Markov Chain Monte Carlo for Reliability Analysis: A Concept Exploration

[1]Jeremiah Mbazor, [2]Marco Torbol
Ulsan national Institute of Science and Technology
mbazorjc@unist.ac.kr, mtorbol@unist.ac.kr

Introduction

Markov chain and Markov process are two common group of methods applied to analysing complex engineering systems to compute their probabilities of failure. The main advantage of Markov chain is its ability to track the evolution of the reliability of a system in time domain. Markov chain Monte Carlo (MCMC) computes the probability of failure by running many simulations through the transition matrix until the final state is reached or enough time has passed. As opposed to direct sampling, MCMC can exhibit large errors. However, it is rich because it can accommodate large set of variables.

Recent developments and design requirements have pushed the limits of reliability methods like first order reliability method (FORM) and second order reliability method (SORM) to the point where new methods must be developed[1, 2] . For example, to estimate the probability of failure of a multivariate system represented with an indicator function $I_{f(x)}$.

$$p_F = p(x \in F) = \int_{R^d} \pi(x) I_F(x) dx$$

Indicator function $I_F(x)$ must be computed for all x. We obtain the classical definition of a Markov process: then the transition probability matrix is the cross product of the state vector. The Markov probability is defined as:

$$P(X) = \pi(x_0) * \prod_{t=0}^{T} P(x_t \mid x_{t-1}) \qquad (1)$$

Where,

$P(x_t \mid x_{t-1})$ is the probability of transitioning to stat x_t given that it was in state x_{t-1} . we multiply the values of the transition probability matrix form t = 0 to T. $\pi(x_0)$ is the transition probability at t = 0.

Markov Chain Concept

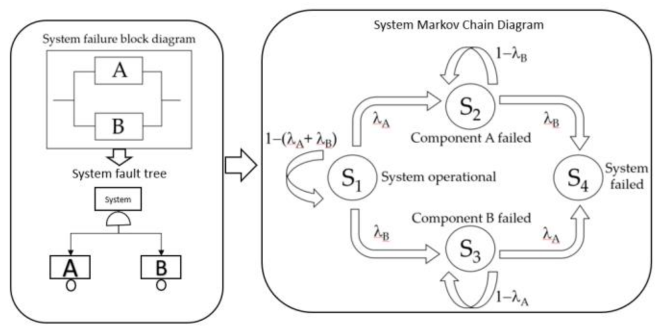


Figure 1: Two Stage Markov Chain

Fig. 1, shows a two-stage Markov chain whose transition probability matrix is

$$P = \begin{bmatrix} 1-(\lambda_A+\lambda_B) & 0 & 0 & 0 \\ \lambda_A & -\lambda_B & 0 & 0 \\ \lambda_B & 0 & -\lambda_A & 0 \\ 0 & \lambda_B & \lambda_A & 0 \end{bmatrix}$$

[1]

Where,

$1-(\lambda_A+\lambda_B)$ = probability of being in state S_1 at time t,

λ_A = probability of moving to state S_2 from S_1.

λ_B = probability of moving to S_3 from S_1

The negatives signify remaining in the same state. Thus, figure 1 gives a complete picture of all possible state of the system when they are to move from a known initial state in time t. This is a static transition matrix. The properties of the static Markov chain Monte Carlo are

- Reversibility and
- Stationarity

It is reversible because we can move between states irrespective of where we are currently and stationary because the transition matrix is independent of the number of steps we take in moving in between the states.
Therefore, the Monte Carlo realization from the pdf of the mean μ_m is expressed as [3]:

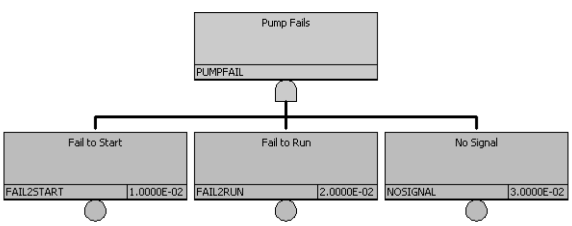
$$\mu_{MC} = \frac{1}{M} \sum_{i=1}^{M} f(x^{(i)}) \tag{2}$$

Where $\left\{x^{(i)}\right\}_{i=1}^{M}$ are the samples from the pdf $f_X(x)$.

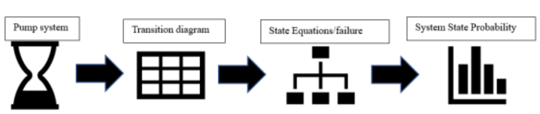
Application

MCMC can be applied in the analysis of pump failure by fault tree decomposition

Pump failure



To apply MCMC to the pump system shown above, we follow the following steps:



The following transition state diagram is obtained:

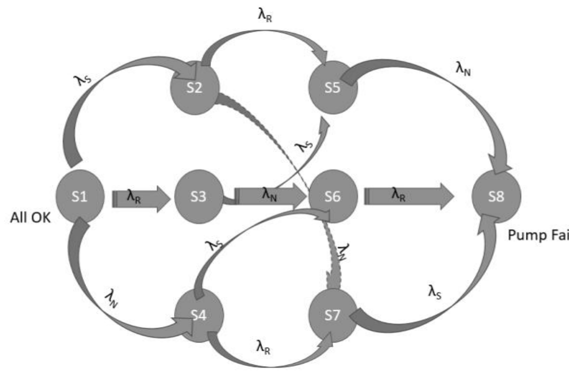


Figure 2: Markov State Transition Diagram

$$\mathrm{P}\left\{x_{n+1} = x_i \mid x_n = x_i\right\} = P_{ij}; \quad i = 1, \ldots n; \; j = 1, \ldots n \tag{3}$$

Equation 3 drives the realisation of the transition: λ_S is the probability of failure, when the pump fails to start; λ_R is the probability of failure, when the pump fails to run; and λ_N is the probability of failure, when there is no signal. The probability of failures is combined by an AND gate to lead to the total failure of the system (pump). It is assumed that the pump failure due to AND gate is the ideal case but for the Markov decomposition and analysis, we consider it as an OR gate. This will enable us to obtain the probability of failure of the system as a function of the three basic events: ‘failure to start’, ‘failure to run’ and ‘no signal.’

Acknowledgment

This work was supported by the 2017 Research Fund (1.180015.01) of UNIST (Ulsan National Institute of Science and Technology).

REFERENCES

[1] L. Cizelj, B. Mavko, and H. Riesch-Oppermann, "Application of first and second order reliability methods in the safety assessment of cracked steam generator tubing," *Nuclear Engineering and Design*, vol. 147, no. 3, pp. 359-368, 1994/04/01/ 1994.

[2] H. Shoji, M. Shinozuka, and S. Sampath, "A Bayesian evaluation of simulation models of multiple-site fatigue cracks," *Probabilistic Engineering Mechanics*, vol. 16, no. 4, pp. 355-361, 2001/10// 2001.

[3] E. Zio, *The Monte Carlo Simulation Method for System Reliability and Risk Analysis* (Springer Series in Reliability Engineering). London: Springer London, 2013.