A Study on the Uncertainty of Flow-Induced Vibration in a Cross Flow over Staggered Tubes

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1. Introduction

Flow-induced vibration (FIV) in a cross flow is an important problem in various engineering fields and it is known to have caused many failures in industrial components including nuclear systems. Especially, cross-flow in many support columns of very high temperature reactor (VHTR) lower plenum would have FIV issues under high speed flow jetting from the core.

For a group of multiple circular cylinders subjected to a cross-flow, three types of potential vibration mechanisms may exist: (1) Vortex-induced vibration (VIV), (2) Fluid-elastic vibration (FEV) and (3) Turbulence-induced vibration (TIV) [1].


The FIV evaluation is usually performed with computational fluid dynamic (CFD) analysis to obtain unknown frequency of oscillation of the multiple objects under turbulent flow and thus the uncertainty residing in the turbulence model used should be quantified. In this paper, potential FIV uncertainty arising from the turbulence phenomena are evaluated for a typical cross flow through staggered tube bundles resembling the VHTR lower plenum support columns.

2. Methods of Analysis

2.1 CFD Analysis of Cross Flow over Staggered Tubes

Major parameters in FIV evaluation for the cross flow given a flow geometry are the mean flow velocity and frequencies of drag and lift force oscillations, and they should be obtained from an experiment or validated CFD analysis. In this paper CFD method is utilized and the CFD validation is validated to prove the vortex shedding behavior past submerged body is correctly captured. And the reference test used for the CFD validation is the staggered tube bundle array by Simonin and Barcouda [5] for which CFD validations are performed previously [6-9]. This staggered tube bundle and a computational domain is shown in Fig. 1 [5] and the dimension is presented in Table 1.

ANSYS FLUENT is used as a solver of the 2D unsteady Reynolds averaged Navier-Stokes (URANS) equations. The URANS uses Standard k-ω turbulence model considering the characteristics of strong low Reynolds effect near the wall [10]. For all the analyses, second order upwind scheme is used.

![Fig. 1. Staggered tube array of Simonin and Barcouda [5].](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reynolds number, Re</td>
<td>18000</td>
</tr>
<tr>
<td>Cylinder Pitch, P (m)</td>
<td>0.0450</td>
</tr>
<tr>
<td>Cylinder Diameter, D (m)</td>
<td>0.0217</td>
</tr>
<tr>
<td>Cylinder Length, L (m)</td>
<td>0.1000</td>
</tr>
<tr>
<td>Cylinder Volume, Vc (m³)</td>
<td>3.696E-04</td>
</tr>
<tr>
<td>Cylinder Density, ρc (kg/m³)</td>
<td>7000.0</td>
</tr>
<tr>
<td>Cylinder mass, M (kg)</td>
<td>2.588</td>
</tr>
<tr>
<td>Pitch/Diameter (P/D)</td>
<td>2.074</td>
</tr>
<tr>
<td>Fluid Density(water), ρ</td>
<td>998.2</td>
</tr>
</tbody>
</table>

The Standard k-ω model is an empirical model based on the transport equation of the turbulence kinetic energy (k) and the specific dissipation rate (ω). The equation for k contains additional turbulent fluctuation terms that is linked to the mean flow and ω is an inverse time scale that is associated with the turbulence. The
coefficient of the TKE ($\alpha_\sigma$) determines the energy in the turbulence and the SDR ($\sigma_o$) does the scale of the turbulence and both coefficients are related to diffusivities of the k-\(\omega\) model. Coefficients of the TKE and SDR Prandtl numbers have effect on the turbulence diffusivity then the other coefficients [10-13]. In this study, therefore, the TKE and SDR Prandtl numbers are considered for evaluation of the FIV uncertainty arising from the turbulence models.

In order to quantify FIV uncertainty arising from these turbulence phenomena, the coefficients used in the Standard k-\(\omega\) turbulence model are manipulated based on the ranges of $\sigma_k$ and $\sigma_o$ available from the literature: the ranges reported are $0.5 \sim 1.25$ and $0.5 \sim 0.856$, respectively. The code-recommended values for the two numbers are $2.0$ [10]. Therefore, following two cases are considered in this paper:

- Case 1: Code-recommended value
- Case 2: Lower values to expedite the vortex shedding and considering uncertainty.

The turbulence model coefficients are summarized in Table 2. The coefficients other than TKE and SDR Prandtl Numbers are nominal values of Standard k-\(\omega\) turbulence model in the FLUENT code [10] as follows: $\alpha_{w}$ and $\beta_{i}$ are related with damping of the turbulent viscosity, causing a low-Reynolds number correction. $\alpha_{x}$ has an effect of producing $\omega$. $\beta_{x}$ and $\zeta$ are related with the dissipation of $k$ and $\omega$. $M_{e0}$ is related with compressibility [10].

Table 2: Standard k-\(\omega\) turbulence model coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Case 1</th>
<th>Case 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{w}$</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>$\alpha_{x}$</td>
<td>0.072</td>
<td>0.072</td>
</tr>
<tr>
<td>$\beta_{x}$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_k$</td>
<td>2.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$\sigma_o$</td>
<td>2.0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

The fluid is water and the thermal properties used for water are: $\rho=998.2$ kg/m$^3$, $\mu=1.003 \times 10^{-5}$ kg/m/s at the reference temperature of 293.15 K (20 °C). Boundary conditions are all periodic. The Reynolds number is 18,000 based on the diameter and the properties of liquid water used in the experiment [6-8]. The mass flow rate is thus set to 40.75 kg/s.

The computational mesh is shown in Fig. 2. The cell size is basically 0.0002 m and the cell size near the cylinder wall is halved to 0.0001 m. Total number of the cells is 63,466. The time step size is fixed at 0.0001 seconds.

The data that are needed for the present FIV evaluations but are not available from the Simonin and Barcouda [5] are obtained from the other references.

2.2 Method of FIV evaluation

There are three types of potential vibration mechanisms of FIV and they are VIV, FEV and TIV. When the natural frequency of the structure is close to the vortex-shedding frequency, VIV occurs. VIV is usually considered for a single cylinder.

On the other hand, arrays of multiple cylinders can oscillate with large amplitudes when they are exposed to high velocity fluid flow. In this situation, FEV may occur and this is the most severe vibration mechanism that would cause structural failure. TIV usually results in arrhythmic pressure drop around structures and leads to vibration [14-16]. Among these vibrational mechanisms, VIV and FEV phenomena are evaluated in the present paper.

In FIV, vibrations occur in two directions, transverse and parallel to the flow. In the transverse direction, the excitation force has a dominant frequency called the Karman vortex shedding frequency. The vortex shedding is generally expressed in terms of Strouhal Number [1] as following:

$$St = \frac{f_s D}{V} \quad (1)$$

where $f_s$ is the vortex shedding frequency, $D$ is the characteristic length of the object and $V$ is the flow speed.

In evaluating the structural stability under VIV, natural frequency ($f_n$) and vortex shedding frequency ($f_s$) are required for the frequency ratio, $f_s$, and this ratio is expressed in the following form [1].
On the other hand, evaluating the structural stability under FEV needs reduced velocity and mass damping. The reduced velocity is calculated with pitch-to-diameter velocity. The velocity can be expressed as follows:

\[ V_{pc} = \frac{VP}{P-D} \]  

where \( P \) is the cylinder pitch and \( D \) is the diameter of a cylinder [17].

The reduced velocity \( (V_R) \) is a dimensionless value expressed in the following form:

\[ V_R = \frac{V_{pc}}{f_s D} \]  

where \( f_s \) is the natural frequency of a structure.

The mass damping \( (m_d) \) is given by:

\[ m_d = m \delta / \rho D^2 \]  

where \( m \) is the mass per unit length, \( \delta \) is the damping ratio and \( \rho \) is the fluid density [18].

According to the section III of the ASME Boiler and Pressure Vessel Code [19], following should be satisfied in order to avoid VIV:

\[ 0.7 \text{ (or } 0.8) \leq f_s \leq 1.3 \text{ (or } 1.2) \]  

FEV instability criterion using the mass damping and the reduced velocity was proposed by Connors [20] as follows:

\[ \frac{V_c}{f_s D} = K \left[ \frac{m \delta}{\rho D^2} \right]^{1/2} \]  

where \( V_c \) is the critical flow velocity and \( K \) is the dimensionless factor. Connors [20] also proposed a two-dimensional stability map.

3. Result and Discussion
3.1 Cross Flow over Staggered Tubes

Figures 3 and 4 show subsequent velocity magnitude contours for the two cases obtained from the present URANS computations with Standard k-\( \omega \) turbulence. Accuracy of these computations in terms of mean velocity profiles and vortex shedding frequency was validated by Choi and Park [6,7] and thus will not be reproduced in this paper.

The in-flow mean velocities for each case obtained are 1.777 and 1.830 m/s, respectively. The velocity magnitude ranges from 0 to 3.165 m/s for the Case 1 and from 0 to 4.011 m/s for the Case 2 and the oscillation periods are 0.0582 and 0.0258 seconds, respectively. It can be found that the turbulence coefficients surely affect the frequency of the oscillations. The larger is the velocity, the larger is the frequency (smaller periods).

Figures 5 and 6 present the drag and the lift force variations. Each force is obtained at the left and the bottom of the central cylinder, respectively. For the
Case 1, the periods of drag and lift forces are 0.0146 and 0.0219 seconds, and the vortex shedding frequencies are 68.73 and 45.64, respectively. For the Case 2, the periods are 0.0129 and 0.0259 seconds and the vortex shedding frequencies are 77.52 and 38.61, respectively. The results are summarized in Table 3.

The flow characteristics and thus FIV parameters are truly affected by the turbulence coefficients such as TKE and SDR Prandtl Numbers. It can be thus stated that these coefficients would be the origin of the uncertainty in FIV evaluation for unknown flow conditions [12].

For the FEV evaluation, the reduced velocities are obtained by using Eq.(4) and the values are 8.3 for the Case 1 and 8.6 for the Case 2. The uncertainty in the reduced velocity is about 5%. For the calculation of the mass damping in Eq.(5), damping factor $\delta=0.0344$ is obtained from the literature [21] and the value of $m$, the mass per unit length, used is 25.88 kg/m, $\rho$ is 998.2 kg/m³ and $D$ is 21.7 mm. Thus, the mass damping finally obtained is 11.89 and this value is the same for all the cases.

Table 4: Spreadsheet for VIV and FEV evaluations

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Case 1</th>
<th>Case 2</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Drag force</td>
<td>Lift force</td>
<td>Drag force</td>
<td>Lift force</td>
<td></td>
</tr>
<tr>
<td>Strouhal No., St</td>
<td>0.8394 0.5574</td>
<td>0.9190 0.4577</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch velocity, $V_p$ (s/m)</td>
<td>3.432 3.432</td>
<td>3.535 3.535</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Natural frequency, $f_r$ (Hz)</td>
<td>18.99 18.99</td>
<td>18.99 18.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Frequency Ratio, $f_r=f_p/D$</td>
<td>3.62 2.40</td>
<td>4.08 2.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vortex Shedding Instability</td>
<td>Stable $f_r&gt;1.3$ Stable $f_r&gt;1.3$ Stable $f_r&gt;1.3$ Stable $f_r&gt;1.3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reduced Velocity, $V_k=V_p/D$</td>
<td>8.3 8.3</td>
<td>8.6 8.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass per unit length, $m=ML$ (kg/m)</td>
<td>25.88 25.88</td>
<td>25.88 25.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damping factor, $\epsilon$</td>
<td>0.0344 0.0344</td>
<td>0.0344 0.0344</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Damping ratio, $\delta=2\epsilon\rho D$</td>
<td>0.2160 0.2160</td>
<td>0.2160 0.2160</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass Damping, $m\epsilon/D^2$</td>
<td>11.89 11.89</td>
<td>11.89 11.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fluid-Elastic Instability</td>
<td>Marginally stable Marginally stable Marginally stable Marginally stable</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.2 Evaluation of Flow Induced Vibration Uncertainty

The spreadsheet of the present stability evaluations for VIV and FEV are presented in Table 4. For VIV evaluation, Strouhal number ($St$) is calculated from Eq.(1). The drag and the lift force Strouhal numbers thus calculated for the Case 1 are 0.8394 and 0.5574, and those for the Case 2 are 0.9100 and 0.4577, respectively.

For the natural frequency, previous cross-flow experiment for the square array [21] is referred to and the value used is 18.99 Hz. The frequency ratios for the drag and lift forces obtained by using these data are 3.62 and 2.40 for the Case 1, and 4.08 and 2.03 for the Case 2, respectively. Thus these two cases are stable with respect to VIV according to the stability criteria of Eq.(6). However, it can be found that the effect of turbulence parameters is not negligible and the uncertainty is about 10%.

![Fig. 6. Lift force variations from URANS with Standard k-ω (0.5s ~ 0.6s).](image)

![Fig. 7. Location of FEV evaluation parameters.](image)
however, FEV is the more realistic FIV mechanism than VIV.

4. Conclusion

Flow induced vibration (FIV) is one of the important mechanical and fatigue issues in nuclear systems. Especially, cross-flow in many support structures of VHTR lower plenum would have FIV issues under highly turbulent jet flows from the core. Through the computational fluid dynamic analysis, FIV uncertainty evaluations in terms of turbulence model are performed against an existing cross-flow experiment. The results show that the effect of turbulence parameters on FIV is not negligible and the uncertainty is 5 to 10%. Present method can be applied to future FIV evaluations of nuclear systems. More extensive studies on flow induced vibration in a plant scale by using more rigorous computational methods are under way.

Acknowledgements

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REFERENCES