Performance Evaluation of Moving Mesh Method with Higher-Order Numerical Scheme Applied to 1D Thermal-Hydraulic System Analysis Code

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Introduction

▣ The existing nuclear system analysis codes such as RELAP5, COBRA-TF, TRAC, MARS and SPACE use the first-order numerical scheme in both space and time discretization.

▣ The 1st order numerical scheme is very robust and stable. But it is highly diffusive and less accurate. These characteristics are critical drawback in modeling the dramatically fluctuated situation like LOCA (Loss Of Coolant Accident).

▣ First, the 1st order numerical scheme on the fixed grid can occur excessive numerical diffusion problem in simulation of accident condition due to the dramatic fluctuation. So, the prediction is less accurate and conservative than reality.

▣ Second is very strict global requirement on the time step for the dramatic fluctuation. The time step should be extremely small in order to reduce the error near the regions where the gradients should be high during the analysis. This results in inefficient computational cost. And even the code is dead.

TWICE code with Moving Mesh Method

▣ A single phase transient analysis code which is possible to calculate in the first-order and the higher-order scheme but mimics MARS solver is built in MATLAB environment. This code is called TWICE code (Transient Water system analysis code with ICE method). In this code, the moving mesh method is applied to compare the performance of the moving mesh method and the higher-order numerical schemes.

▣ Moving mesh method

$$\frac{\partial f}{\partial t}+\frac{\partial (fu)}{\partial x}=S,\ \ x_i^n=X(\xi_i,t_n)$$

So, the differential equations (1) are transformed to an equation in (ξ, t) using a smooth mapping function $X(\xi, t)$.

$$(\tilde{f}u)_\xi=(fu)_x X_\xi \rightarrow (fu)_x=\frac{(\tilde{f}u)_\xi}{X_\xi}$$

$$\tilde{f}_t=f_t+X_t f_x=f_t+\frac{X_t\tilde{f}_\xi}{X_\xi}\rightarrow f_t=\tilde{f}_t-\frac{X_t\tilde{f}_\xi}{X_\xi}$$

$$(X_\xi\tilde{f})_t+\left((u-X_t)\tilde{f}\right)_\xi=X_\xi\tilde{S}$$

$$k_{i+1/2}^{n+1}f_{i+1/2}^{n+1}=k_{i+1/2}^{n}f_{i+1/2}^{n}-\frac{\Delta t}{\Delta\xi}\left[\left(u_{i+1}^{n+1}-\dot{x}_{i+1}^{n}\right)\dot{f}_{i+1}^{n}-\left(u_i^{n+1}-\dot{x}_i^n\right)\dot{f}_i^n\right]$$

where $k_{i+1/2}^n=X_\xi(\xi_i,t_n)$, $\dot{x}_i^n=X_t(\xi_i,t_n)$ and

$$\dot{f}_i^n=f_{i-1/2}^n+\phi(1-\nu_i)\frac{f_{i+1/2}^n-f_{i-1/2}^n}{2}\ \text{if}\ u_{i-\frac{1}{2}}^{n+1}\ge 0$$

$$=f_{i+1/2}^n-\phi(1-\nu_i)\frac{f_{i+1/2}^n-f_{i-1/2}^n}{2}\ \text{if}\ u_{i-1/2}^{n+1}\le 0$$

Numerical scheme for the spatial	
1st order upwind scheme	ϕ=0
2nd order upwind scheme	ϕ=3, ν=0
Lax-Wendroff scheme	ϕ=1
Centered differencing scheme	ϕ=1, ν=0

▣ Moving mesh PDE

To determine the movement of mesh points, the moving mesh PDE approach by Huang et al. [1] is used.

$$\frac{\partial}{\partial\xi}\left(M\frac{\partial\dot{x}}{\partial\xi}\right)=-\frac{1}{\tau}\frac{\partial}{\partial\xi}\left(M\frac{\partial x}{\partial\xi}\right)$$

where M is the monitor function, τ is temporal smoothing parameter.

A commonly used form of the monitor function is the arclength monitor function [1].

$$M_{i+1/2}=\sqrt{1+\frac{1}{\alpha}\left|\frac{\bar{Q}_{i+1}-\bar{Q}_i}{x_{i+1}-x_i}\right|^2}$$

To smooth the mesh, a regularized version $\widetilde{M}$ is used.

$$\widetilde{M}_{i+1/2}=\sqrt{\sum_{k=i-i_p}^{k=i+i_p}M_{k+\frac{1}{2}}^2\left(\frac{\gamma}{1+\gamma}\right)^{|k-i|}\Big/\sum_{k=i-i_p}^{k=i+i_p}\left(\frac{\gamma}{1+\gamma}\right)^{|k-i|}}$$

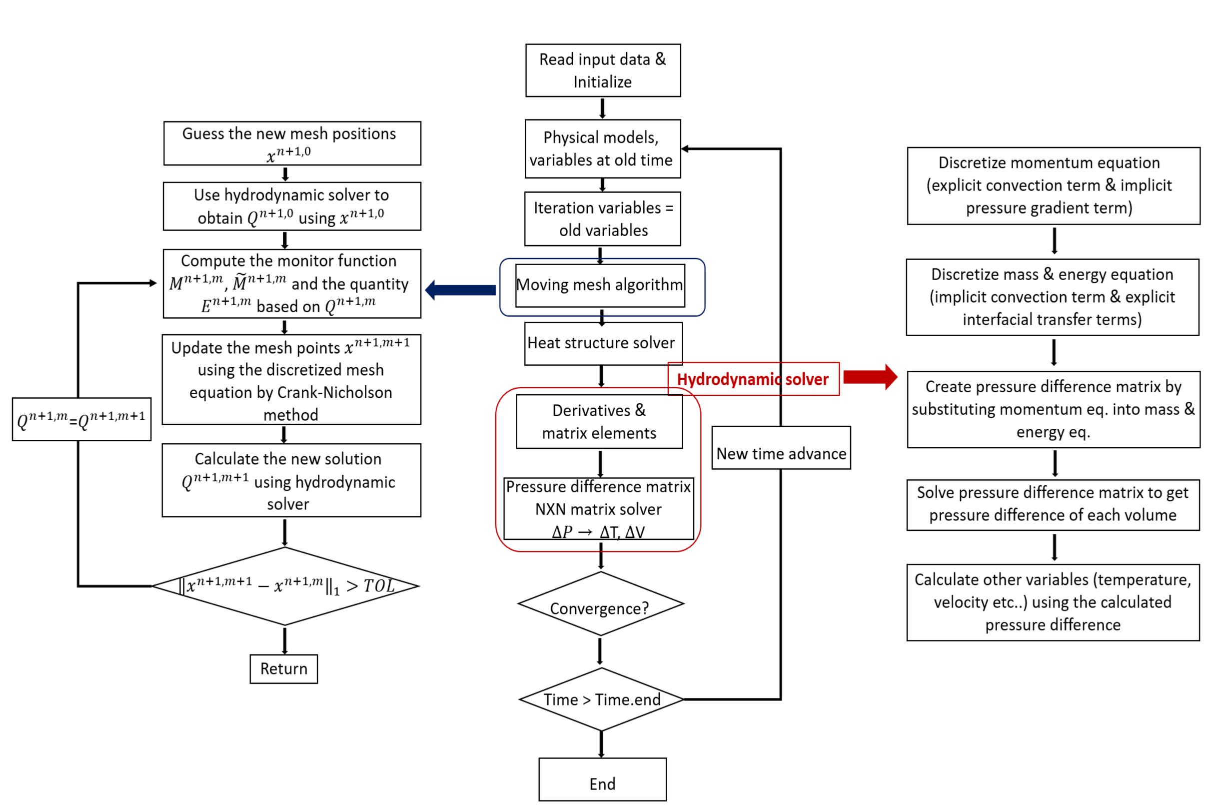
where γ and i_p are the spatial smooth factors. $\gamma=2$ and $i_p=4$ are recommended in [1].

By Crank-Nicholson discretization,

$$\widetilde{M}_{i+\frac{1}{2}}^{n+1}\left(x_{i+1}^{n+1}-x_i^{n+1}\right)-\widetilde{M}_{i-\frac{1}{2}}^{n+1}\left(x_i^{n+1}-x_{i-1}^{n+1}\right)=\widetilde{M}_{i+\frac{1}{2}}^{n+1}(x_{i+1}^n-x_i^n)-\widetilde{M}_{i-\frac{1}{2}}^{n+1}(x_i^n-x_{i-1}^n)-\frac{\Delta t^n}{2\tau}\left(E_i^{n+1}+E_i^n\right)$$

where E_i is a centered approximation to the term on the right hand side of eq. (6) given by

$$E_i=\widetilde{M}_{i+\frac{1}{2}}(x_{i+1}-x_i)-\widetilde{M}_{i-\frac{1}{2}}(x_i-x_{i-1})$$

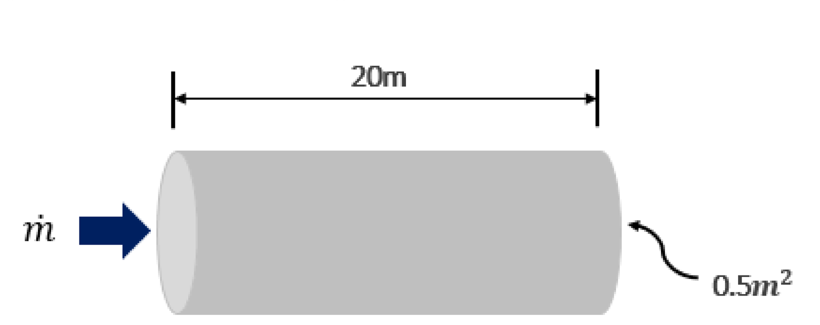


▲Algorithm of TWICE code with the moving mesh method

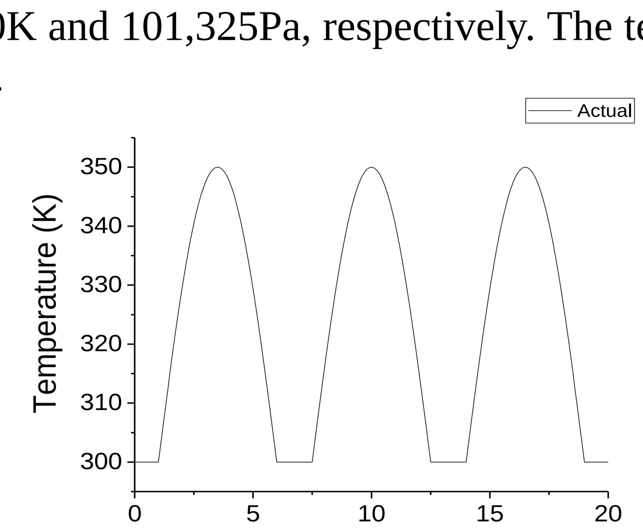
Numerical Test Problem

▣ A single phase pipe flow with a sine pulse of temperature is modeled by MARS and the TWICE codes with several higher-order numerical schemes separately and the results are compared to each other.

▣ The initial temperature and pressure of the fluid is 300K and 101,325Pa, respectively. The temperature of the injected fluid is changed with time as shown in Fig. 3.



▲ Configuration of single phase pipe flow with sine pulse of temperature

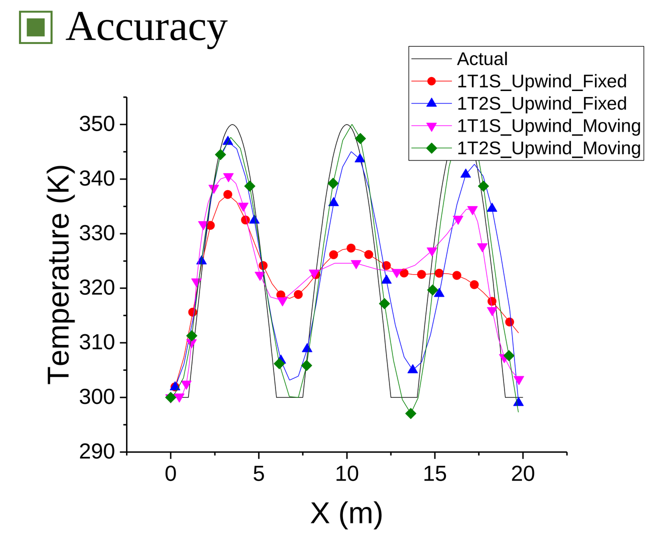


▲ Temperature profile of fluid injected at pipe inlet

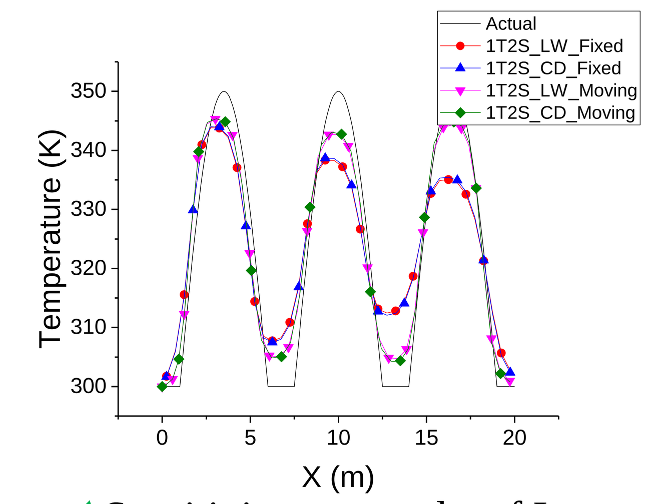
▣ This simulation is performed for several numbers of meshes to evaluate the accuracy improvement and compare the computational efficiency of the moving mesh grid compared to the fixed grid. A sensitivity test for several combinations of spatial and temporal higher-order schemes is conducted.

Numerical Results

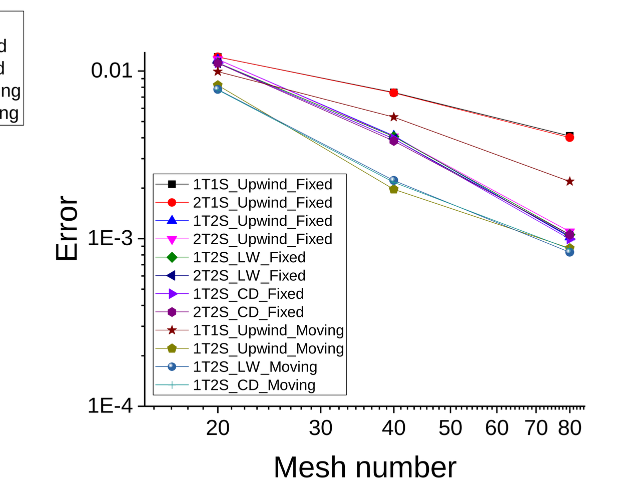
▣ Accuracy



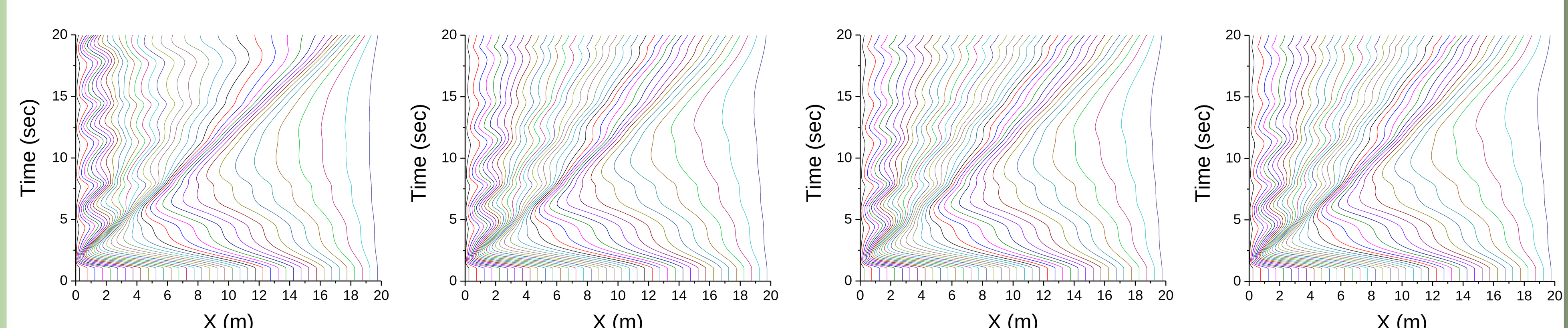
▲Sensitivity test results of the 1st order and 2nd order upwind scheme on the fixed mesh and moving mesh



▲Sensitivity test results of Lax-Wendroff (LW) and centered differencing (CD) scheme on the fixed mesh and moving mesh



▲Estimated error for each numerical schemes along the mesh number



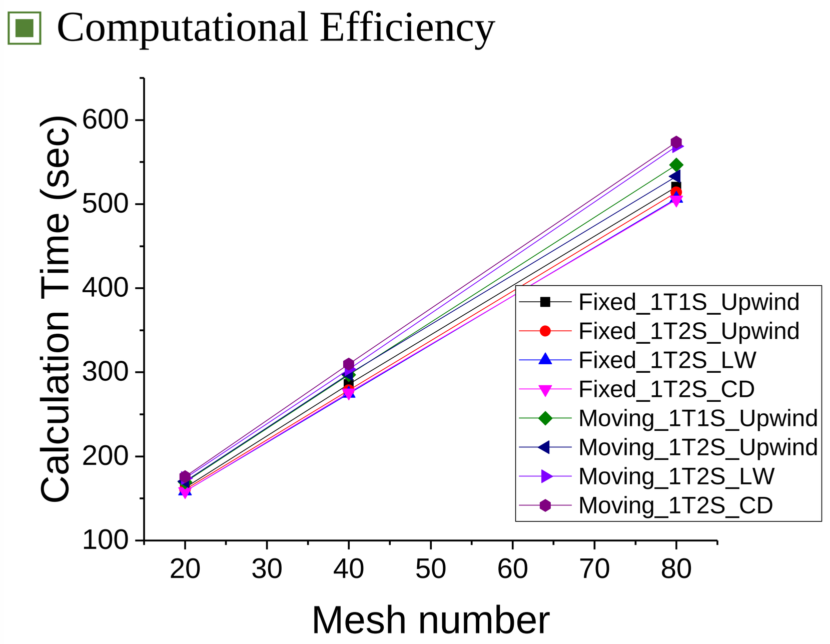
(a) 1T1S upwind (b) 1T2S upwind (c) 1T2S Lax-Wendroff (d) 1T2S centered differencing

▲Mesh movement along the time for each numerical schemes

▲Estimated error and convergence rate for error

Mesh number	1T1S_Upwind_Fixed	2T1S_Upwind_Fixed	1T2S_Upwind_Fixed	2T2S_Upwind_Fixed	1T2S_LW_Fixed	2T2S_LW_Fixed	1T2S_CD_Fixed	2T2S_CD_Fixed	1T1S_Upwind_Moving	1T2S_Upwind_Moving	1T2S_LW_Moving	1T2S_CD_Moving
20	0.01213	0.01212	0.01174	0.01174	0.01121	0.01117	0.01118	0.01115	0.00992	0.00824	0.00776	0.00772
40	0.00745	0.00741	0.00411	0.00406	0.00407	0.00394	0.00395	0.00382	0.00532	0.00197	0.00222	0.00217
80	0.00409	0.00401	0.00102	0.0011	0.00106	0.00103	9.95E-04	0.00105	0.0022	8.75E-04	8.29E-04	8.68E-04
Convergence rate	0.7842	0.79786	1.7624	1.70793	1.70133	1.71946	1.74516	1.70429	1.08642	1.61724	1.61349	1.57668

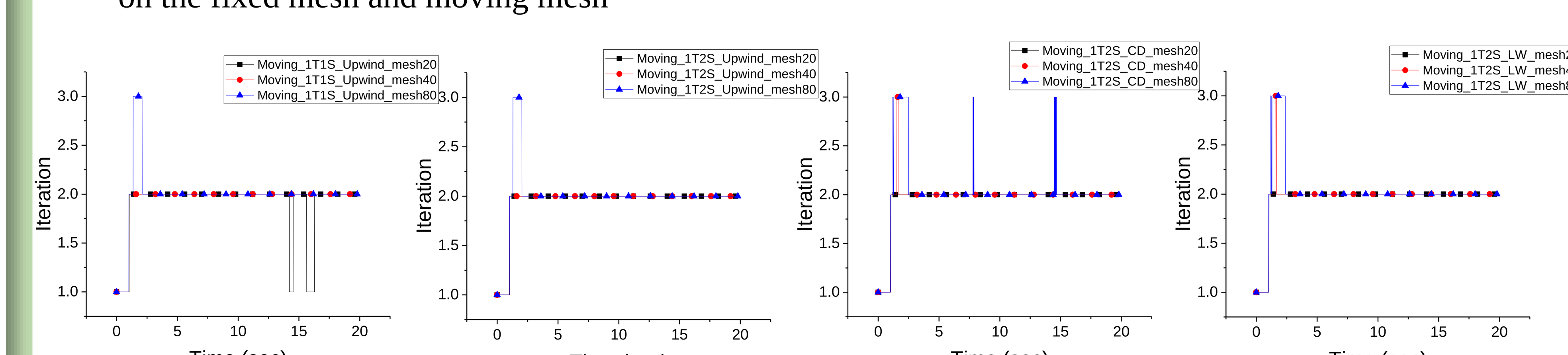
▣ Computational Efficiency



▲Calculation time of each numerical schemes on the fixed mesh and moving mesh

▲Calculation time of each numerical schemes on the fixed mesh and moving mesh (τ=0.1)

Mesh Number.	Fixed_1T1S_Upwind	Fixed_1T2S_Upwind	Fixed_1T2S_LW	Fixed_1T2S_CD
20	162.7	160.8	158.7	158.4
40	286.0	278.8	274.5	275.9
80	520.7	514.1	506.5	505.4
Mesh Number.	**Moving_1T1S_Upwind**	**Moving_1T2S_Upwind**	**Moving_1T2S_LW**	**Moving_1T2S_CD**
20	169.5	170.3	174.1	176.1
40	297.0	298.1	303.5	309.8
80	546.5	532.9	568.7	573.9



(a) 1T1S upwind (b) 1T2S upwind (c) 1T2S Lax-Wendroff (d) 1T2S centered differencing

▲Number of iteration in the moving mesh algorithm for each numerical schemes (τ=0.1)

Conclusions

▣ This study evaluated the performance of the moving mesh method with the higher-order numerical schemes for the next generation nuclear system analysis code.

▣ The accuracy is slightly improved in the moving mesh than the fixed mesh since the mesh points move depending on the propagation of the temperature pulse along time. However, the convergence rate for the error becomes lower on the moving mesh.

▣ The number of the iteration for determining the movement of the meshes is small. So, the calculation time on the moving mesh is not much different with the fixed mesh. Also, there is no difference between the higher-order numerical schemes on the calculation time.

▣ Since the time step control on the fixed mesh is not carried out, the moving mesh method applied to the nuclear system analysis code has the possibility of improvement for the calculation efficiency.

▣ For further works, the performance of the moving mesh algorithm depends on the temporal smoothing factor τ, which determines the concentration of the meshes. So, depending on this factor, the accuracy and the computational efficiency will be evaluated.

▣ In case of the wall heat transfer, the coupling between the hydrodynamic mesh and the heat structure mesh should be considered. If the hydrodynamic meshes move axially along the pipe in one dimension, the movement of the heat structure meshes should be considered in two dimensions. So, the coupling between the hydrodynamic mesh and the heat structure mesh will be studied.