Method to Calculate Accurate Top Event Probability in a Seismic PSA

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1. Introduction

In order to calculate more accurate top event probability from cutsets, ACUBE (Advanced Cutset Upper Bound Estimator) software [1] had been developed several years ago and shortly became a vital tool for calculating the core damage frequency (CDF) of nuclear power plants in the seismic Probabilistic Safety Assessment (PSA) [2]. As explained in Section 4, ACUBE calculates the top event probability and importance measures from cutsets by (1) dividing cutsets into major and minor groups depending on the cutset probability, where the cutsets that have higher cutset probability are included in the major group and the others in minor cutsets, (2) converting major cutsets into a Binary Decision Diagram (BDD) [3,4]. By applying the ACUBE algorithm to the seismic PSA cutsets, the accuracy of a top event probability and importance measures can be significantly improved [2].

2. Approximate CDF Calculation

In most PSAs, the use of Rare Event Approximation (REA) is justified, since the number of basic events with high probability is limited. However, this approximation is not useful when events with high failure probabilities are considered.

$$P(S) \approx P_{REA} = \sum_{i=1}^{n} P(C_i) \cdot \tag{1}$$

Many tools employ the Min Cut Upper Bound (MCUB) approximation to calculate probability measures such as a top event probability and importance measures.

$$P(S) \approx P_{MCUB} = 1 - \prod_{i=1}^{n} (1 - P(C_i)) \cdot \tag{2}$$

3. Overestimated CDF in a PSA

In this Section, it is proposed that the cutset can be categorized into three groups that are according to the event probabilities and the success terms in cutsets. It is necessary to choose appropriate method to calculate top event probability from cutsets by estimating overestimation level before calculating top event probability. Let us categorize cutsets into Types I to III as
(a) Type I cutets have events that have low probabilities,
(b) Type II cutets have events that have large probabilities, and
(c) Type III cutets have events have success events.

Table I: Category of MCS structures

Type	I	II	III
Cutsets	$S=AB+AC$	$S=AB+AC$	$S = A\overline{B} + A\overline{C}$
Event probabilities	$P(A)$=0.01 $P(B)$=0.01 $P(C)$=0.01	$P(A)$=0.01 $P(B)$=0.50 $P(C)$=0.50	$P(A)$=0.01 $P(B)$=0.01 $P(C)$=0.01
$P(S)$ P_{MCUB} P_{REA}	1.9900e-04 1.9999e-04 2.0000e-04	7.5000e-03 9.9750e-03 1.0000e-02	9.9990e-03 1.9702e-02 1.9800e-02
ΔP_{MCUB} (a) ΔP_{REA} (b)	0.50% 0.50%	33.00% 33.33%	97.04% 98.02%
PSA applications	Level 1 PSA	Level 2 PSA Fire PSA Flooding PSA	Seismic PSA [2]

(a) $(P(S)-P_{MCUB})/P(S))$ * 100.0 (b) $(P(S)- P_{REA})/P(S))$ * 100.0

The typical sample calculation are illustrated in Table I. As shown in Table I,

(a) The overestimation of cutset probability in Type I can be ignored, where all event probabilities are small and no success events exist in cutsets.

(b) Some effort to calculate the accurate top event probability should be done since the amount of overestimation in Type II is not small.

(c) The approximate top event probability calculation method in Section 2 cannot be accepted and one exact top event probability calculation method such as the BDD should be employed to minimize or eliminate the high overestimation in the top event probability calculation. The success events in cutsets results in highly overestimated cutset probability. Seismic cutsets in EPRI method [2] have many success events. ACUBE in Section 4 plays an important role to minimize the overestimation in the CDF calculation of a seismic PSA [2].

4. ACUBE Algorithm

The objective of the ACUBE algorithm is to improve upon the traditional approximation in the CDF calculation with given cutsets. These approximations are limited, and normally conservative, based on not fully accounting for the dependence between cutsets. Empirical evidence has shown that for most practical models, the bulk of the contribution to the over-estimate of the cutsets appears in the ‘top-most’ (highest probability) cutsets. This conclusion makes practical sense in that the largest cross products are present in the largest probabilistic cutsets. While lower order cutsets may experience the same phenomena, the absolute value of the cross product decreases and in lower order cutsets is very small as the cross product is a subset of the probability of the lower order cutset.

Therefore, as described in more detail below, ACUBE calculates the top event probability and importance measures from cutsets by (1) dividing cutsets into major and minor groups depending on the cutset probability, where the cutsets that have higher cutset probability are included in the major group and the others in minor cutsets, (2) converting major cutsets into a Binary Decision Diagram (BDD), (3) calculating the exact probability of the BDD (the probability of major group), (4) calculating the probability of minor group with an conventional approximate method such as Min Cut Upper Bound (MCUB), and (5) combining the two results of major and minor groups (see Eq. (6)). In a similar manner, ACUBE can calculate importance measures more accurately by calculating the importance measures after separating the cutsets into the two groups.

ACUBE works by dividing the given cutsets into two groups, and calculating one group exactly whereas the second group is calculated with an approximation. When there are no initiator events, the top event probability is calculated in the following manner:

(1) The set of cutsets are divided into two groups:

$$S = (C_1 \cup C_2 \cup ...) = S_1 \cup S_2 \tag{3}$$

$$S_1 = C_1 \cup C_2 \cup ... \cup C_m \tag{4}$$

$$S_2 = C_{m+1} \cup C_{m+2} \cup ... \quad . \tag{5}$$

ACUBE selects major cutsets that have higher cutset probabilities than the remaining minor cutsets. Here, major cutsets S_1 have higher cutset probabilities than minor cutsets S_2.

(2) The function of major cutsets S_1 is converted into a BDD structure. Then, $P(S_1)$, $P(S_1 | x)$, and $P(S_1 | \bar{x})$ are calculated exactly using BDD equations.

(3) $P(S_2)$, $P(S_2 | x)$, and $P(S_2 | \bar{x})$ are calculated using the Min Cut Upper Bound (MCUB) approximation.

(4) The system probability is calculated as the approximation:

$$P(S) \approx p_{ACUBE} = 1 - (1 - P(S_1))(1 - P(S_2)) \tag{6}$$

(5) The system conditional probabilities for each basic event x are calculated as

$$P(S | x) = 1 - (1 - P(S_1 | x))(1 - P(S_2 | x)) \tag{7}$$

$$P(S | \bar{x}) = 1 - (1 - P(S_1 | \bar{x}))(1 - P(S_2 | \bar{x})) . \tag{8}$$

(6) Importance measures [5,6] are calculated using the conditional probabilities in Eqs. (7) and (8). The importance measures for a basic event x are then calculated using the conditional probabilities as:

$$BI(S,x) = P(S | x) - P(S | \bar{a}) \tag{9}$$

$$FV(S,x) = \frac{P(x) \cdot BI(S,x)}{P(S)} \tag{10}$$

$$RRW(S,x) = \frac{P(S)}{P(S | \bar{x})} \tag{11}$$

$$RAW(S,x) = \frac{P(S | x)}{P(S)} \tag{12}$$

The ACUBE method results in a better estimation of a top event probability and importance measures than the calculations in a current PSA. In particular, the following inequalities hold for the coherent model.

$$P(S) \leq P_{ACUBE} \leq P_{MCUB} \leq P_{REA} \; . \tag{13}$$

5. Conclusions

ACUBE works by dividing the cutsets into two groups (higher and lower cutset probability groups), calculating the top event probability and importance measures in each group, and combining the two results from the two groups. Here, ACUBE calculates the top event probability and importance measures of the higher cutset probability group exactly. On the other hand, ACUBE calculates these measures of the lower cutset probability group with an approximation such as MCUB.

The ACUBE algorithm is useful for decreasing the conservatism that is caused by approximating the top event probability and importance measure calculations with given cutsets. By applying the ACUBE algorithm to the seismic PSA cutsets, the accuracy of a top event probability and importance measures can be significantly improved.

This study shows that careful attention should be paid and an appropriate method be provided in order to avoid the significant overestimation of the top event probability calculation. Due to the strength of ACUBE that is explained in this study, the ACUBE became a vital tool for calculating more accurate CDF of the seismic PSA cutsets than the conventional probability calculation method.

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