1. Introduction

An idea of the supercritical carbon dioxide (S-CO₂) power cycle was first proposed by Sulzer [1] and later reassessed by Feher [2] but some technical limits of that era made S-CO₂ cycles to be unsuccessful. However, since the technical capability for manufacturing power cycle components such as heat exchanger and turbomachinery has been advanced and due to recent study conducted by MIT on the S-CO₂ cycles for GEN IV reactors concepts [3], S-CO₂ cycles have received again a lot of attention as a promising future power cycle. This is because S-CO₂ cycles have relatively high thermal efficiency in moderate temperature range than any cycles, and it is quite simple and compact in terms of the cycle layout and components compared to other power cycles [4].

Nevertheless, advantages of the S-CO₂ cycles were purely evaluated based on the theoretical thermodynamic analysis. Therefore, an integral test facility of the S-CO₂ cycle should be available to accumulate real operation data and validate controllability. Because of this necessity, KAIST has built an integral test S-CO₂ loop called SCO₂PE (Supercritical CO₂ Pressurizing Experiment) [5]. SCO₂PE is equipped with a compressor, a water cooled precooler, and an expansion valve. At first, SCO₂PE was connected to a spiral heat exchanger and the experimental data was validated with modified GAMMA+ code [6]. GAMMA+ code is a system code for gas cooled systems to simulate various transient conditions [7]. In 2014 the spiral heat exchanger was replaced with a printed circuit heat exchanger (PCHE) and the experimental data was also compared to modified GAMMA+ code which is revised to model PCHE [8]. Although, the demonstrations using GAMMA+ code were successful but there is a limit of GAMMA+ code in the view of controller design and optimization. The reason is GAMMA+ code does not provide a characteristic equation of a plant or system which is a basic property for controller design. On the other hand, Simulink which is widely known as a dynamic simulator in MATLAB environment supports the characteristic equation of a plant or system and various controller design toolboxes. Therefore, modeling SCO₂PE using Simulink gives a potential possibility to analyze the characteristic equations of S-CO₂ cycles and design controllers based on the characteristic equations.

2. Methods and Results

2.1. Simplified governing equations

Mass, momentum and energy conservation equations are the basic governing equation to model the fluid system. Usually, system codes solve these governing equations with discretization for exact solutions. However, as it is mentioned earlier, the purpose of Simulink modeling of fluid system is to assess the characteristic equation for controller design rather than performing an exact simulation of the S-CO₂ cycle. Thus, these governing equations are simplified with adequate assumption in Simulink modeling [9].

- Mass conservation equation

\[
\frac{\partial \rho}{\partial t} + \frac{\partial m}{\partial x} = 0 \rightarrow \dot{m} = c(t)
\]

During the Simulink simulation, only mild transient scenarios such as load following situation can be modeled. As a result, abrupt density change does not occur during simulation so that \( \frac{\partial \rho}{\partial t} \) term can be ignored. This assumption leads to the constant mass flow rate in space.

- Energy conservation equation of constant mass flow rate

\[
\frac{\partial h}{\partial t} + G \frac{\partial h}{\partial x} = A_k \dot{q}' \frac{\partial h}{\partial x} \quad \rightarrow \quad A \int \left( \frac{\partial h}{\partial t} + G \frac{\partial h}{\partial x} \right) dx = A \int A_k \dot{q}' \frac{\partial h}{\partial x} dx
\]

\[
\rho A \Delta l \frac{\partial h}{\partial t} + m(h_{out} - h_{in}) = A_k \dot{q}'
\]

For conversion, upwind scheme is used in the energy conservation equation.

- Momentum conservation equation of constant mass flow rate
\[
\frac{\partial G}{\partial t} + \frac{\partial}{\partial x} \left( G^2 \frac{\partial (G^2)}{\partial x} \right) = -\frac{\partial p}{\partial x} - f \frac{G^2}{2D_k \rho} + \rho g \cos \theta
\]
\[
\rightarrow \int_{\text{comp}}^{\text{outlet}} \frac{\partial G}{\partial t} dx = \int_{\text{comp}}^{\text{inlet}} \left( -\frac{\partial p}{\partial x} - f \frac{G^2}{2D_k \rho} \right) dx
\]
\[
\rightarrow \sum_{k} L_k \frac{\partial h_k}{\partial t} = \Delta P_{\text{comp}} - \Delta P_{\text{terb}} - \Delta P_{\text{fric}}
\]
\[
\frac{\partial h_k}{\partial t} = \frac{\Delta P_{\text{comp}} - \Delta P_{\text{terb}} - \Delta P_{\text{fric}}}{\sum_{k} L_k / A_k}
\]

Since SC\textsubscript{O}2PE piping lines have equal height, gravity effect can be ignored. To obtain change of the mass flow rate with respect to time, momentum conservation equation is integrated from compressor outlet to inlet. The results of right hand side after integration become net pressure in the fluid system. If pressure is balanced, the mass flow rate is kept constant, which means steady state.

2.2. Component modeling in Simulink

As equations (1)–(3), the governing equations are simplified for the first order differential equation. Since Simulink provides analytic solver for differential equations, the simplified equations (1)–(3) can be directly solved.

- Mass flow rate modeling (Momentum equation solver)

- Pipe modeling with pressure drop (Energy equation solver)

In Figure 2, each block diagram shows equation (2). The fluid properties can be obtained by tabulating the properties based on the REFPROP database. Except for the PCHE flow path, the pressure drop of other flow paths can be calculated with Gnielinski correlation.

\[
f = (0.7904 \ln(\text{Re}) - 1.64)^2
\]

- Precooler modeling

One of the important components in SC\textsubscript{O}2PE is the precooler. Commonly, S-CO\textsubscript{2} fluid approaches to the critical point when the fluid passes through the precooler flow path. Near the critical point the thermal properties of S-CO\textsubscript{2} is stiffly changed so that the general LMTD method is not fitted to calculate heat transfer amount between cold and hot side of S-CO\textsubscript{2} precooler. Hence, modified LMTD method should be applied in the precooler modeling [10]. Basic form of the modified LMTD method can be expressed in equation (5). Here, \(Q_{\text{off}}\) stands for heat transfer amount.

\[
Q_{\text{off}} = \frac{(UA)_{\text{off}} (\text{LMTD})_{\text{off}} F_{\text{off}}}{(UA)_{\text{on}} (\text{LMTD})_{\text{on}} F_{\text{on}}} \cdot Q_{\text{true-on}}
\]

In equation (5), \(Q_{\text{true-on}}\) means the real heat transfer amount in the precooler at the design point, which can be evaluated by using KAIST-HXD. The HXD code divides a heat exchanger flow path into a few hundred meshes, and it makes the total heat transfer amount converges to the real value by iteratively calculating energy balance in the divided meshes. Figure 3 shows how KAIST-HXD code calculates the heat transfer amount.

- Precooler modeling

Table I: Friction factor and Nusselt number of PCHE

<table>
<thead>
<tr>
<th></th>
<th>Water</th>
<th>CO\textsubscript{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction factor</td>
<td>(f = 6.9982 \text{Re}^{-0.766})</td>
<td>(f = 0.0748 \text{Re}^{-0.19})</td>
</tr>
<tr>
<td>Nusselt number</td>
<td>(\text{Nu} = 0.2829 \text{Re}^{0.666} \text{Pr}^{0.18})</td>
<td>(\text{Nu} = 0.8405 \text{Re}^{0.5784} \text{Pr}^{1.18})</td>
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Correction factor \( F \) can be expressed in equation (6)

\[
Z = \left[ \frac{UA}{\text{min}} \left( \frac{1}{(nC_p)_{\text{hot}}} + \frac{1}{(nC_p)_{\text{cold}}} \right) \right]_{\text{min}} - \left[ \frac{UA}{\text{max}} \left( \frac{1}{(nC_p)_{\text{hot}}} + \frac{1}{(nC_p)_{\text{cold}}} \right) \right]_{\text{max}}
\]

\( F = -0.09Z + 1.21 \)

By introducing correction factor, error of heat transfer amount using LMTD method can be substantially reduced. The following figure shows precooler modeling in Simulink.

Fig. 4. Precooler modeling in Simulink

- Compressor and expansion valve modeling

For \( \text{SCO}_2\text{PE} \) compressor, constant isentropic efficiency and pressure ratio with respect to mass flow rate can be proposed because \( \text{SCO}_2\text{PE} \) compressor has very small pressure ratio: \( \sim 1.05 \).

In case of expansion valve, since enthalpy is conserved, expansion ratio should be applied in the expansion valve.

- \( \text{SCO}_2\text{PE} \) layout modeling

Integrating previous components (pipe, precooler, compressor, expansion valve), \( \text{SCO}_2\text{PE} \) layout can be modeled in Simulink. An experimental dataset which was gained from reducing water flow rate in the precooler will be validated with \( \text{SCO}_2\text{PE} \) modeling in Simulink.

Fig. 5. \( \text{SCO}_2\text{PE} \) layout modeling using Simulink

Fig. 6. Nodalization of \( \text{SCO}_2\text{PE} \) and comparison between Simulink and experimental data

Comparison of steady state data between experimental data and Simulink is shown in figure 6. There are maximally 2\% error in temperature and 0.6\% error in pressure.

- Condition boundary for transient

From steady state transient simulation is implemented with a boundary condition which is reduction of flow rate in precooler of water side. Figure 7 shows the mass flow rate of water side of precooler which is kept 0.1kg/s until 64sec after that mass flow rate of water side of precooler is abruptly reduced.

Fig. 7. Comparison of mass flow rate of water side of precooler as the boundary condition
The results from figure 8 to figure 10 show pressure, temperature and mass flow rate data of CO2 in transient condition. Steady state showed quite low deviation but transient results show relatively large difference to experimental data (Pressure: 19%, Temperature: 23%). This is because bold assumptions and cancelations in governing equations (1) to (3). Moreover, performance map of SCO2PE compressor is not obtainable so that it might be another reason that makes the errors to be large. For further works governing equation will be enhanced and it is expected to show a better simulation result later.

For controllability and operability of S-CO2 power cycle, KAIST has built SCO2PE device which is an integral experimental S-CO2 loop. Until now a system code for gas cooled cycle called GAMMA+ code was used to model the device but the modeling of SCO2PE with GAMMA+ code has a limitation in controller design. Therefore, Simulink is used to simulate SCO2PE in this paper to recognize characteristic equation of real S-CO2 system. Even though steady state results of Simulink well match with experimental data, the deviation between Simulink and experimental data begin to be widen when mass flow rate of water side is decreasing. However, the results might be improved after governing equations are enhanced.

4. Conclusions

REFERENCES
