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Verification of a COPA Mechanical Analysis Module

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1. Introduction

The mechanical analysis module of the COPA code [1,2] treats the mechanical analysis for the coating layers of a coated fuel particle (CFP) of a high temperature reactor (HTR) such as an inner high-density pyrocarbon (IPyC) layer, a silicon carbide (SiC) layer, and an outer high-density pyrocarbon (OPyC) layer. It uses a finite element method utilizing the Galerkin form of the weighted residuals procedure [3] to calculate the stress and displacements of the coating layers.

According to the American Institute of Aeronautics and Astronautics (AIAA) and the American Society of Mechanical Engineers (ASME), verification is defined as "the process of determining that a model implementation accurately represents the developer's conceptual description of the model and the solution to the model" [4,5]. In verification (of the code and the solution), the association or relationship of the simulation to the real world is not an issue [6].

This study treats the verification of the COPA mechanical analysis module using the previously performed benchmark problems and their theoretical solutions and other code solutions.

2. Verification

The normal operation benchmark problems of a coordinated research program (CRP) of the International Atomic Energy Agency (IAEA), IAEA CRP-6 [7] was selected as problems for COPA verification, The COPA results are compared with analytical solutions, semi-analytical solutions [8], and other countries' code solutions [7].

2.1. A single spherical layer

Cases 1 and 2 of the IAEA CRP-6 normal operation benchmark problems test simple thermo-mechanical behaviors of the inner pyrocarbon (IPyC) or silicon carbide (SiC) layer of a CFP. Fig. 1 shows a single spherical layer. The layer is assumed to be elastic only. Constant internal and ambient pressures are acting on the inner and outer surface of the layer, respectively.

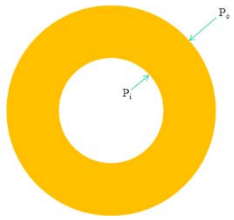

Fig. 1. A single spherical layer.

In this case, the analytical stresses and radial displacement of a layer are given by:

$$\sigma_r(r) = P_i \frac{r_i^3\left(r_o^3 - r^3\right)}{r^3\left(r_o^3 - r_i^3\right)} - P_o \frac{r_o^3\left(r_i^3 - r^3\right)}{r^3\left(r_o^3 - r_i^3\right)}, \tag{1}$$

$$\sigma_\theta(r) = -P_i \frac{r_i^3\left(2r^3 + r_o^3\right)}{2r^3\left(r_o^3 - r_i^3\right)} + P_o \frac{r_o^3\left(2r^3 + r_i^3\right)}{2r^3\left(r_o^3 - r_i^3\right)}, \tag{2}$$

$$u(r) = -P_i \frac{ar^3 r_i^3 + br_i^3 r_o^3}{2r^2\left(r_o^3 - r_i^3\right)} + P_o \frac{ar^3 r_o^3 + br_i^3 r_o^3}{2r^2\left(r_o^3 - r_i^3\right)}, \tag{3}$$

where r_i and r_o are the inner and outer radii of a layer (μm), P_i (≤ 0) and P_o (≤ 0) are the pressures acting on the inner and outer surfaces of a layer (MPa), and $a = 2(1-2\nu)/E$, $b = (1+\nu)/E$, ν is Poisson's ratio (unitless), E is Young's modulus (MPa), σ is the stress (MPa), u is the radial displacement (μm), and the subscripts r and θ indicate the radial and tangential directions, respectively.

Table I lists the maximum tangential stresses acting on a single spherical layer which are calculated using the COPA and the above analytical solution. They are in very good agreement.

Table I: Maximum tangential stresses acting on a single spherical layer.

Layer	Maximum tangential stress (MPa)	
	Analytical solution	COPA
IPyC	50.30	50.13
SiC	125.24	125.22

2.2. A double spherical layer

This particle has two coating layers, IPyC and SiC, as shown in Fig. 2. The two coating layers are assumed to be elastic only. Constant internal and ambient pressures are acting on the inner surface of IPyC and the outer surface of SiC, respectively.

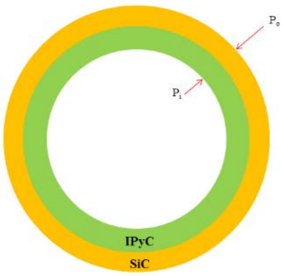

Fig. 2. IPyC/SiC composite.

The analytical stresses and radial displacements of the IPyC and SiC layers are given by:

$$\sigma_{r,IPyC}(r) = P_i \frac{r_2^3\left(r_3^3 - r^3\right)}{r^3\left(r_3^3 - r_2^3\right)} - q\frac{r_3^3\left(r_2^3 - r^3\right)}{r^3\left(r_3^3 - r_2^3\right)}, \qquad (4)$$

$$\sigma_{\theta,IPyC}(r) = -P_i \frac{r_2^3\left(2r^3 + r_3^3\right)}{2r^3\left(r_3^3 - r_2^3\right)} + q\frac{r_3^3\left(2r^3 + r_2^3\right)}{2r^3\left(r_3^3 - r_2^3\right)}, \qquad (5)$$

$$u_{IPyC}(r) = m_1 P_i + m_2 q, \qquad (6)$$

$$\sigma_{r,SiC}(r) = q\frac{r_3^3\left(r_4^3 - r^3\right)}{r^3\left(r_4^3 - r_3^3\right)} - P_o\frac{r_4^3\left(r_3^3 - r^3\right)}{r^3\left(r_4^3 - r_3^3\right)}, \qquad (7)$$

$$\sigma_{\theta,SiC}(r) = -q\frac{r_3^3\left(2r^3 + r_4^3\right)}{2r^3\left(r_4^3 - r_3^3\right)} + P_o\frac{r_4^3\left(2r^3 + r_3^3\right)}{2r^3\left(r_4^3 - r_3^3\right)}, \qquad (8)$$

$$u_{SiC}(r) = n_1 q + n_2 P_o, \qquad (9)$$

where

$$q = \frac{-m_1(r_3)P_i + n_2(r_3)P_o}{m_2(r_3) - n_1(r_3)}, \qquad (10)$$

= the contact stress at the interface between IPyC and SiC (MPa),

$$m_1 = -\frac{a_{IPyC}r^3r_2^3 + b_{IPyC}r_2^3r_3^3}{2r^2\left(r_3^3 - r_2^3\right)},\ m_2 = \frac{a_{IPyC}r^3r_3^3 + b_{IPyC}r_2^3r_3^3}{2r^2\left(r_3^3 - r_2^3\right)},$$

$$n_1 = -\frac{a_{SiC}r^3r_3^3 + b_{SiC}r_3^3r_4^3}{2r^2\left(r_4^3 - r_3^3\right)},\ n_2 = \frac{a_{SiC}r^3r_4^3 + b_{SiC}r_3^3r_4^3}{2r^2\left(r_4^3 - r_3^3\right)},$$

$$a = 2(1-2\nu)/E,\ b = (1+\nu)/E,$$

r_2 is the radial coordinate of the inner surface of IPyC (μm), r_3 is the radial coordinate of the inner surface of SiC (μm), r_4 is the radial coordinate of the outer surface of SiC (μm), E is Young's modulus (MPa), ν is Poisson's ratio (unitless), σ is the stress (MPa), u is the radial displacement (μm), and the subscripts r and θ indicate the radial and tangential directions, respectively.

Solving Eqs. (4) through (10) gives the analytical maximum tangential stresses of 8.72 and 104.41 MPa for IPyC and SiC, respectively. The stresses calculated using COPA are 8.78 and 104.42 MPa. They are in very good agreement.

2.3. A TRISO experiencing a cyclic temperature history

Fig. 3 shows a TRISO particle experiencing a cyclic temperatures. This problem is Case 8 of the IAEA CRP-6 normal operation benchmark problem [7]. It is assumed that the particle experiences ten cycles where the temperature is initially 873 K and increases linearly to 1273 K, and then decreases immediately back to 873 K. The period for each cycle is one-tenth the total irradiation time, or 100 days. Fig. 4 shows the maximum tangential IPyC and SiC stresses as a function of fast fluence which are calculated using the COPA and the semi-analytical method [8]. They are in very good agreement. Fig. 5 shows code-to-code comparisons for maximum stresses of IPyC and SiC as a function of fluence. The other countries' code results are extracted from the IAEA report [7]. They are in good agreement.

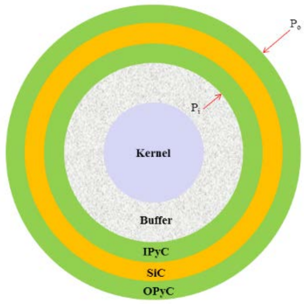

Fig. 3. A TRISO.

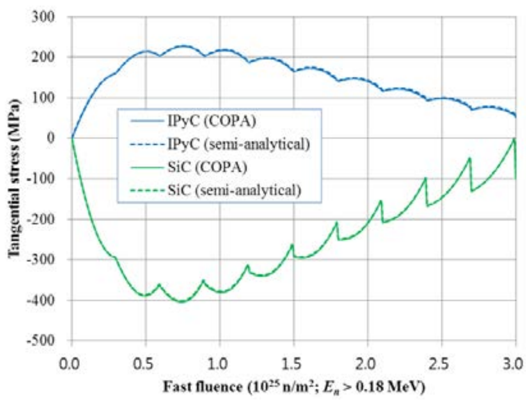

Fig. 4. Maximum stresses of IPyC and SiC as a function of fluence.

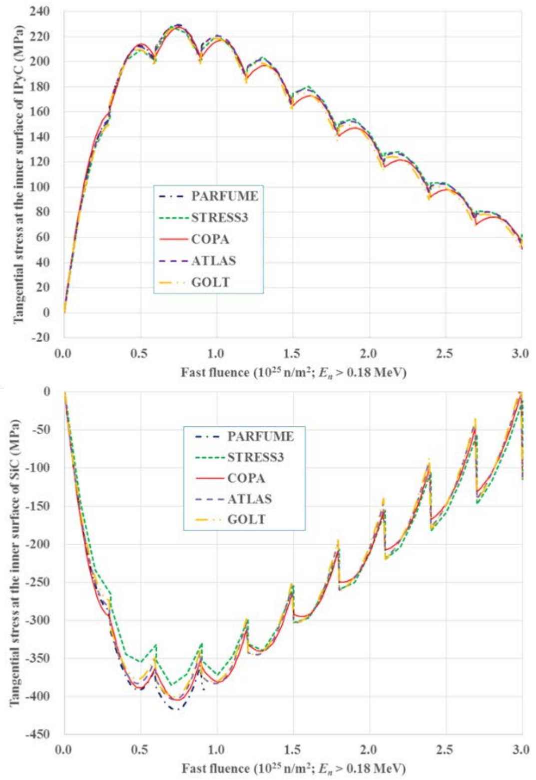

Fig. 5. Code-to-code comparison for maximum stresses of IPyC and SiC as a function of fluence.

3. Summary

The COPA mechanical analysis module has been verified for mechanical analysis problems treating single-layer particles, double-layer particles, and TRISOs simulating realistic conditions. The stresses calculated using COPA were in very good agreement with the analytical solutions and the semi-analytical solutions. A code-to-code comparison showed the COPA stresses had been in good agreement with other countries' code solutions. It is judged that the COPA mechanical model accurately represents the conceptual description of the model and the solution to the model.

ACKNOWLEDGEMENTS

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