Proceedings of the Korean Nuclear Society Autumn Meeting
Seoul, Korea, October 1998

Evaluation of Higher-Order Bounded Convection Schemes for Oscillatory Natural Convection of Liquid Metal

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Abstract

A study on the evaluation of higher-order bounded convection schemes for simulation of oscillatory natural convection of liquid metal in a square cavity is presented. Two higher-order bounded convection schemes, SOUCUP and COPLA are evaluated, together with HYBRID and QUICK, to test their capability for predicting oscillatory natural convection. Calculations are performed for $Gr = 10^7, \Pr = 0.005$ employing 42*42 and 82*82 nonuniform grids. The COPLA and QUICK schemes are shown to be capable of predicting the oscillatory motion while the HYBRID and SOUCUP schemes are not.

1. Introduction

Natural convection of liquid metal has become an important topic of study due to its applications in material processing, semiconductor crystal growth and decay heat removal in a liquid metal reactor. The understanding of natural convection in a liquid metal reactor is very important in securing the structural integrity of the reactor.

The problem of predicting natural convection of liquid metal is not well understood. The flow is highly nonlinear, and the shape of streamlines is nearly circular. The numerical false diffusion occurs when the numerical grids are a rectangular shape so that the flow is oblique to the grid lines. The low-Prandtl-number fluids have a tendency to oscillate at relatively low Rayleigh numbers. Some of the numerical issues associated with prediction of oscillatory natural convection will be the treatment of an unsteady term, the choice of a convection scheme to avoid numerical false diffusion, and the use of momentum interpolation method for unsteady flows involving large body forces.

Mohamad and Viskanta [1] evaluated four different convection schemes for natural convection of low Prandtl number fluids. They concluded that the first order schemes were incapable of predicting oscillatory convection and recommended the central difference scheme for transient simulation. However, the central difference scheme is unstable when the grid Peclet number is high and is not well used in the general purpose code. Mohamad and Viskanta [1] used this scheme in the transient calculations using a very small time step.

The higher-order schemes such as the central difference scheme, the QUICK scheme [2] and the second-order upwind scheme [3] have been successful in increasing the accuracy of the solution, but all suffer from the boundedness problem; that is, the solutions display unphysical undershoots and overshoots in regions of steep gradient, which can lead to numerical instability. In the practical turbulent calculations, the undershoots behavior of higher-order convection schemes may produce negative value of turbulent quantities that should be always positive, such as the turbulent kinetic energy and the rate of dissipation of turbulent kinetic energy. Such a phenomenon can cause the numerical instability and can occur in the analysis of turbulent natural convection or mixed convection of liquid metal flows in a liquid metal reactor.

Many higher-order bounded schemes, such as the SOUCUP scheme [4], the HLPA scheme [5], the SMARTER scheme [6] and the COPLA scheme [7], have been developed to resolve the forementioned boundedness problems. Recently Choi and Lee [8] have evaluated these higher-order bounded schemes and showed that the HLPA, SMARTER and COPLA schemes result in nearly the same solution behaviors both in accuracy and convergence, while the SOUCUP scheme is more diffusive than the other schemes. Thus, it suffices to test the SOUCUP and COPLA schemes in the present study.

In the present study the higher-order bounded convection schemes are evaluated for transient calculations of oscillatory natural convection of liquid metal. Among the various higher-order bounded schemes, the SOUCUP scheme and the COPLA scheme are chosen in the present study. The solutions of HYBRID scheme and QUICK scheme [2] are also included for comparison.

2. Governing Equations

We consider a time-dependent, two-dimensional natural convection of liquid metal in a square cavity, shown in Fig.1. The isothermal vertical walls are kept at constant but different temperatures, where the upper and lower walls are adiabatic. Initially the fluid is at the cold wall temperature. At time t = 0, the temperature of one of the vertical walls is raised to a constant value $T_H > T_C$. The fluid is assumed to be Newtonian with constant properties and the Boussinesq approximation is assumed to be valid. The governing equations for the transport of mass, momentum and energy can be written in a dimensionless form as follows using L, $(L/g\beta\triangle T)^{1/2}$, $(g\beta\triangle TL)^{1/2}$, and $\triangle T = T_H - T_C$ for the length, time, velocity and temperature scales, respectively

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{\sqrt{Gr}}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + \theta \tag{3}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\Pr\sqrt{Gr}}\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \tag{4}$$

where Gr is the Grashof number and Pr is the Prandtl numbers and $\theta = (T - T_0)/\triangle T$. The initial and boundary conditions are as follows ;

$$\begin{array}{lll} u = v = \theta = 0 & & t = 0 \\ u = v = 0 & \text{on the walls} & t \geq 0 \\ \theta(0, y) = 1, \ \theta(1, y) = 0 & & t \geq 0 \\ \dfrac{\partial \theta}{\partial y} = 0 & \text{at } y = 0, 1 & t \geq 0 \end{array} \tag{5}$$

3. Results and Discussions

Calculations were performed for $Gr=10^7, \Pr=0.005$ employing 42*42 and 82*82 nonuniform grids. Dimensionless time step of 1/80 was used for all calculations and this time step size was small enough to resolve the oscillatory transient behaviour. Iterations were performed for each time step until the maximum of the absolute sum of dimensionless residuals of momentum equations, energy equation and pressure correction equation was smaller than 10^{-6}. Relaxation factors of 0.7 were used for momentum equations and 1.0 was used for the energy equation.

Fig.2 shows the transients of average Nusselt number at the hot wall predicted by four different convection schemes. We observe that the HYBRID and SOUCUP schemes result in steady state solutions. These two schemes are not capable of predicting the oscillatory behaviour, even if the numerical grids are increased to 82*82. The HYBRID scheme underpredicts the average Nusselt number severely. The prediction does not much improve with grid refinement. The SOUCUP scheme predicts the average Nusselt number much better than the HYBRID scheme, however, fails to predict the oscillatory behaviour. It is noted that the SOUCUP scheme is a composite of upwind, second order upwind and central difference scheme and is second order accurate. The QUICK and COPLA schemes predict the oscillatory behaviour of the average Nusselt number well. The predictions by both schemes are nearly the same, while the QUICK scheme results in slightly better results when the grid is coarse (42*42). It is not a surprising result if we note that the bounded scheme COPLA employs the QUICK scheme in a certain range in the normalized variable diagram. The results show that the bounded scheme COPLA is as accurate as the QUICK scheme and the QUICK scheme does not show wiggling in the present problem.

Fig.3 shows the predicted streamlines predicted by the HYBRID and SOUCUP schemes in the final stages of steady state. Only the results for 82*82 grids are presented. The streamlines predicted by the HYBRID scheme are rather square shaped compared with those by the SOUCUP scheme. The HYBRID scheme predicts rather large vortices at the four corners and it was found that these vortices are weak and do not change with time. The streamlines predicted by the SOUCUP scheme are nearly circular. There are two very small vortices at upper-right and bottom-left corners. It was also checked that these vortices do not vary with time.

Fig.4 shows the unsteady motion of streamlines during one cycle of oscillation predicted by the COPLA scheme employing a 82*82 numerical grid. The streamlines reveal nearly periodic growth and decay of corner vortices. The evolution of corner vortices during one cycle of oscillation can be described as follows. The evolution of vortices begins at the upper-right and bottom-left corners where the temperature gradient is large (t=35.3135). The upper-right and bottom-left vortices appear first, and the vortices at the other corners also evolved (t=35.4760). Then the strength of the vortices increased (t=35.6510) and the corner vortices sheared with the main vortices (t=35.8010). The strength of the corner vortices weakened and broke into small vortices (t=35.9510). Then the corner vortices disappeared and evolved into main vortices (t=35.1510). This ends one cycle of the evolution process. When these predictions are compared with the predictions of streamlines by the HYBRID and SOUCUP schemes, the oscillation of the average Nusselt number is due to the unsteady motion of corner vortices.

4. Conclusions

The evaluation of the higher-order bounded convection scheme for simulation of natural convection of low Prandtl number fluid is studied. Four convection schemes are tested for natural convection of liquid metal in a square cavity in the case of $Gr=10^7, \mathrm{Pr}=0.005$ employing 42*42 and 82*82 nonuniform grids. The results of numerical experiments conducted by the present study show that the HYBRID and

SOUCUP schemes should not be used for prediction of oscillatory natural convection of liquid metal since they fail to predict the experimentally observed oscillatory motion. The QUICK scheme and the central difference scheme tested by Mohamad and Viskanta [1] are capable of predicting the oscillatory motion. However, these two schemes are unbounded and can cause numerical instability in the turbulent flow simulations. The results of present study and those of Choi et al. [7] show that the COPLA scheme is as accurate as the QUICK scheme, while preserving the boundedness of the problem. Thus, the higher-order bounded scheme like COPLA scheme can be used confidently for practical simulation of both laminar and turbulent natural convection of liquid metal such as the decay heat removal in a liquid metal reactor.

5. Acknowledgement

This work has been carried out under the nuclear research and development program by the Ministry of Science and Technology.

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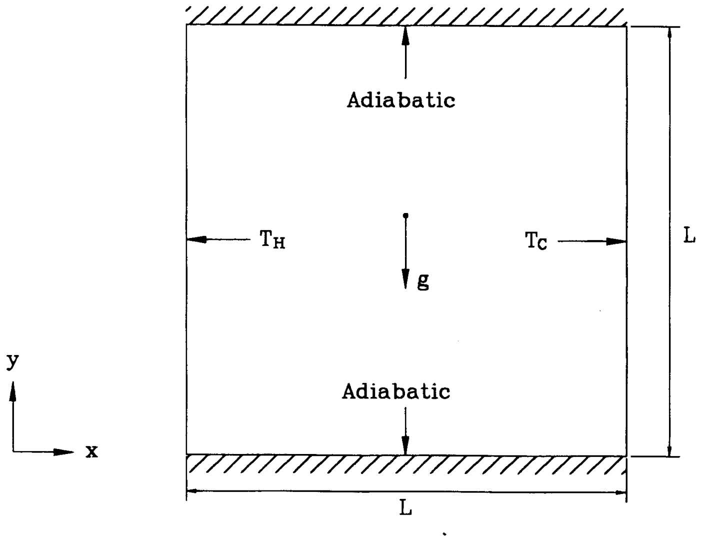
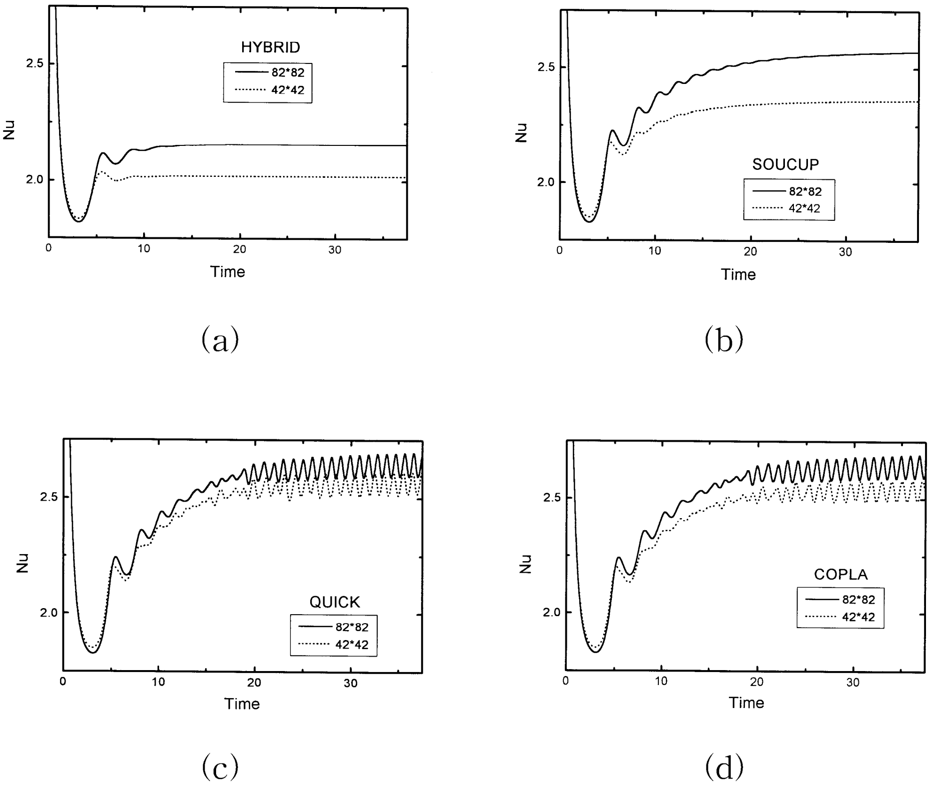


Fig. 1 Schematic of the cavity



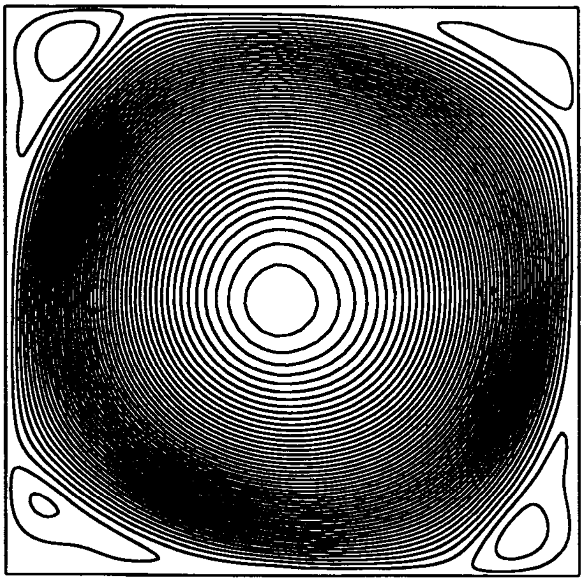
(a)

(b)

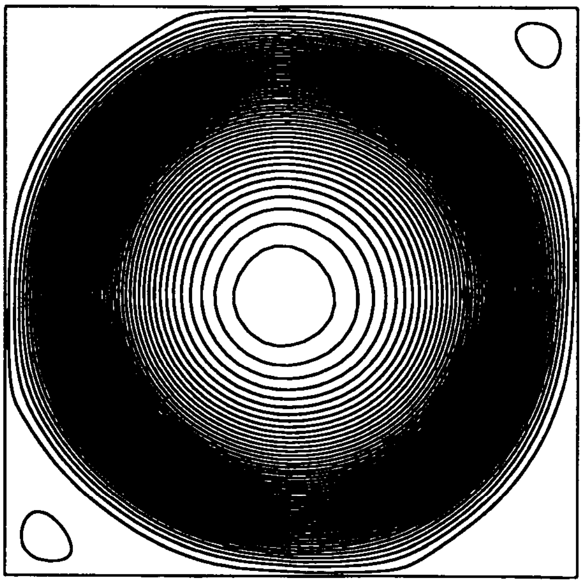
(c)

(d)

Fig. 2 Transients of average Nusselt number at hot wall : (a) HYBRID, (b) SOUCUP, (c) QUICK, (d) COPLA.

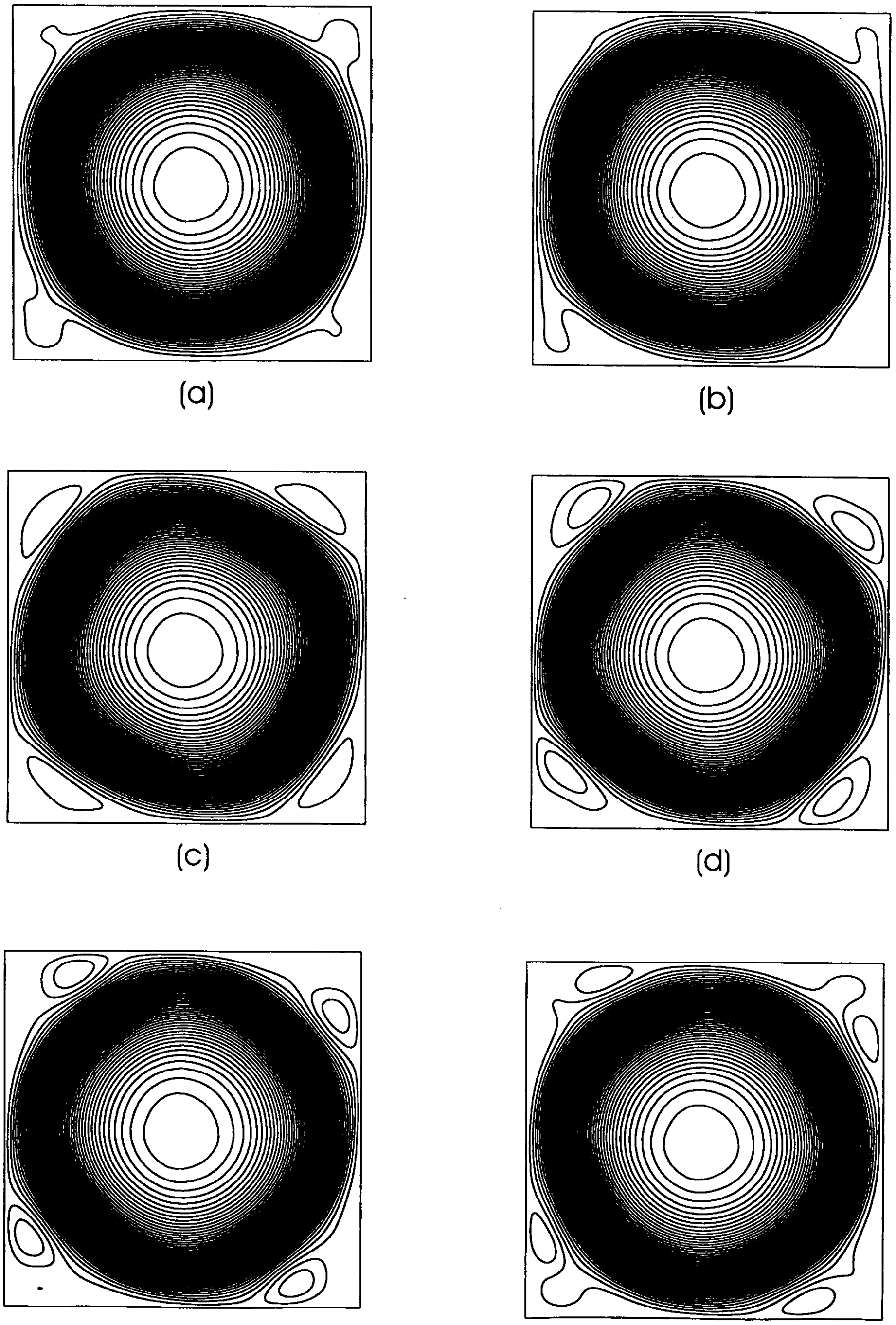


(a) HYBRID



(b) SOUCUP

Fig. 3 Streamlines predicted by HYBRID and SOUCUP.



(a) (b) (c) (d) (e) (f)

Fig. 4 Streamlines during one cycle of oscillation predicted by COPLA : (a) t=35.1510, (b) t=35.3135, (c) t=35.4760, (d) t=35.6510, (e) t=35.8010, (f) t=35.9510.