*Transactions of the Korean Nuclear Society Autumn Meeting*
*Gyeongju, Korea, October 26-27, 2017*

# Preliminary Study on the Applicability of Adjoint Based Optimization Method to Single Phase Thermal-Hydraulic System Analysis Code for Node Optimization

Jae Jun Lee, Min-Gil Kim, Seongmin Son, Jeong Ik Lee*
*Department of Nuclear and Quantum engineering, Korea Advanced Institute of Science and Technology (KAIST)*
*291 Daehak-ro, (373-1, Guseong-dong), Yuseong-gu, Daejeon 34141, Republic of KOREA*
**Corresponding author: jeongiklee@kaist.ac.kr*

## 1. Introduction

In nuclear engineering, the best estimate plus uncertainty approach is becoming more and more standard approach of the nuclear safety analyses globally. In general, 1-D thermal hydraulic system analysis code simulates the system by dividing it into arbitrarily determined nodes. The arbitrarily determined nodes sometimes cannot effectively reflect sudden changes or local extremal value. Although conventional approaches to solve such problems are to divide the system into more number of nodes, the computational cost goes up and the time step is limited by the length of the nodes as well for the semi-implicit time progress schemes. Therefore, while maintaining reasonable computational cost, the nodalization can be further optimized to yield improved accuracy compared to the user arbitrarily determined nodalization scheme. For a large system that has many parameters governing the system, i.e. nuclear system, gradient based optimization using the adjoint method can be an efficient method for this type of optimization.

In this paper, the authors would study the applicability of the adjoint method for 1-D thermal hydraulic system analysis code for getting the node sensitivity needed for node optimization.

## 2. Methods

In this section, governing equations and discretized equations of 1-D single-phase thermal hydraulic system analysis code (NTS code) written in MATLAB by W. W. Lee and J. I. Lee [1] are presented in section 2.1. Applying the adjoint method to discretized governing equations is presented in section 2.2. In section 2.3, implementation process for the code is described.

### *2.1 Governing equations*

NTS code is consisted of following three equations. Continuity equation is Eq.1. Momentum and energy equation are Eq.2 and Eq.3 respectively. The three equations are solved for pressure, internal energy and velocity in the code.

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial(\rho v A)}{\partial x} = 0 \qquad (1)$$

$$\rho A\frac{\partial v}{\partial t} + \frac{1}{2}\rho A\frac{\partial v^2}{\partial x} = -A\frac{\partial P}{\partial x} + \rho B_x A - \rho FWF(v)A \qquad (2)$$

$$\frac{\partial(\rho U)}{\partial t} + \frac{1}{A}\frac{\partial(\rho v A U)}{\partial x} = -\frac{P}{A}\frac{\partial(vA)}{\partial x} + Q_w + DISS_f \qquad (3)$$

Semi-implicitly discretized equations are Eq.4, Eq.5 and Eq.6.

$$\frac{\rho_i^{n+1} - \rho_i^n}{\Delta t} + \frac{1}{A_i}\frac{(\dot{\rho}_{j+1}^n v_{j+1}^{n+1} A_{j+1} - \dot{\rho}_j^n v_j^{n+1} A_j)}{\Delta x_i} = 0 \qquad (4)$$

$$\rho_j^n \frac{v_j^{n+1} - v_j^n}{\Delta t} + \frac{1}{2}\rho_j^n \frac{(v_i^n)^2 - (v_{i-1}^n)^2}{\Delta x_{j+1}} - \frac{1}{2}\rho_j^n \frac{VISF_j^n}{\Delta x_{j+1}} = -\frac{P_i^{n+1} - P_{i-1}^{n+1}}{\Delta x_{j+1}} + \rho_j^n B_x - \rho_j^n FWF_j^n (v_j^{n+1}) \qquad (5)$$

$$\frac{(\rho U)_i^{n+1} - (\rho U)_i^n}{\Delta t} + \frac{1}{A_i}\frac{v_{j+1}^{n+1} A_{j+1}(\dot{\rho}_{j+1}^n \dot{U}_{j+1}^n + P_i^n)}{\Delta x_i} - \frac{1}{A_i}\frac{v_j^{n+1} A_j(\dot{\rho}_j^n \dot{U}_j^n + P_i^n)}{\Delta x_i} = Q_w^n + DISS_f^n \qquad (6)$$

Above equations can be expressed in Eq.7 with dependent variable vector set $\mathbf{X}$ and parameter vector $\mathbf{P}$.

$$N(\mathbf{X},\mathbf{P}) + S(\mathbf{P}) = 0 \quad \mathbf{X} = (P, v, U)$$

$$\begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix}\begin{pmatrix} P^{n+1} \\ v^{n+1} \\ U^{n+1} \end{pmatrix} + \begin{pmatrix} N_{44} & N_{45} & N_{46} \\ N_{54} & N_{55} & N_{56} \\ N_{64} & N_{65} & N_{66} \end{pmatrix}\begin{pmatrix} P^n \\ v^n \\ U^n \end{pmatrix} = \begin{pmatrix} S_1^n \\ S_2^n \\ S_3^n \end{pmatrix} \qquad (7)$$

### *2.2 Application of adjoint method to discretized governing equations*

When objective function as $g \sim g(\mathbf{X}^{n+1}, \mathbf{P})$, parameters i.e. node position as $\mathbf{P}$. The sensitivity is defined as Eq.9.

$$\frac{dg}{d\mathbf{P}} = g_{\mathbf{P}} + g_{\mathbf{X}^{n+1}} \mathbf{X}_{\mathbf{P}}^{n+1} \qquad (8)$$

In Eq.9, differentiating the discretized governing equations with respect to the parameters, an approach called discrete adjoint approach, is used. Discrete adjoint approach is closely tied to the numerical solution method, so it guarantees a duality with the discretized governing equations and has a benefit that it can be applied to the code directly.

$$N(\mathbf{\Phi},\mathbf{P})+N_{\mathbf{P}}(\mathbf{X},\mathbf{P})+S_{\mathbf{P}}(\mathbf{P})=0$$

$$\mathbf{\Phi}=(\Phi_1,\Phi_2,\Phi_3)=(\frac{\partial P}{\partial \mathbf{P}},\frac{\partial v}{\partial \mathbf{P}},\frac{\partial U}{\partial \mathbf{P}}) \qquad (9)$$

$$\begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix}\begin{pmatrix} \Phi_1^{n+1} \\ \Phi_2^{n+1} \\ \Phi_3^{n+1} \end{pmatrix}+\begin{pmatrix} N_{44} & N_{45} & N_{46} \\ N_{54} & N_{55} & N_{56} \\ N_{64} & N_{65} & N_{66} \end{pmatrix}\begin{pmatrix} \Phi_1^{n} \\ \Phi_2^{n} \\ \Phi_3^{n} \end{pmatrix}=\begin{pmatrix} S_1^* \\ S_2^* \\ S_3^* \end{pmatrix}$$

Eq.9 can be expressed with identity matrix $\mathbf{I}$ in Eq.10.

$$\begin{pmatrix} N_{n+1} & N_n \\ 0 & \mathbf{I} \end{pmatrix}\begin{pmatrix} \mathbf{\Phi}^{n+1} \\ \mathbf{\Phi}^{n} \end{pmatrix}=\begin{pmatrix} \mathbf{S}^* \\ \mathbf{\Phi}^{n} \end{pmatrix} \qquad (10)$$

Eq.12 shows that the node sensitivity can be obtained with adjoint function $\boldsymbol{\lambda}$ that satisfying Eq.11. In other words, adjoint method can get the node sensitivity by bypassing the recalculation for $\mathbf{X}_{\mathbf{P}}$.

$$\begin{pmatrix} N_{n+1} & N_n \\ 0 & \mathbf{I} \end{pmatrix}^T\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}=\begin{pmatrix} g_{\mathbf{x}^{n+1}} \\ 0 \end{pmatrix} \qquad (11)$$

$$\frac{dg}{d\mathbf{P}}=g_{\mathbf{P}}+g_{\mathbf{x}^{n+1}}\mathbf{X}_{\mathbf{P}}^{n+1}=g_{\mathbf{P}}+\begin{pmatrix}\lambda_1 & \lambda_2\end{pmatrix}\begin{pmatrix} \mathbf{S}^* \\ \mathbf{\Phi}^{n} \end{pmatrix} \qquad (12)$$

### *2.3 Implementation process for single-phase transient analysis code*

Fig. 1 shows algorithm of NTS code and Fig. 2 shows an algorithm for node optimization. For node optimization, adjoint module and optimization module would be added into the code. Determining the appropriate objective function and getting the sensitivity are processed in adjoint module. Applying the calculated sensitivity to the optimization is done in optimization module.

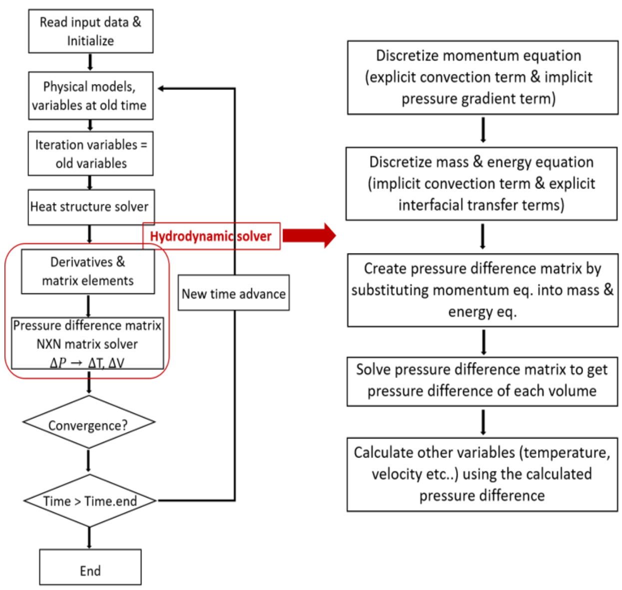


Fig. 1. Algorithm of single phase transient analysis code [1].

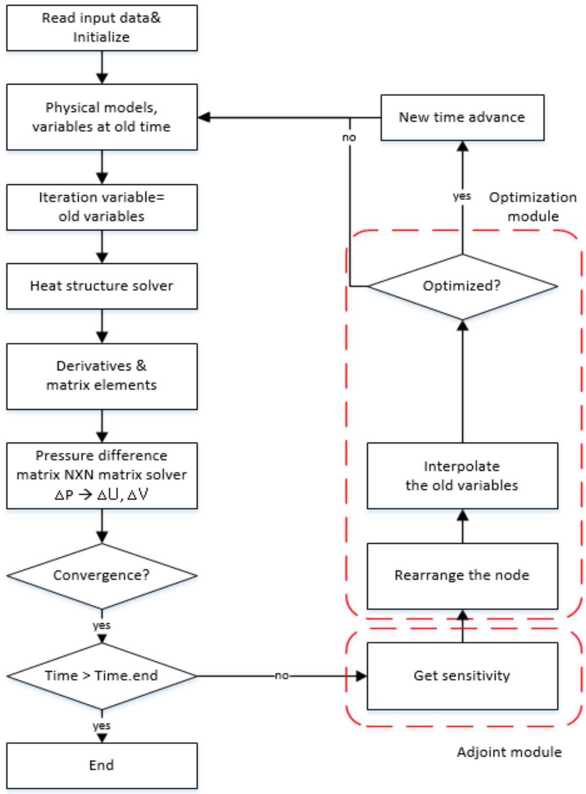


Fig. 2. Algorithm for node optimization.

## 3. Further Works

Applying that to simple case of 1-D thermal hydraulic system simulation and evaluating the node sensitivities will be presented.

## ACKNOWLEDGEMENT

This work was supported by the Nuclear Safety Research Program through the Korea Foundation Of Nuclear Safety (KoFONS), granted financial resource from the Nuclear Safety and Security Commission (NSSC), Republic of Korea. (No. 1603010)

## REFERENCES

[1]W. W. Lee and J. I. Lee, “Performance Comparison of Higher-Order Numerical Schemes Applied on 1D Thermal-Hydraulic System Analysis Code”, Transactions of the Korean Nuclear Society Spring Meeting, May.18-19, 2017, Jeju, Korea.

[2]M. G. Kim, W. W. Lee and J. I. Lee, “Application of Adjoint Sensitivity Analysis Procedure (ASAP) to Single Phase Thermal-Hydraulic System Analysis Code”, Transactions of the Korean Nuclear Society Spring Meeting, May.18-19, 2017, Jeju, Korea.

[3]D. G. Cacuci, Sensitivity and Uncertainty Analysis: Volume I Theory, Chapman & Hall/CRC, 2003.

[4]S. M. Son and J. I. Lee. "Preliminary study of applying adjoint-based mesh optimization method to nuclear power plant safety analysis." Annals of Nuclear Energy 109, 2017, 405-418.