Scaling analysis of various simulators for sodium thermal-hydraulic experiment in a reduced-height scale test facility

Dong Eok Kim*, Jae Min Choi†, Jaehyuk Eoh‡, Moo Hwan Kim§

*Department of Precision Mechanical Engineering, Kyungpook National University, Sangju, South Korea
†Korea Atomic Energy Research Institute, Daejon, South Korea
‡Division of Advanced Nuclear Engineering, POSTECH, Pohang, South Korea
§Corresponding author: dekim@knu.ac.kr

1. Introduction

Sodium-cooled Fast Reactor (SFR) is one of the promising reactor types for Generation IV (Gen IV) nuclear reactor technology. During the past decades, several countries with advanced nuclear reactor technology had constructed and operated the SFR, and currently, China, France, India, Korea, and Russia have actively conducted the R&D works for advanced SFR development.

In the design of a SFR safety system, thermal-hydraulic behavior of liquid sodium (Na) flow inside the reactor is essential. Especially in an accident condition, such as LOOP (Loss Of Off-site Power), passive Decay Heat Removal System (DHRS) should be readily operated with sufficient natural circulation sodium flow induced by gravity. Since the natural circulation behavior of a fluid system is determined under the closely coupled mechanism of heat transfer and hydraulic effects, the experimental validation of the natural circulation system based on rigorous scaling analysis is needed. In practice, since the reduced geometrical scaling is unavoidable for simulating the real phenomena in a large-scale reactor, the selection of same fluid (sodium) as an experimental working fluid would be recommended for reliability and simplification of the analysis and application of the experimental results. However, in the lab-scale experiments by academic or small research group, it is difficult to use liquid sodium as a working fluid due to the risk of sodium-water reaction (SWR) and thereby high safety cost. Hence, the use of simulant fluids and its examination in terms of scaling effect are required.

In this study, we simply introduce the scaling analysis method for single-phase natural circulation system, and investigate the scaling characteristics of various simulants for practical application as working fluids.

2. Methods and Results

First of all, we postulate a simple loop of single-phase natural circulation with a heating region, a cooling region, and the pipe lines (Fig. 1). In this loop, the natural circulation flow is generated by gravity effect due to the density difference between the heating and cooling regions placed in lower and higher positions, respectively. For establishing the non-dimensional mass, momentum, and energy conservation equations, and deriving the key similarity parameters, we utilized the one-dimensional and single-phase dimensional analysis approach by Heisler [1], and Ishii and Kataoka [2].

![Fig. 1. Schematics of a single-phase natural circulation loop.](image)

2.1 Scaling analysis

Eq. (1)-(5) are the one-dimensional simplified conservation equations and the boundary condition [2].

Continuity equation

\[ u_n = \frac{a_n}{a_s} u_s \]  

(1)

Integral momentum equation

\[ \rho \frac{du_n}{dt} + \sum_{a_n} \frac{a_n}{a_s} \mathbf{f}_n = \rho g \beta \Delta T \left[ \frac{d}{d} \left( \frac{B + K}{d} \right) \right] \left( \frac{a_n}{a_s} \right) \]  

(2)

Fluid-side energy equation for nth section

\[ \rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} \right) = \frac{4h}{d} (T_s - T) \]  

(3)

Solid-side energy equation for nth section

\[ \rho c_m \frac{\partial T}{\partial t} + k T_s V^2 - q_s = 0 \]  

(4)

Boundary condition at the solid-liquid interface

\[ -k \frac{\partial T}{\partial x} = h(T_s - T) \]  

(5)
where \( u, a, \rho, t, g, \beta, \Delta T, l, d, f, K, c_p, T, h, k \), and \( \dot{q}_i \) are the fluid velocity, cross-sectional area, density, time, gravity acceleration, volume expansion coefficient, fluid temperature rise, length, hydraulic diameter, friction factor, minor loss coefficient, specific heat capacity, temperature, heat transfer coefficient, thermal conductivity, and volumetric heat generation rate inside solid, respectively. And, subscripts \( o, r, h, \text{ and } s \) denote the reference constant value, representative variable, heating region, and solid, respectively. From the above five equations, we can obtain the non-dimensional equations, and then the key similarity groups can be defined in Table I.

### Table I. Non-dimensional numbers for the similarity

<table>
<thead>
<tr>
<th>Richardson number ( Ri )</th>
<th>Friction number ( F_s )</th>
<th>modified Stanton number ( St_r )</th>
<th>Time ratio number ( T^*_r )</th>
<th>Biot number ( Bi_n )</th>
<th>heat source number ( Q_{sm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{g\Delta T_l J_s}{u_o^2} )</td>
<td>( \frac{f l / d + K}{n} )</td>
<td>( \frac{4hl_o}{(\rho c_p u_o d)} )</td>
<td>( \alpha_i l_o / (\delta^2 u_o) )</td>
<td>( (h\delta / k_o) )</td>
<td>( \left( \frac{q_i l_o}{(\rho c_p u_o \Delta T_s)} \right) )</td>
</tr>
</tbody>
</table>

where \( \alpha_i \) and \( \delta \) are the thermal diffusivity of solid and the conduction (wall) thickness in heating region. In addition to Table I, non-dimensional axial scales, \( l_o = l_o / l_o \) and \( l_h = l_h / l_h \), and cross-sectional flow area scale, \( A_o = a_o / a_o \) can be defined. In Table I, the reference velocity \( u_o \) and temperature rise \( \Delta T_o \) scales are utilized, and those should be obtained in advance for the scaling analysis. Ishii and Kataoka [2] had presented the solutions for \( u_o \) and \( \Delta T_o \) by setting the heating region as a representative section. Thus, the solution of \( u_o \) could be obtained with steady-state calculation of Eq. (2), and that of \( \Delta T_o \) was given by steady-state energy balance consideration between solid and fluid regions

\[
\begin{align*}
    u_o &= \left( \frac{\beta}{\rho c_p} \frac{q_i l_o}{A_o} \right) \left( \frac{a_o}{a_o} \right) \left[ \frac{F_s}{2g \sum \frac{F_s}{A_o^2}} \right]^{1/3}, \\
    \Delta T_o &= \frac{q_i l_o}{\rho c_p u_o a_o} \left( \frac{a_o}{a_o} \right),
\end{align*}
\]

where \( A_o \) is the solid cross-sectional area.

For satisfying complete similarity between model and prototype systems, the ratio of dimensionless numbers in Table I should be

\[
Ri_n = F_{stn} = St_{t} = T^*_r = Bi_{n} = Q_{sm} = 1
\]

where subscript \( R \) denotes the ratio of model to prototype.

Assuming that the similarity condition, \( F_{st} = 1 \) can be unconditionally satisfied by inserting suitable orifice into the model loop, the geometrical similarities for the axial length \( L_{st} = 1 \) and flow cross-sectional area \( A_{st} = 1 \) give the complete kinematic and dynamic similarity. In this case, by substituting Eq. (10) and (11) into Eq. (12), \( Ri_n = 1 \) can be automatically established.

\[
\begin{align*}
    u_{st} &= \frac{u_{om}}{u_{op}} = \left( \frac{q_i}{\rho c_p} \frac{\beta}{a_o} \frac{\delta_{st}}{d_{st}} \right)^{1/3}, \\
    \Delta T_{st} &= \frac{\Delta T_{om}}{\Delta T_{op}} = \left( \frac{1}{\rho c_p} \right) \frac{l_{st}}{u_{op} d_{st}}, \\
    Ri_n &= \beta_s \Delta T_{st} / u_{st}.
\end{align*}
\]

where subscripts \( m \) and \( p \) denote the model and prototype.

In the next stage, the energy similarity conditions \( St_{t} = T^*_r = Bi_{n} = Q_{sn} = 1 \) should be considered. It is noted that the Stanton number similarity is automatically satisfied when other three requirements for \( T^*_r, Bi_{n} \), and \( Q_{sn} \) are established. Through using the same solid materials between the model and prototype, and satisfying the above geometrical similarities, we can get the detailed energy similarity conditions

\[
\begin{align*}
    T^*_r &= l_{st} / (u_{st} / \delta^2_{st}) = 1, \\
    Bi_n &= h_{st} \delta_{st} = 1, \\
    Q_{sn} &= (\rho c_p) s d_{st} / \delta_{st} = 1.
\end{align*}
\]

From eqs. (13)-(15), the scaling ratios for key parameters can be obtained

\[
\begin{align*}
    \delta_{st} &= \frac{l_{st}}{u_{op}}, \\
    d_{st} &= \frac{1}{(\rho c_p) s} \sqrt{\frac{l_{st}}{u_{op}}}, \\
    u_{st} &= \left( \frac{q_i}{\rho c_p} \right)^{1/3}, \\
    \Delta T_{st} &= \frac{\Delta T_{st}}{u_{st}}.
\end{align*}
\]
\[ t_s = \frac{l_s}{u_s}, \quad (20) \]
\[ h_s = \frac{1}{d_s} = \frac{u_s}{l_s}. \quad (21) \]

where \( t_s \) is the time scaling ratio, and subscript \( o \) was omitted for simplicity.

The heat transfer coefficient scaling ratio in Eq. (21), \( h_s \) is closely related to the temperature change across the thermal boundary layer at liquid-solid interface in the heating region, and the value is a strong function of the flow structure and fluid thermo-physical properties. Generally, a correlation for \( h \) is represented in terms of the Nusselt number, \( Nu = \frac{hd}{k} \). In this study, through the approximation of the laminar flow or the liquid metal flow with low velocity, it is assumed that the \( Nu \) is a constant value. From this assumption, the real scaling ratio of the heat transfer coefficient can be determined as Eq. (22). Therefore, for the experimental design of reduced-scale model, the heat transfer similarity by Eqs. (21) and (22) should be carefully considered.

\[ h_{s,\text{cor}} = \frac{k_s}{d_s} = \left( \frac{\rho c_p k_s}{\sqrt{\beta}} \right) \sqrt{l_s}. \quad (22) \]

2.2 Selection of various simulant fluids

In this study, for selecting the simulant fluids of liquid sodium, melting and boiling point, toxicity, fluid-to-water reactivity, procurement cost, and thermo-physical properties with temperature were considered. The selected simulants are water (H\(_2\)O), Galinstan (Ga-In-Sn eutectic alloy), Lead-Bismuth eutectic (LBE, Pb-Bi), Bismuth (Bi), Tin-Bismuth alloy (Sn-Bi), Gallium (Ga), Tin (Sn), and Dowtherm A fluid by Dow company. All fluids except water and Dowtherm A [3-8] are liquid metals. Table II shows the thermo-physical properties of the simulants, and the property data were selected as the values at atmospheric pressure and moderate temperature range between the boiling and melting points. In this table, \( T_m \) and \( T_b \) are the melting and boiling point of the material.

<table>
<thead>
<tr>
<th>Table II. Thermo-physical properties of selected simulants</th>
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<tbody>
<tr>
<td>( T_m ) (°C)</td>
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<tr>
<td>----------------</td>
</tr>
<tr>
<td>Sodium [3] (( T = 500^\circ \text{C} ))</td>
</tr>
<tr>
<td>Water [4] (( T = 60^\circ \text{C} ))</td>
</tr>
<tr>
<td>Galinstan [6] (( T = 300^\circ \text{C} ))</td>
</tr>
<tr>
<td>LBE, Pb-Bi [3] (( T = 500^\circ \text{C} ))</td>
</tr>
<tr>
<td>Bi [3] (( T = 500^\circ \text{C} ))</td>
</tr>
<tr>
<td>Sn-Bi [4] (( T = 300^\circ \text{C} ))</td>
</tr>
<tr>
<td>Ga [5] (( T = 300^\circ \text{C} ))</td>
</tr>
<tr>
<td>Sn [7] (( T = 500^\circ \text{C} ))</td>
</tr>
<tr>
<td>Dowtherm A [8] (( T = 150^\circ \text{C} ))</td>
</tr>
</tbody>
</table>

2.3 Scaling analysis results for the various simulants

For the reduced-scale experiment of a large-scale thermo-hydraulic system, the preceded determination of length scaling ratio, \( l_s \) is necessary, and also the available heating power is an important factor for the experimental design. In this study, for observing the scaling behaviors on the various simulants, the following assumptions are given as

- \( l_s = 0.1 \)
- \( d_s = 1 \)
- same solid material usage

Fig. 2 shows the scaling ratios of velocity and the temperature rise for the simulants. From Eq. (18), the natural circulation velocity and the temperature rise scales of a fluid strongly depend on \( \beta_h \). The two parameters in the model experiments would be significantly lower than those in the prototype as well. Fig. 3 shows the scaling ratios of the conduction thickness and the hydraulic diameter. Both \( d_s \) and \( d_s \) values are smaller than that of the unity in all test simulants, i.e. for thermal-hydraulic similarity condition, the thickness of the heating wall and the hydraulic diameter should be reduced in the design of model experiments. Especially in the case of water, the hydraulic diameter is significantly smaller than that of the prototype. In the reduced-scale experiment, it can rise an excessive scale reduction problem, and the unconditional satisfaction for friction number similarity might not be achieved in a sodium-to-water simulation experiment.
In this study, the thermal and hydraulic scaling characteristics of various simulants were investigated for the purpose of replicating natural circulation system in an SFR. Since the natural circulation phenomenon is generated by the closely coupled effects of both hydrodynamic and thermal behaviors of working fluid, the attentive selection of the simulant fluids based on the rigorous scaling considerations is needed. And, since these works were based on the simple one-dimensional approach, CFD analyses for verifying the results obtained from this study would be recommended as a future work.

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