

Equilibrium Xenon Feedback Function in Monte Carlo Code MCS

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1. Introduction

¹³⁵Xe has an enormous impact on thermal neutrons due to its large absorption cross section in the thermal energy range. It is known that numerical xenon oscillation can occur in Monte Carlo (MC) burnup simulation [1] induced by statistical uncertainty of MC result which is used for burnup. The solution of numerical xenon oscillation is well known. It can be reduced by updating the xenon number density at the end of every transport cycle [2-4]. In this paper, the xenon oscillation phenomena and the effect of equilibrium xenon feedback is reviewed against pin and assembly geometry using Monte Carlo code MCS [5].

2. Xenon Oscillation

2.1 Numerical Xenon Oscillation

The numerical oscillation is induced by the statistical uncertainty of MC simulation. When the neutron flux at n^{th} step is overestimated, the xenon number density computed with n^{th} step flux will be overestimated. The overestimated xenon number density will be used at $n+1^{\text{th}}$ step, and this leads to the underestimation of flux.

2.2 Equilibrium Xenon Feedback

The xenon oscillation can be eliminated when the xenon number density is updated at the end of every transport cycle. MCS uses the equilibrium xenon approximation to update the xenon number density as follows:

$$n_I = \frac{\sum_{i=\text{nuclides}} \gamma_{i,I} R_{i,f}}{\lambda_I}, \quad (1)$$

$$n_{Xe} = \frac{\sum_{i=\text{nuclides}} (\gamma_I + \gamma_{Xe}) R_f}{\lambda_{Xe} + \sigma_{Xe,a} \phi}, \quad (2)$$

where n_I is the number density of ¹³⁵I in equilibrium state, n_{Xe} is the number density of ¹³⁵Xe in equilibrium state, γ_I is the cumulative fission yield of ¹³⁵I, and γ_{Xe} is the cumulative fission yield of ¹³⁵Xe, λ_I is the decay

constant of ¹³⁵I, λ_{Xe} is the decay constant of ¹³⁵Xe, R_f is fission reaction rate, and $\sigma_{Xe,a}$ is the absorption cross-section of ¹³⁵Xe. MCS updates the xenon number density at the end of every transport cycle by using the tallied quantities during each cycle.

2.3 Effect of Xenon Feedback

The effect of xenon oscillation can be significant in high dominance ratio problems. In high dominance ratio problems, MC tally result of each transport cycle has strong inter-cycle correlation. Because of inter-cycle correlation, the flux distribution can be tilted a lot and the real Standard Deviation (SD) can be very huge compared to the apparent SD. This tilted flux distribution will lead to the distorted xenon number density distribution. The equilibrium xenon feedback can reduce the inter-cycle correlation by changing the xenon number density during every transport cycle, and it can also reduce the xenon oscillation effect at every burnup step by removing the error propagation of xenon number density.

3. Results

3.1 Verification of EQXE

When the xenon number density is saturated, the solution with and without equilibrium xenon feedback (EQXE) must be identical. VERA benchmark 1C pin problem [6] is selected to verify the solution with EQXE.

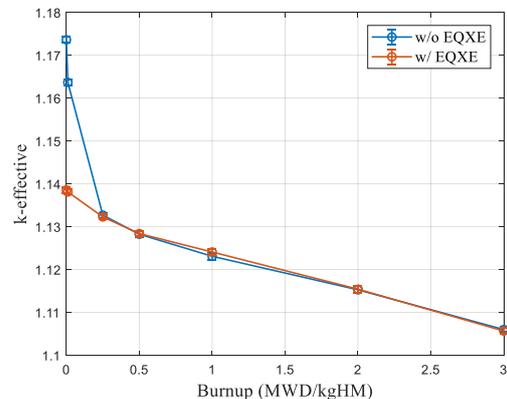


Fig. 1. Solution of VERA-1C with and without EQXE.

The simulations were performed with 10 inactive cycles, 40 active cycles, and 10,000 histories per cycle. Fig. 1 shows the multiplication factor of two cases. As shown in the figure, the two results are identical within 2sigma after the third burnup step of 6.25 days.

3.2 Pin Model

To see the xenon oscillation phenomena and to confirm the effect of EQXE, the pin cell model which is used in Dufek's paper [2] is tested. The problem specifications and simulation parameters are listed below:

- Fuel: 3.1% enriched UO₂
- Cladding: Zircaloy-4
- Moderator: light water
- Radius of fuel: 0.41cm
- Outer radius of cladding: 0.475cm
- Pin pitch: 1.26cm
- Height: 400cm
- Power: 0.064MW
- Boundary condition: reflective (axially and radially)
- # of burnup step: 10 (predictor)
- Burnup interval: 60 days
- # of axial nodes: 8
- # of inactive cycles: 1,000
- # of active cycles: 5,000
- # of histories per cycle: 1,000

It should be mentioned here that the true axial distribution is flat since there are no changes of material along axial direction and the reflective boundary conditions are imposed.

Figs. 2-3 show the flux and ¹³⁵Xe density distribution without EQXE. As shown in the fig. 2, the flux distribution at step 1 (beginning of simulation) is distorted a lot. This leads to the distortion of ¹³⁵Xe density distribution at the next step. The flux distribution at step 2 is more distorted than step 1 with the distorted xenon density.

Fig.4 shows the relative SD at step 1 with and without EQXE. The real SD in the figure is calculated with 30 independent simulations. The apparent SD of both cases is about 1%. However, the real SD without EQXE is about 10% on average. The real SD can be reduced with EQXE. Since the xenon oscillation is induced by statistical uncertainty, the effect of xenon oscillation can be reduced by reducing the SD. Fig. 5 shows the SD at step 2.

The SD at step 2 shows bigger SD since it contains number density uncertainty. The uncertainty of density can be simply estimated by comparing the real SD of step 1 and step 2. The uncertainty without EQXE is about 10% while the uncertainty with EQXE is about 2%. This result shows that the equilibrium xenon

feedback can reduce the uncertainty of xenon density by removing error propagation of xenon density.

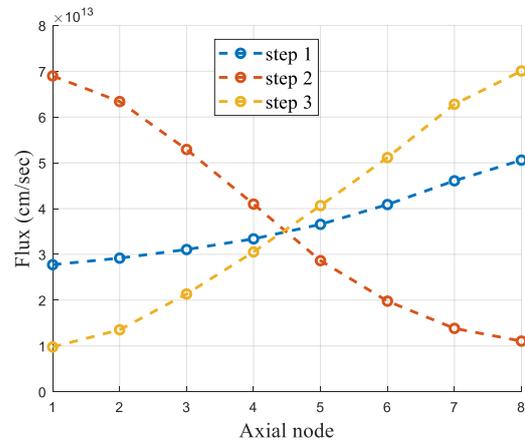


Fig. 2. Flux distribution without EQXE.

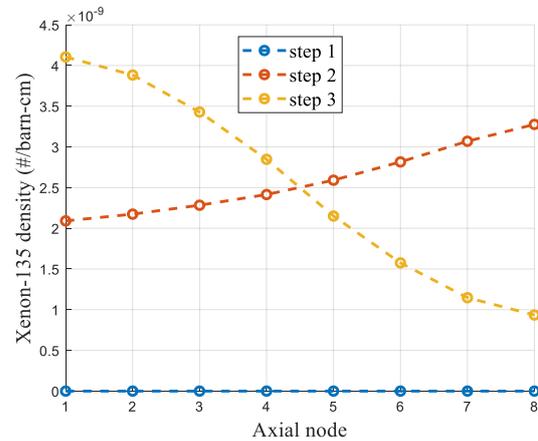


Fig. 3. Xenon density distribution without EQXE.

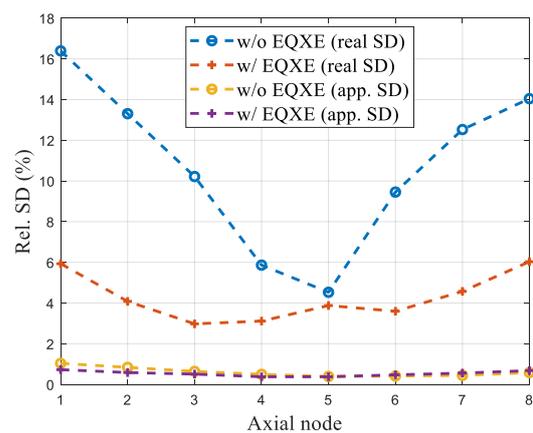


Fig. 4. Real and apparent SD of flux at step 1.

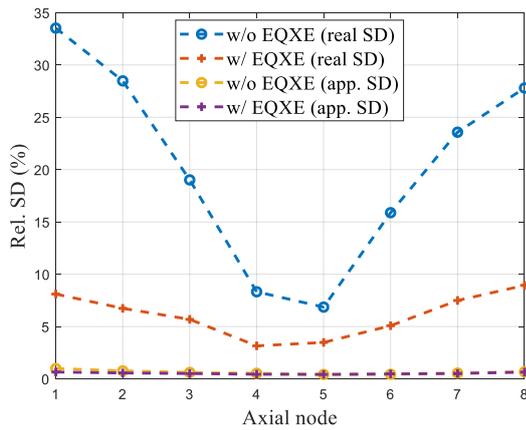


Fig. 5. Real and apparent SD of flux at step 2.

Figs. 6-7 shows the flux distribution at every burnup step with and without EQXE. As shown in the figure, the oscillation of flux can be dramatically reduced by adopting EQXE.

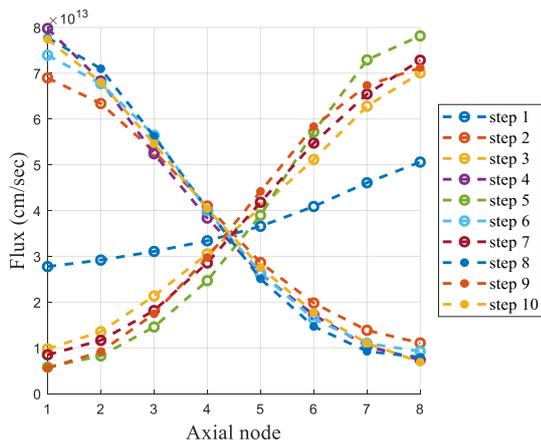


Fig. 6. Flux distribution at burnup steps without EQXE.

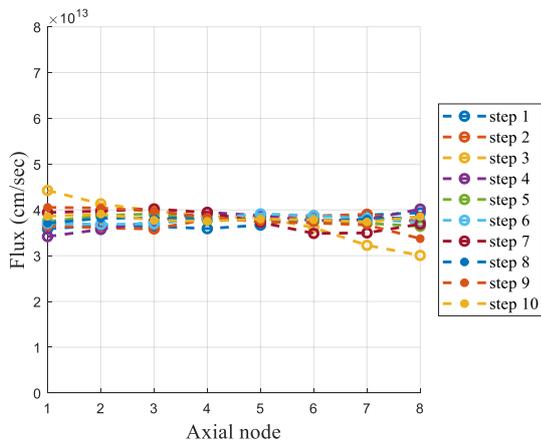


Fig. 7. Flux distribution at burnup steps with EQXE.

3.3 Assembly Model

The effect of equilibrium xenon feedback is tested on a 16 x 16 3-dimensional assembly as shown in Fig. 5. The assembly contains 168 UO_2 pins and 58 Gd_2O_3 pins. All the pins are divided into 24 axial meshes. MCS simulations were performed with 30 inactive cycles, 30 active cycles, 10 sub-cycles and 10,000 histories per sub-cycle.

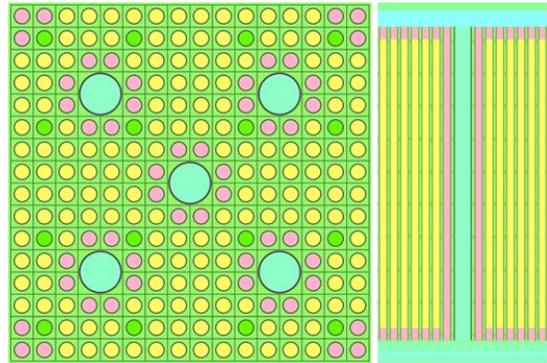


Fig. 5. 16x16 assembly model.

Five independent simulations with different random seed were performed without EQXE. Fig. 6 shows the multiplication factor of five cases. As an effect of xenon oscillation and inter-cycle correlation, the uncertainty of multiplication is much larger than the apparent SD. Fig. 7 shows the result of three independent simulations with EQXE. It is shown in the figures that the uncertainty of the MC result is reduced by adopting EQXE.

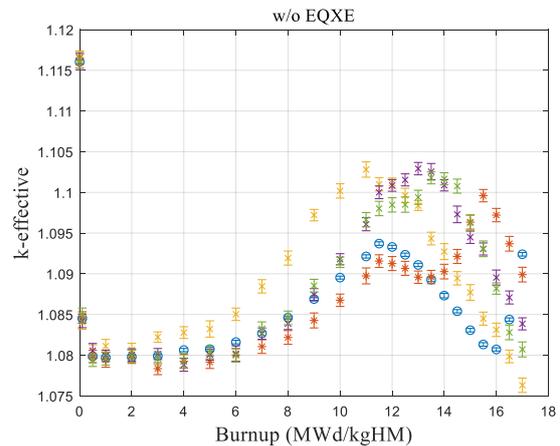


Fig. 6. Multiplication factor for assembly model without EQXE.

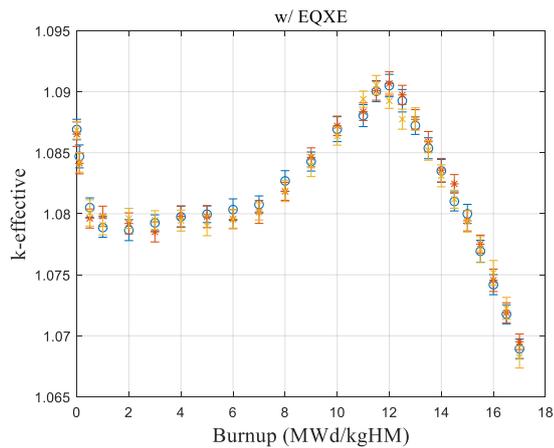


Fig. 7. Multiplication factor for assembly model with EQXE.

4. Conclusions

The equilibrium xenon feedback function was implemented into MCS. The xenon oscillation phenomena and the effect of equilibrium xenon feedback function was reviewed on a fuel pin and assembly model test cases. Two simulations were performed with and without equilibrium xenon feedback.

It is shown that the equilibrium xenon feedback function is very effective to prevent numerical xenon oscillation by reducing the inter-cycle correlation and removing error propagation of xenon density.

Acknowledgements

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