The Behavior of Axial Nodes Relative Power Distribution in SMART by using Different Cubic Spline Method Function Sets in SCOPS System

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1. Introduction

In Order to synthesize axial power distribution, the spline function set is determined and the magnitude of spline function is also determined. Detector response is ultimately proportional to the core peripheral power, so the axial shape can be predicted by comparing the sizes of each response. The shape of the power distribution is can be determined within 8 types, which those types goes in the categories (middle-peaked, saddle-shaped, or flat) depending on the relative magnitude of detector responses. Using shape index (k) depending on the type of power distribution, the magnitude of the spline function is calculated.

Here, the behavior of axial nodes relative power in different spline function sets (k=1~8) will be shown.

2. Selection and Magnitude of Spline Function Set

The shape index (k) which indicates the shape of power distribution is determined as shown in Table 1.

Table 1 Shape index (k) depending on Power Distribution

Table 2

k	Shape	P_2''' (%)	$\Delta_{P'''} = \lvert P_1''' - P_3'''\rvert$ (%)	Function Set
1	Middle-Peaked	> PWRM2	≤ PWRC1	2882
2		> PWRM2	> PWRC1	2873
3	Flat	PWRM1 ≤ P_2'''≤ PWRM2	< PWRF1	2882
4		PWRM1 ≤ P_2'''≤ PWRM2	PWRF1 ≤ $\Delta_{P'''}$ ≤ PWRF2	2873
5		PWRM1 ≤ P_2'''≤ PWRM2	> PWRF2	2837
6	Saddle	< PWRM1	< PWRS1	2882
7		< PWRM1	PWRS1 ≤ $\Delta_{P'''}$ ≤ PWRS2	2882
8		< PWRM1	> PWRS2	2882

Where, PWRM1,2= Breakpoint power to distinguish axial power distribution shape (middle-peaked, flat, saddle).
PWRC1 = Breakpoint to test the degree of asymmetry of center-peaked power distribution.
PWRF1,2 = Breakpoint to test the degree of asymmetry of flat power distribution.
PWRS1,2 = Breakpoint to test the degree of asymmetry of saddle power distribution.

Normally, the value of spline function is expressed as shown below.

$$\overline{A} = \overline{H_k}^{-1} \cdot \overline{B} \qquad (1)$$

where,
$\overline{A}$ = Vector of spline function magnitude
$\overline{H_k}^{-1}$ = 5x5 matrix associated with shape index (k)
$\overline{B}$ = B_1 to B_5 (Element of power vector)

The 5x5 matrix ($\overline{H_k}^{-1}$) matrices for all k values are pre-calculated and saved in the database, and used when k is determined depending on the characteristics of detector responses. Therefore, the actual algorithm is expressed as follows:

$$A_i = \sum_{j=1}^{5} B_j \cdot HC_n \quad \text{for } i=2,6 \qquad (2)$$

Where A_i is the magnitude of the spline functions (i=2~6), the spline functions (A_i, i=2,...,6) calculated from the formula above is a 5x1 vector, and boundary values are not included. Therefore, they are determined as shown below.

$$A_1 = - A_2 /4$$
$$A_7 = - A_6 /4$$

The HC_n matrix has 25 elements for each k, which are used for both KDIR=1 and KDIR=-1. KDIR represent the element order. In other words, for k=1, if KDIR=1, then n has the value of 1~25 as i and j increase; if KDIR= -1, then n has the value of 25~1 as i and j increase.

3. Cubic Spline Methodology

It is assumed that the core axial power distribution is a sum of spline functions as shown below.

$$\phi(z) = \sum a_i \mu_i(z) \qquad (3)$$

where,

$\phi(z)$ = Neutron flux at axial location z.
a_i = Amplitude coefficients.
$\mu_i(z)$ = Cubic spline basis function.

For calculating the relative axial nodes power, a total of seven spline functions are used.

$$\mathrm{FZ(i)} = \sum_{J=1}^{7} A_J \cdot \mu_J(i) \qquad (4)$$

where,
FZ (i) = Relative axial power of i-th node, i=1,20
μ_J (i) = Value of J-th spline function at i-th node, (i=1,20, J=1,7)
A_J = Magnitude of J-th spline function, J=1,7

Under this method, however, μ_J (i) has to be calculated every time or values of seven spline functions (A_J, J=1,7) of eight shape indices (k=1,8) for all nodes (n=1,20) are to be calculated and saved in advance, which requires considerable calculating time and memory. Hence, an alternative method is used instead. First, the axial region is divided into 4 regions where the spline function is to be applied. The number of nodes for each region is predetermined in Table 1 depending on the type of power shape. Then, the equations are solved for each axial node.

The four regions in one spline function (basis function) are shown in Figure 1. Each region, the μ_i can be determined differently according to the following:

$$\mu_i(z) = f_1(\eta_1) \quad for \quad z_{i-2} \le z \le z_{i-1}$$
$$\mu_i(z) = f_2(\eta_2) \quad for \quad z_{i-1} \le z \le z_i$$
$$\mu_i(z) = f_1(\eta_3) \quad for \quad z_i \le z \le z_{i+1}$$
$$\mu_i(z) = f_2(\eta_4) \quad for \quad z_{i+1} \le z \le z_{i+2}$$

Where

$$\eta_1 = \frac{z - z_{i-2}}{z_{i-1} - z_{i-2}}$$

$$\eta_2 = \frac{z - z_{i-1}}{z_i - z_{i-1}}$$

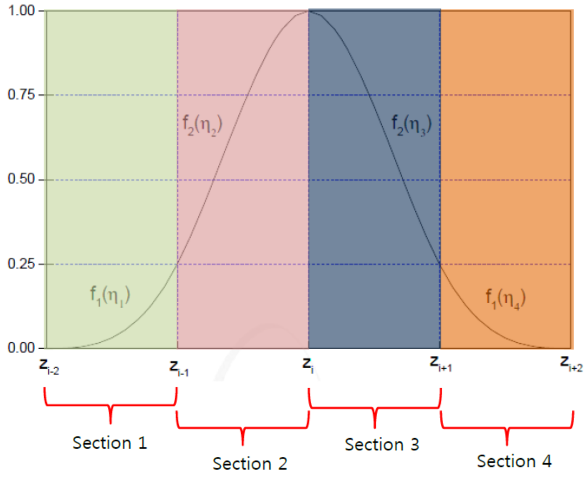
$$\eta_3 = \frac{z_{i+1} - z}{z_{i+1} - z_i}$$

$$\eta_3 = \frac{z_{i+2} - z}{z_{i+2} - z_{i+1}}$$

And

$$f_1(\eta) = \frac{\eta^3}{4}$$
$$f_2(\eta) = \frac{1}{4} + \frac{3}{4}(\eta + \eta^2 - \eta^3)$$

Figure 1



4. Assumptions and Conditions for the Methodology

a) Detector Responses are divided into three regions. The length of each detector 1/3 of the active core length. Hence, D_1 covers z = 0 to z = 0.3333, D_2 covers z = 0.3333 to z = 0.6666, and D_3 covers z = 0.6666 to z = 1.0 of the active core length. The detectors responses are then the summation of each region detector response.

$$D_i = \int_i \phi(z)dz \qquad i = 1,2,3$$

b) As for the empirical boundary point powers;

$$\phi(0) = \alpha_1 D_1 + \alpha_2$$
$$\phi(H) = \alpha_3 D_3 + \alpha_4$$

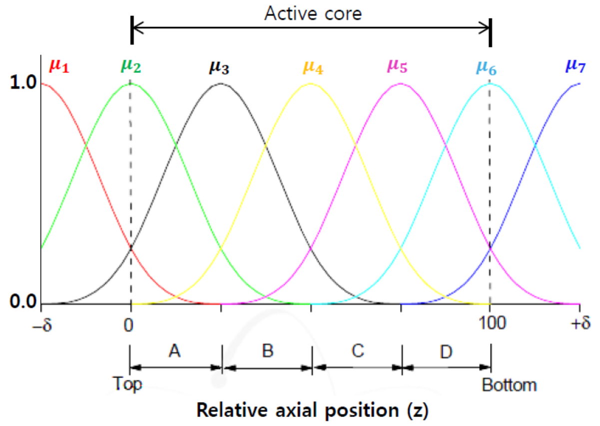
c) As for the extrapolated boundary conditions;

$$\phi(-\delta) = 0$$
$$\phi(H + \delta) = 0$$

Where
α_{1-4}: Empirically correlated coefficients for boundary point powers.
δ: Extrapolated length.

It is assumed that the core axial power distribution is the of the spline functions as shown in Figure 2.

Figure 2



As it is shown in the above figure, the active core is divided into four subsections. The length of each subsection is determined by the shape index (k). For example, if KDIR = 1 and k = 2, then the number of axial nodes in A, B, C, D will be 2, 8, 7, 3 respectively. Therefore, the relative axial length of each subsection will be 10, 40, 35, 15.

The next step is to transform all the integration terms into 7x7 matrix by applying the following:

- **at z=−δ** $\quad \phi(-\delta) = 0.0 = a_1\mu_1(-\delta) + a_2\mu_2(-\delta) + 0 + \cdots + 0$
- **at z=0** $\quad \phi(0) = a_1\mu_1(0) + a_2\mu_2(0) + a_3\mu_3(0) + 0 + \cdots + 0$
- **Detector 1** $\quad D_1 = \int_0^{33.33\cdots} a_1\mu_1(z) + a_2\mu_2(z) + a_3\mu_3(z) + a_4\mu_4(z) + a_5\mu_5(z)\,dz$
- **Detector 2** $\quad D_2 = \int_{33.33\cdots}^{66.66\cdots} a_2\mu_2(z) + a_3\mu_3(z) + a_4\mu_4(z) + a_5\mu_5(z) + a_6\mu_6(z)\,dz$
- **Detector 3** $\quad D_3 = \int_{66.66\cdots}^{100} a_3\mu_3(z) + a_4\mu_4(z) + a_5\mu_5(z) + a_6\mu_6(z) + a_7\mu_7(z)dz$
- **at z=100** $\quad \phi(100) = 0 + \cdots + 0 + a_5\mu_5(100) + a_6\mu_6(100) + a_7\mu_7(100)$
- **at z= +δ** $\quad \phi(+\delta) = 0.0 = 0 + \cdots + 0 + a_6\mu_6(+\delta) + a_7\mu_7(+\delta)$

$$\underbrace{\begin{pmatrix} 0 \\ \phi(0) \\ D_1 \\ D_2 \\ D_3 \\ \phi(100) \\ 0 \end{pmatrix}}_{B} = \begin{pmatrix} h_{11}a_1 & h_{12}a_2 & 0 & 0 & 0 & 0 & 0 \\ h_{21}a_1 & h_{22}a_2 & h_{23}a_3 & 0 & 0 & 0 & 0 \\ h_{31}a_1 & h_{32}a_2 & h_{33}a_3 & h_{34}a_4 & h_{35}a_5 & 0 & 0 \\ 0 & h_{42}a_2 & h_{43}a_3 & h_{44}a_4 & h_{45}a_5 & h_{46}a_6 & 0 \\ 0 & 0 & h_{53}a_3 & h_{54}a_4 & h_{55}a_5 & h_{56}a_6 & h_{57}a_7 \\ 0 & 0 & 0 & 0 & h_{65}a_6 & h_{66}a_6 & h_{67}a_7 \\ 0 & 0 & 0 & 0 & 0 & h_{76}a_6 & h_{77}a_7 \end{pmatrix} = \underbrace{\begin{pmatrix} h_{11} & h_{12} & 0 & 0 & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & 0 & 0 & 0 & 0 \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & 0 & 0 \\ 0 & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} & 0 \\ 0 & 0 & h_{53} & h_{54} & h_{55} & h_{56} & h_{57} \\ 0 & 0 & 0 & 0 & h_{65} & h_{66} & h_{67} \\ 0 & 0 & 0 & 0 & 0 & h_{76} & h_{77} \end{pmatrix}}_{H\ (7x7)} \cdot \underbrace{\begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix}}_{A}$$

Since a_1 and a_7 can be described with respect to a_2 and a_7 respectively, the coefficient matrix (H) can be changed into 5x5.

$$\begin{pmatrix} 0 \\ \phi(0) \\ D_1 \\ D_2 \\ D_3 \\ \phi(100) \\ 0 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & 0 & 0 & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & 0 & 0 & 0 & 0 \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & 0 & 0 \\ 0 & h_{42} & h_{43} & h_{44} & h_{45} & h_{46} & 0 \\ 0 & 0 & h_{53} & h_{54} & h_{55} & h_{56} & h_{57} \\ 0 & 0 & 0 & 0 & h_{65} & h_{66} & h_{67} \\ 0 & 0 & 0 & 0 & 0 & h_{76} & h_{77} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \\ a_7 \end{pmatrix}$$

$$\begin{pmatrix} \phi(0) \\ D_1 \\ D_2 \\ D_3 \\ \phi(100) \end{pmatrix} = \begin{pmatrix} h_{11}a_1 & h_{12}a_2 & 0 & 0 & 0 \\ h_{21}a_1 & h_{22}a_2 & h_{23}a_3 & h_{24}a_4 & 0 \\ h_{31}a_1 & h_{32}a_2 & h_{33}a_3 & h_{34}a_4 & h_{35}a_5 \\ 0 & h_{42}a_2 & h_{43}a_3 & h_{44}a_4 & h_{45}a_5 \\ 0 & 0 & 0 & h_{54}a_4 & h_{55}a_5 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & 0 & 0 & 0 \\ h_{21} & h_{22} & h_{23} & h_{24} & 0 \\ h_{31} & h_{32} & h_{33} & h_{34} & h_{35} \\ 0 & h_{42} & h_{43} & h_{44} & h_{45} \\ 0 & 0 & 0 & h_{54} & h_{55} \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix}$$

5. Results

The results shown in the following figures are for shape indices (k=1~8) for KDIR = -1. The results will show the behavior of the spline functions and relative Axial Nodes Relative Power (FZ) according to reactor height. Where the core height is from top to bottom (0% ~100%).

Figure 3: k = 1

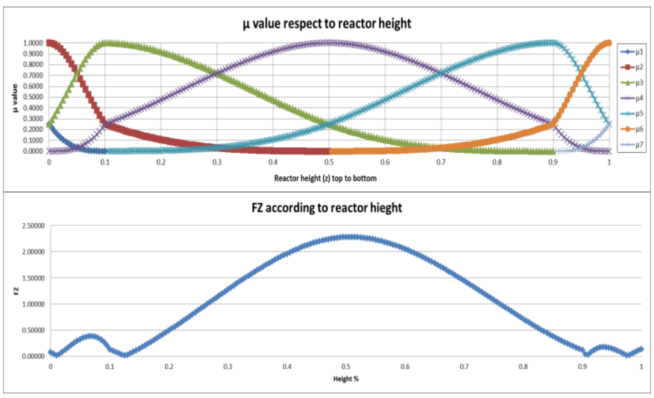


Figure 4: k = 2

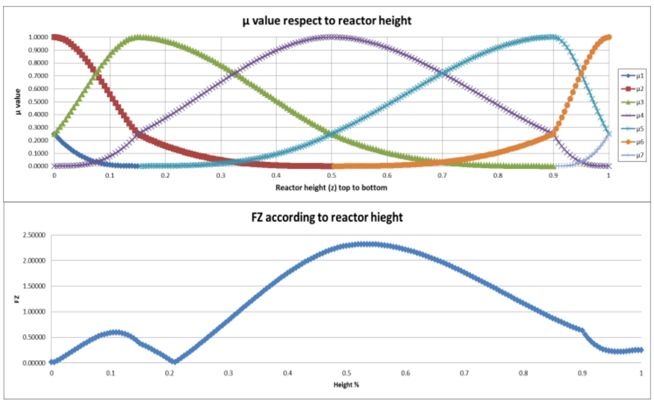


Figure 6: k = 3

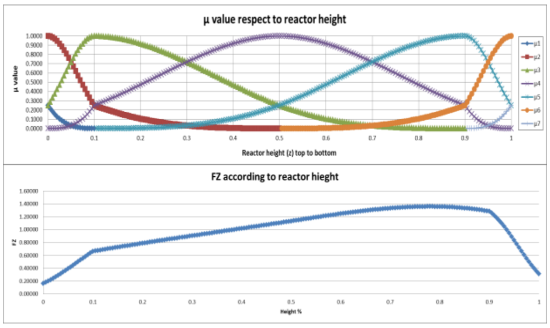


Figure 7: k = 4

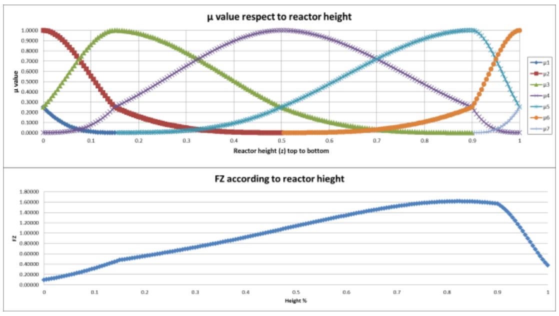


Figure 8: k = 5

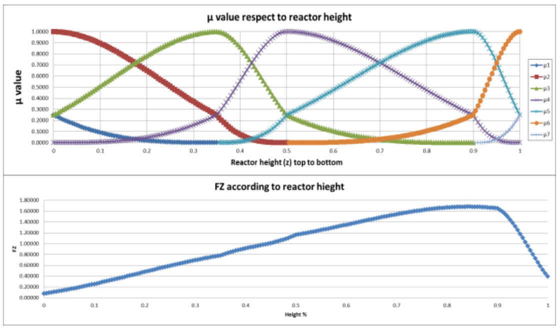


Figure 9: k = 6

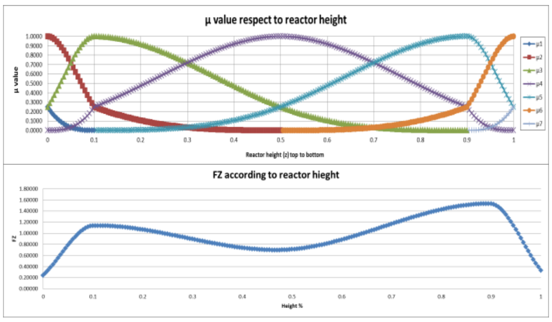


Figure 5: k = 7

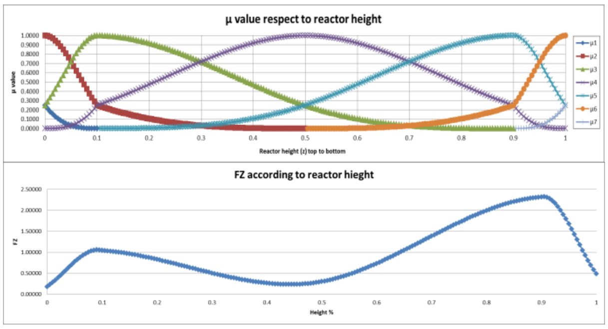
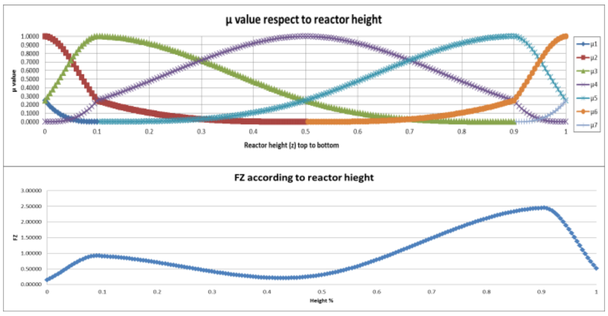


Figure 10: k = 8



6. Conclusion

The cubic spline synthesis method is used in axial power distribution synthesis. The core is axially divided into 4 spline function regions, and the shape of cubic spline function in each region is determined by the number of breakpoints. There are 8 power distribution shapes which are called shape indices that give the axial nodes distribution type.

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REFERENCES

[1] S. R. Shin, "Functional Design Requirements for SMART Core Protection System", S-004-NR444-013, Rev. 04, KAERI, 2018.02.