Kinetics Parameter Estimations for Accelerator-Driven Subcritical Systems in the Monte Carlo Fixed Source Calculations

Donghun Kim, Hyung Jin Shim
Seoul National University, 1 Gwanak-ro, Gwanak-gu, Seoul 08826, Korea
**Corresponding author: shimhj@snu.ac.kr*

1. Introduction

Since the early 1990's, accelerator-driven subcritical systems (ADSs) [1] have been proposed and tested throughout the world by its merits of the high flexibility in nuclear fuel cycles as well as the unique safety concept. It is well known that the spatial distribution and energy spectrum of neutron flux calculated from the *k*-mode eigenvalue equation can be significantly different from those for a highly subcritical system with an external source. One of the related subjects is the point kinetics analysis for the initially subcritical system with kinetics parameters weighted by an adequate adjoint function.

There have been several works [2-5] to develop a point kinetics model for the ADS analysis with varying the corresponding adjoint equation. In this paper, we develop Monte Carlo (MC) algorithms to estimate the kinetics parameters of the point kinetics equation (PKE) based on the inhomogeneous adjoint equation [6] in the MC fixed source calculations. The developed method is verified in an infinite homogeneous two-group problem by comparing its numerical results with analytic solutions.

2. Methods

For the completeness, the "exact" PKE [6] for the subcritical system is reviewed in Section 2.1 and its practical form and the MC algorithms for its kinetics parameters are derived in the following sub-sections.

2.1 Exact PKE for ADS

The time-dependent neutron transport equation and the delayed neutron precursor density equation can be expressed as

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = -\mathbf{M}\Phi + \mathbf{F}_p\Phi + \sum_i \lambda_i c_i + s_{ext} , \tag{1}$$

$$\frac{\partial c_i}{\partial t} = \mathbf{F}_{di}\Phi - \lambda_i c_i ; \tag{2}$$

$$\begin{aligned}\mathbf{M}\Phi = &\left[\mathbf{\Omega}\cdot\nabla + \Sigma_t(\mathbf{r},E,t)\right]\Phi(\mathbf{r},E,\mathbf{\Omega},t)\\ &-\int dE'\int d\mathbf{\Omega}'\Sigma_s(E',\mathbf{\Omega}'\to E,\mathbf{\Omega}\,|\,\mathbf{r},t)\Phi(\mathbf{r},E',\mathbf{\Omega}',t),\end{aligned} \tag{3}$$

$$\mathbf{F}_p\Phi = \frac{\chi_p(\mathbf{r},E,t)}{4\pi}\int dE'\int d\mathbf{\Omega}'\,\nu_p(\mathbf{r},E',t)\Sigma_f(\mathbf{r},E',t)\Phi(\mathbf{r},E',\mathbf{\Omega}',t), \tag{4}$$

$$\mathbf{F}_{di}\Phi = \frac{\chi_{di}(\mathbf{r},E,t)}{4\pi}\int dE'\int d\mathbf{\Omega}'\,\nu_{di}(\mathbf{r},E',t)\Sigma_f(\mathbf{r},E',t)\Phi(\mathbf{r},E',\mathbf{\Omega}',t), \tag{5}$$

where Φ denotes the time-dependent angular flux, $\Phi(\mathbf{r},E,\mathbf{\Omega},t)$. The subscripts *p* and *d* indicates operator or functions for prompt and delayed neutrons, respectively. λ_i is the decay constant of the delayed neutron precursors of group *i*. $c_i(\mathbf{r},E,\mathbf{\Omega},t)$ is defined by $\left(\chi_{di}(\mathbf{r},E,t)/4\pi\right)C_i(\mathbf{r},t)$ where χ_{di} and C_i are the energy spectrum and the precursor density, respectively, of the *i*-th group delayed neutrons. s_{ext} is the external source density, $s_{ext}(\mathbf{r},E,\mathbf{\Omega},t)$. Other notations follow the standard.

For the further derivations, here, the fission production operator is defined by

$$\mathbf{F}\Phi \equiv \left(\mathbf{F}_p + \sum_i \mathbf{F}_{di}\right)\Phi . \tag{6}$$

In this study, the solution of the following inhomogeneous adjoint equation is used as the adjoint function for the subcritical system: [6]

$$\mathbf{M}_0^\dagger\Phi_{det}^\dagger = \mathbf{F}_0^\dagger\Phi_{det}^\dagger + \Sigma_{det} , \tag{7}$$

where $\Phi_{det}^\dagger$ denotes the adjoint flux, $\Phi_{det}^\dagger(\mathbf{r},E,\mathbf{\Omega})$, which is dependent on an arbitrary detector cross section function of $\Sigma_{det}(\mathbf{r},E)$. The adjoint operators of $\mathbf{M}_0^\dagger$ and $\mathbf{F}_0^\dagger$, where the subscript "0" indicates the nominal state of the nuclear reactor, are defined as

$$\begin{aligned}\mathbf{M}_0^\dagger\Phi_{det}^\dagger = &\left[-\mathbf{\Omega}\cdot\nabla + \Sigma_{t0}(\mathbf{r},E)\right]\Phi_{det}^\dagger(\mathbf{r},E,\mathbf{\Omega})\\ &-\int dE'\int d\mathbf{\Omega}'\Sigma_{s0}(E,\mathbf{\Omega}\to E',\mathbf{\Omega}'\,|\,\mathbf{r})\Phi_{det}^\dagger(\mathbf{r},E',\mathbf{\Omega}'),\end{aligned} \tag{8}$$

$$\mathbf{F}_0^\dagger\Phi_{det}^\dagger = \int dE'\int d\mathbf{\Omega}'\frac{\chi_0(\mathbf{r},E')}{4\pi}\nu_0(\mathbf{r},E)\Sigma_{f0}(\mathbf{r},E)\Phi_{det}^\dagger(\mathbf{r},E',\mathbf{\Omega}') . \tag{9}$$

By multiplying Eq. (1) by $\Phi_{det}^\dagger$ and Eq. (7) by Φ, subtracting the resulting two equations, and integrating it over $(\mathbf{r},E,\mathbf{\Omega})$, one can obtain

$$\begin{aligned}\left\langle \Phi_{det}^\dagger, \frac{1}{v}\frac{\partial\Phi}{\partial t}\right\rangle = &-\left\langle\Phi_{det}^\dagger,\mathbf{M}\Phi\right\rangle + \left\langle\Phi_{det}^\dagger,\mathbf{F}_p\Phi\right\rangle\\ &+\left\langle\Phi_{det}^\dagger,\sum_i\lambda_i c_i\right\rangle + \left\langle\Phi_{det}^\dagger,s_{ext}\right\rangle\\ &+\left\langle\Phi_{det}^\dagger,\mathbf{M}_0\Phi\right\rangle - \left\langle\Phi_{det}^\dagger,\mathbf{F}_0\Phi\right\rangle - \left\langle\Sigma_{det}\Phi\right\rangle,\end{aligned} \tag{10}$$

where the angle bracket < > implies the inner product of two components in it over $(\mathbf{r},E,\mathbf{\Omega})$. Note that the property of the adjoint operators on the neutron flux, i.e., $\left\langle \Phi, \mathbf{M}_0^\dagger \Phi_{det}^\dagger \right\rangle = \left\langle \Phi_{det}^\dagger, \mathbf{M}_0 \Phi \right\rangle$, $\left\langle \Phi, \mathbf{F}_0^\dagger \Phi_{det}^\dagger \right\rangle = \left\langle \Phi_{det}^\dagger, \mathbf{F}_0 \Phi \right\rangle$, and $\left\langle \Phi_{det}^\dagger, \Sigma_{det} \right\rangle = \left\langle \Sigma_{det} \Phi \right\rangle$, are used in Eq. (10).

Now let us separate the angular flux Φ into an amplitude function $P(t)$ and a shape function $\psi(\mathbf{r}, E, \mathbf{\Omega}, t)$ as [6]

$$\Phi(\mathbf{r}, E, \mathbf{\Omega}, t) = P(t) \cdot \psi(\mathbf{r}, E, \mathbf{\Omega}, t), \qquad (11)$$

where the shape function satisfies the normalization condition of

$$\frac{\partial}{\partial t} \left\langle \Phi_{det}^\dagger, \frac{\psi}{v} \right\rangle = 0. \qquad (12)$$

As in the derivation of the conventional PKE for the critical reactor [6], by inserting $\left\langle \Phi_{det}^\dagger, \sum_i \mathbf{F}_{di} \Phi \right\rangle - \left\langle \Phi_{det}^\dagger, \sum_i \mathbf{F}_{di} \Phi \right\rangle$ in the right side of Eq. (10) and using Eqs. (7) and (12), introductions of Eq. (11) into Eq. (10) yields the exact PKE for Eq. (1) as

$$\frac{dP(t)}{dt} = \frac{\rho(t) - \beta(t)}{\Lambda(t)} P(t) + \sum_i \lambda_i C_i(t) + S_{ext}(t) - q(t) P(t); \qquad (13)$$

$$\rho(t) \equiv \frac{-\left\langle \Phi_{det}^\dagger, (\mathbf{M} - \mathbf{M}_0)\psi \right\rangle + \left\langle \Phi_{det}^\dagger, (\mathbf{F} - \mathbf{F}_0)\psi \right\rangle}{\left\langle \Phi_{det}^\dagger, \mathbf{F}\psi \right\rangle}, \qquad (14)$$

$$\beta(t) \equiv \sum_i \beta_i(t), \qquad (15)$$

$$\beta_i(t) \equiv \left\langle \Phi_{det}^\dagger, \mathbf{F}_{di}\psi \right\rangle \Big/ \left\langle \Phi_{det}^\dagger, \mathbf{F}\psi \right\rangle, \qquad (16)$$

$$\Lambda(t) \equiv I(t) \Big/ \left\langle \Phi_{det}^\dagger, \mathbf{F}\psi \right\rangle, \qquad (17)$$

$$C_i(t) \equiv \left\langle \Phi_{det}^\dagger, c_i \right\rangle \Big/ I(t), \qquad (18)$$

$$S_{ext}(t) \equiv \left\langle \Phi_{det}^\dagger, s_{ext} \right\rangle \Big/ I(t), \qquad (19)$$

$$q(t) \equiv \left\langle \Sigma_{det}\psi \right\rangle \Big/ I(t), \qquad (20)$$

$$I(t) = \left\langle \Phi_{det}^\dagger, \psi / v \right\rangle, \qquad (21)$$

where $\rho(t)$ is named the generalized reactivity [15] using the weighting function of $\Phi_{det}^\dagger$ because the conventional reactivity [19] is defined with the adjoint flux obtained at the critical state.

In the same manner to derive Eq. (13), one can obtain the PKE for Eq. (2) as

$$\frac{dC_i(t)}{dt} = \frac{\beta_i(t)}{\Lambda(t)} P(t) - \lambda_i C_i(t). \qquad (22)$$

2.2 Practical PKE for ADS

As the time-dependent shape function is approximated by the fundamental-mode solution of the *k*-mode eigenvalue equation in the critical reactor analysis, we apply the solution, Φ_0, to the steady-state neutron transport equation for the nominal state of the subcritical reactor with the external source:

$$\mathbf{M}_0 \Phi_0 = \mathbf{F}_0 \Phi_0 + s_{ext0}. \qquad (23)$$

$$\begin{aligned} \mathbf{M}_0 \Phi_0 &= \left[\mathbf{\Omega} \cdot \nabla + \Sigma_{t0}(\mathbf{r}, E) \right] \Phi_0(\mathbf{r}, E, \mathbf{\Omega}) \\ &\quad - \int dE' \int d\mathbf{\Omega}' \Sigma_{s0}(E', \mathbf{\Omega}' \to E, \mathbf{\Omega} \mid \mathbf{r}) \Phi_0(\mathbf{r}, E', \mathbf{\Omega}'), \end{aligned} \qquad (24)$$

$$\mathbf{F}_0 \Phi_0 = \frac{\chi_0(\mathbf{r}, E)}{4\pi} \int dE' \int d\mathbf{\Omega}' \nu_0(\mathbf{r}, E') \Sigma_{f0}(\mathbf{r}, E') \Phi_0(\mathbf{r}, E', \mathbf{\Omega}'), \qquad (25)$$

where S_{ext0} denotes the external source density at the nominal state, $s_{ext0}(\mathbf{r}, E, \mathbf{\Omega})$.

By using Φ_0 as the shape function ψ and the operators at the nominal state, the integral parameters in the exact PKE can be approximated as

$$\rho(t) \cong \left[-\left\langle \Phi_{det}^\dagger, (\mathbf{M} - \mathbf{M}_0)\Phi_0 \right\rangle + \left\langle \Phi_{det}^\dagger, (\mathbf{F} - \mathbf{F}_0)\Phi_0 \right\rangle \right] \Big/ F_0, \qquad (26)$$

$$\beta(t) \cong \beta_{eff} \equiv \sum_i \beta_{i,eff}(t), \qquad (27)$$

$$\beta_i(t) \cong \beta_{i,eff} \equiv \left\langle \Phi_{det}^\dagger, \mathbf{F}_{di0} \Phi_0 \right\rangle \Big/ F_0, \qquad (28)$$

$$\Lambda(t) \cong \Lambda_{eff} \equiv I_0 / F_0, \qquad (29)$$

$$q(t) \cong q_0 \equiv \left\langle \Sigma_{det} \Phi_0 \right\rangle \Big/ I_0, \qquad (30)$$

$$I(t) \cong I_0 \equiv \left\langle \Phi_{det}^\dagger, \Phi_0 / v \right\rangle, \qquad (31)$$

$$F_0 \equiv \left\langle \Phi_{det}^\dagger, \mathbf{F}_0 \Phi_0 \right\rangle, \qquad (32)$$

2.3 Physical Meaning of the Adjoint Function

In order to derive MC algorithms for calculations of the kinetics parameters of $\beta_{i,eff}$, Λ_{eff}, and q_0 in the fixed source MC calculations, the adjoint flux, $\Phi_{det}^\dagger$, can be expressed in its Neumann Series expansion. We start this derivation from the value equation [7], or the adjoint equation of the outcoming collision density, written as

$$\Phi_{det}^\dagger = \tilde{\Sigma}_{det} + \mathbf{K}_0^\dagger \Phi_{det}^\dagger; \qquad (33)$$

$$\tilde{\Sigma}_{det} \equiv \int d\mathbf{r}' T_0(\mathbf{r} \to \mathbf{r}' \mid E, \mathbf{\Omega}) \left(\frac{\Sigma_{det}(\mathbf{r}', E)}{\Sigma_{t0}(\mathbf{r}', E)} \right), \qquad (34)$$

$$\begin{aligned} \mathbf{K}_0^\dagger \Phi_{det}^\dagger &= \int d\mathbf{r}' \int dE' \int d\mathbf{\Omega}' T_0(\mathbf{r} \to \mathbf{r}' \mid E, \mathbf{\Omega}) \\ &\quad \cdot C_0(E, \mathbf{\Omega} \to E', \mathbf{\Omega}' \mid \mathbf{r}') \Phi_{det}^\dagger(\mathbf{r}', E', \mathbf{\Omega}'), \end{aligned} \qquad (35)$$

where T_0 and C_0 are the free flight kernel and the collision kernel, respectively.

From Eq. (33), its solution, $\Phi_{det}^\dagger$, can be expressed as

$$\Phi_{det}^\dagger = \left(1 - \mathbf{K}_0^\dagger\right)^{-1} \tilde{\Sigma}_{det}. \qquad (36)$$

By expanding $\left(1 - \mathbf{K}_0^\dagger\right)^{-1}$ in its Taylor's series [8], one can obtain the series expansion form of $\Phi_{det}^\dagger$ as

$$\begin{aligned} \Phi_{det}^\dagger &= \frac{1}{1 - \mathbf{K}_0^\dagger} \cdot \tilde{\Sigma}_{det} \\ &= \left[1 + \mathbf{K}_0^\dagger + \left(\mathbf{K}_0^\dagger\right)^2 + \left(\mathbf{K}_0^\dagger\right)^3 + \cdots \right] \tilde{\Sigma}_{det} \\ &= \sum_{j=0}^{\infty} \Phi_{det,j}^\dagger, \end{aligned} \qquad (37)$$

where $\Phi^{\dagger}_{det,j}$ represents the adjont response from the j-th collision (j=0, 1, $\cdots$). Then, using Eqs. (34) and (35), $\Phi^{\dagger}_{det,j}$ can be expressed as

$$\begin{aligned}
\Phi^{\dagger}_{det,j} &= \left(\mathbf{K}_0^{\dagger}\right)^j \tilde{\Sigma}_{det} \\
&= \int d\mathbf{r}' \frac{\Sigma_{det}(\mathbf{r}',E_j)}{\Sigma_{t0}(\mathbf{r}',E_j)} T_0(\mathbf{r}_j \to \mathbf{r}' \mid E_j,\mathbf{\Omega}_j) \\
&\quad \cdot \int d\mathbf{r}_j \int dE_j \int d\mathbf{\Omega}_j C_0(E_{j-1},\mathbf{\Omega}_{j-1} \to E_j,\mathbf{\Omega}_j \mid \mathbf{r}_j) \qquad (38) \\
&\qquad \cdot T_0(\mathbf{r}_{j-1} \to \mathbf{r}_j \mid E_{j-1},\mathbf{\Omega}_{j-1}) \\
&\quad \cdots \int d\mathbf{r}_1 \int dE_1 \int d\mathbf{\Omega}_1 C_0(E,\mathbf{\Omega} \to E_1,\mathbf{\Omega}_1 \mid \mathbf{r}_1) \\
&\qquad \cdot T_0(\mathbf{r} \to \mathbf{r}_1 \mid E,\mathbf{\Omega}).
\end{aligned}$$

From Eqs. (37) and (38), one can clearly find that $\Phi^{\dagger}_{det}(\mathbf{r},E,\mathbf{\Omega})$ means the expected detector response due to a unit neutron introduced at the phase space point $(\mathbf{r},E,\mathbf{\Omega})$. Notice that $q(t)$ in Eq. (13) becomes zero for the critical system because $\Phi^{\dagger}_{det}$, and thus $I(t)$, are infinite from this physical meaning of $\Phi^{\dagger}_{det}$.

2.4 MC Algorithms for Kinetics Parameter Estimation

Using the physical meaning of the adjoint flux, $\Phi^{\dagger}_{det}$, one can estimate F_0 from N external sources MC simulations as

$$\begin{aligned}
\overline{F_0} &= \frac{1}{N}\sum_{n=1}^{N}\sum_{n'}\sum_{j\in D_f^{(n,n')}}^{J^{(n,n')}} \cancel{w_{j+1}^{(n,n')}} \left(\frac{1}{\cancel{w_{j+1}^{(n,n')}}} \sum_{j'=j+1}^{J^{(n,n')}} w_{j'}^{(n,n')} \frac{\Sigma_{det}^{(n,n')j'}}{\Sigma_{t0}^{(n,n')j'}} \right) \qquad (39) \\
&= \frac{1}{N}\sum_{n=1}^{N}\sum_{n'}\sum_{j=1}^{J^{(n,n')}} m_f^{(n,n')j} w_j^{(n,n')} \frac{\Sigma_{det}^{(n,n')j}}{\Sigma_{t0}^{(n,n')j}},
\end{aligned}$$

where n and n' are the neutron source and its branching indices, respectively, where history branches may occur from the multiplicative reactions such as (n,2n) and (n,3n) as well as the fission reaction. j is the collision index. N is the number of neutron sources. $J^{(n,n')}$ and $D_f^{(n,n')}$ are the number of total collisions and a set containing collision indices at which fission reactions happen, respectively, in the n'-th branch of source n. $w_j^{(n,n')}$, $\Sigma_{det}^{(n,n')j}$, and $\Sigma_{t0}^{(n,n')j}$ are the neutron weight, Σ_{det} , and Σ_{t0} , respectively, at the j-th collision of branch n' of source n. Note that the term inside the parentheses in the right side of the first equality of Eq. (39) corresponds to the adjoint flux of a fission neutron generated from the j-th collision with its weight of $w_{j+1}^{(n,n')}$. $m_f^{(n,n')j}$ indicates the number of fissions happened before the j-th collision in the n'-th branch of source n.

In the same manner to derive Eq. (39) for the $\left\langle \Phi^{\dagger}_{det}, \mathbf{F}_0\Phi_0 \right\rangle$ estimation, the numerator of $\beta_{i,eff}$ in Eq. (28), $\left\langle \Phi^{\dagger}_{det}, \mathbf{F}_{di0}\Phi_0 \right\rangle$, can be calculated in the MC neutron tracking as

$$\overline{\left\langle \Phi^{\dagger}_{det}, \mathbf{F}_{di0}\Phi_0 \right\rangle} = \frac{1}{N}\sum_{n=1}^{N}\sum_{n'}\sum_{j=1}^{J^{(n,n')}} m_{di}^{(n,n')j} w_j^{(n,n')} \frac{\Sigma_{det}^{(n,n')j}}{\Sigma_{t0}^{(n,n')j}} \qquad (40)$$

where $m_{di}^{(n,n')j}$ indicates the number of fissions from which delayed neutrons of the i-th precursor group are produced before the j-th collision of branch n' of source n.

Lastly, I_0 defined by Eq. (31) can be estimated by

$$\begin{aligned}
\overline{I_0} &= \frac{1}{N}\sum_{n=1}^{N}\sum_{n'}\sum_{j=1}^{J^{(n,n')}} \cancel{w_j^{(n,n')}} \frac{\Delta l_j^{(n,n')}}{\upsilon_j^{(n,n')}} \left(\frac{1}{\cancel{w_j^{(n,n')}}} \sum_{j'=j}^{J^{(n,n')}} w_{j'}^{(n,n')} \frac{\Sigma_{det}^{(n,n')j'}}{\Sigma_{t0}^{(n,n')j'}} \right) \\
&= \frac{1}{N}\sum_{n=1}^{N}\sum_{n'}\sum_{j=1}^{J^{(n,n')}} w_j^{(n,n')} \frac{\Sigma_{det}^{(n,n')j}}{\Sigma_{t0}^{(n,n')j}} \left(\sum_{j'=1}^{j} \Delta t_{j'}^{(n,n')} \right),
\end{aligned} \qquad (41)$$

where $\Delta l_j^{(n,n')}$ and $\upsilon_j^{(n,n')}$ is the track length and the neutron speed between the (j-1)-th and j-th collisions of branch n' of source n. In the second equality, the flight time, $\Delta t_{j'}^{(n,n')}$, defined by $\Delta l_{j'}^{(n,n')} / \upsilon_{j'}^{(n,n')}$ is introduced.

Then using $\overline{F_0}$, $\overline{\left\langle \Phi^{\dagger}_{det}, \mathbf{F}_{di0}\Phi_0 \right\rangle}$, and $\overline{I_0}$ calculated by Eqs. (39), (40), and (41), $\beta_{i,eff}$, Λ_{eff} , and q_0 defined by Eqs. (28), (29), and (30) can be calculated at the end of MC simulations. Note that $\left\langle \Sigma_{det}\Phi_0 \right\rangle$ of the numerator term of q_0 can be calculated by the standard scoring algorithm in the MC neutron simulations.

3. Numerical Results

The proposed algorithms have been implemented in McCARD [9] and tested for a homogeneous infinite medium problem characterized by two-group cross-sections given by Table I, with changing the differential scattering cross section. In the table, $\Sigma_{sg'g}$ is the scattering cross section from energy group g to g'. In this study, Σ_{s21} is set at 0.0092592, 0.0111894, 0.0131162 or 0.0150397 corresponding to the infinite multiplication factor, k_{inf}, of 0.6, 0.7, 0.8, and 0.9.

Table I: Two-group cross sections for the infinite homogeneous problem

Cross Section	Fast Group (g=1)	Thermal Group (g=2)
Σ_{tg}	0.5	1.3
Σ_{fg}	0.001	0.090
ν_g	2.4	2.4
Σ_{sgg}	0.48	1.09
$\Sigma_{sg'g}$ ($g \neq g'$)	variable	0.0019
$\chi_{p,g}$	1.0	0.0
$\chi_{d,g}$	0.5	0.5
β_{0g} (=ν_d/ν)	0.006	0.006
$1/\upsilon_g$ [s/cm]	2.28626×10^{-10}	1.29329×10^{-6}

The McCARD fixed source calculations are performed using 10,000,000 neutron sources with fast-energy group. In the table, SD stands for the standard deviation. From the table, we can see that the results of the proposed method agree well with the reference values, which are analytically calculated by Eqs. (27)-(30), within 95% confidence intervals.

Table II: Comparisons of kinetics parameter estimates with analytic solutions for the two-group problem

k_{inf}	Para.	Ref.	McCARD (SD[%])	Rel. Err. [%]
0.6	β_{eff}	8.16341×10^{-3}	8.18418×10^{-3} (0.46)	0.25
	Λ_{eff}	8.26690×10^{-6}	8.26898×10^{-6} (0.04)	0.03
	q_0	8.06429×10^{4}	8.05811×10^{4} (0.06)	-0.08
0.7	β_{eff}	7.43095×10^{-3}	7.44250×10^{-3} (0.38)	0.16
	Λ_{eff}	7.34566×10^{-6}	7.34336×10^{-6} (0.03)	-0.03
	q_0	5.83435×10^{4}	5.83163×10^{4} (0.06)	-0.05
0.8	β_{eff}	6.88160×10^{-3}	6.90687×10^{-3} (0.34)	0.37
	Λ_{eff}	6.59941×10^{-6}	6.59943×10^{-6} (0.03)	0.00
	q_0	3.78822×10^{4}	3.78846×10^{4} (0.08)	0.01
0.9	β_{eff}	6.45433×10^{-3}	6.49597×10^{-3} (0.35)	0.65
	Λ_{eff}	5.98625×10^{-6}	5.98326×10^{-6} (0.03)	-0.05
	q_0	1.85611×10^{4}	1.85744×10^{4} (0.11)	0.07

4. Conclusions and Future Works

We have developed a MC method to calculate the kinetics parameters in the PKE for ADS, which requires the adjoint estimation in the MC fixed source calculations. The MC algorithms are derived based on the physical meaning of the adjoint function which is the solution to the inhomogeneous adjoint equation. The validity of the proposed method is demonstrated through a simple two-group problem by showing that the MC results agree very well with the analytic solutions. The proposed method will be applied for the derived PKE for ADS and validated in transient problems of ADS by comparing with experimental results.

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