
Ref. DeCART2D	1.105748	Ref	Ref						
Standard two-step	1.107094	134.55	-6.16 (1.58)						
GPS	1.105825	7.75	-2.06 (0.45)						
Large Variant 4									
Ref. DeCART2D	1.134549	Ref	Ref						
Standard two-step	1.135709	115.98	5.13 (1.22)						
GPS	1.134588	3.89	-2.62 (0.36)						

The FA-wise maximum and RMS pin-power error distribution of each large PWR variant are shown in the Fig. 7. It is noted that the GPS correction reduces the maximum pin-power error to less than 1%, in most of FAs of the inner core regions. However, there are non-negligible maximum errors at the peripheral region where the normalized pin-powers itself are relatively

low such as 0.122, 0.086, 0.060, and 0.058, respectively. Although the impact of this region on the entire core will be small, however, further improvement of the accuracy is currently being studied.



Fig. 7. FA-wise Max and RMS pin power %error distribution of large variants.

3. Conclusions

In this study, the GPS method was applied to the partially MOX-loaded core. Compared with the GPS implementation in the UOX-loaded core, the GPS functionalization in the MOX-loaded core needs additional consideration since the MOX fuel affects quite different spectrum change. In this paper, the wellknown KAIST 1A benchmark and its large variant cores were considered to demonstrate the effect of GPS method on the partially MOX-loaded cores. It can be concluded that the GPS method works well and improves the accuracy of pin-wise core analysis, relative to the standard two-step calculation. The generality of the GPS method is also demonstrated that the GPS functions determined with several color-set problems can be applied to various core configuration without any modifications.

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REFERENCES

[1] K. Koebke, "A New Approach to Homogenization and Group Condensation", IAEA-TECDOC-231, IAEA Technical Committee Meeting on Homogenization Methods in Reactor Physics, Lugano, Italy, November 13-15, 1978.

[2] K. S. Smith, "Assembly Homogenization Techniques for Light Water Reactor Analysis," Progress in Nuclear Energy, 17, 303, 1986.

[3] A. Hebert, "A Consistent Technique for the Pin-by-Pin Homogenization of a Pressurized Water Reactor Assembly", Nucl. Sci. Eng., 113, 227, 1993.

[4] Akio, Yamamoto, et al, "Improvement of the SPH Method for Pin-by-Pin Core Calculation", Journal of Nuclear Science and Technology, Vol. 41, No. 12, p.1155, Dec, 2004.

[5] H. Yu, and Y. Kim, "A Study on in situ Two-group SPH Factor Correction in GET-based Pin-by-Pin Core Analysis", Transactions of the Korean Nuclear Society Autumn Meeting, Jeju, Korea, May 17-18 (2018)

[6] H. Yu, W. Kim, and Y. Kim, "A Leakage Correction with SPH Factors for Two-group Cross-section in GET-based Pinby-Pin Core Analysis", Transactions of the American Nuclear Society, vol 119, will be published.

[7] J. Y. Cho, DeCART2D v1.0 User's Manual, KAERI/TR-5116/2013

[8] S. Song, H. Yu, and Y. Kim, "One-node and two-node hybrid coarse-mesh finite difference algorithm for efficient pin-by-pin core calculation", Nuclear Engineering and Technology, online published, 2018

[9] N. Z. Cho, "Benchmark Problem 1A: MOX Fuel-Loaded Small PWR Core (MOX Fuel with Zoning)", KAIST/NuratT, <u>http://nurapt.kaist.ac.kr/benchmark</u>, June 23 (2000)