

## Calculation of Core Axial Power Shapes Using Alternating Conditional Expectation Algorithm

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### Abstract

We have introduced the alternating conditional expectation (ACE) algorithm in the method of reconstructing 20 node axial power shapes from five level detector powers. The ACE algorithm was used to find the optimal relationships between each plane power and normalized five detector powers. The obtained all optimal transformations had simple forms to be represented with polynomials. The reference axial power shapes and simulated detector powers were drawn out of the 3-dimensional results of Reactor Operation and Control Simulation (ROCS) code for various core states. By the ACE algorithm, we obtained the optimal relationship between dependent variable plane power,  $y$ , and independent variable detector powers,  $\{D_i, i=1, \dots, 5\}$  without any preprocessing, where a total of  $\sim 3490$  data sets per each cycle of YongGwang Nuclear (YGN) Power Plant units 3&4 are used. To test the validity and accuracy of the new method, about 21,200 cases of reconstructed axial power shapes are compared to original ROCS axial power shapes, and they are also contrasted with those obtained by Fourier fitting method (FFM). The average error of root mean square (rms), axial peak ( $\Delta F_2$ ), and axial shape index ( $\Delta SI$ ) of our new method for total 21204 data cases are 0.81%, 0.51% and 0.00204, while FFM 2.29%, 2.37% and 0.00264, respectively. The evaluation results for the data sets not used in the ACE transformations also show that the accuracy of new method is much better than that of FFM.

### I. INTRODUCTION

On-line core monitoring system performs CPU-intensive works such as receiving many measurement data from in-core and ex-core detectors and analyzing them on real-time. It provides the selected typical and important values to operators. Based on these values, operator understands core status appropriately and does right action to the core situation.

ABB-CE(Asia Brown Boveri - Combustion Engineering) type nuclear reactors have a digital on-line core monitoring system, Core Operating Limit Supervisory System (COLSS)<sup>[1]</sup>. COLSS working on YGN (YongGwang Nuclear) unit 3&4 in Korea receives up to 225 in-core detector signals from Plant Data Acquisition System (PDAS) and generates 20 and 40 nodes axial power shapes to estimate DNBR and LHR every 30 seconds, respectively. To compute axial power shapes, COLSS uses Fourier series synthesis method, i.e., Fourier fitting method (FFM). FFM is adjusted with five detector signals and cycle dependent boundary conditions, which are selected to minimize axial peak ( $F_2$ ) difference and root mean square (rms) errors. Although this deterministic method has definite applications, the accuracy of FFM tends to decrease when power shapes are deeply saddled or highly shifted to one end of z-axis. Since overall uncertainty analysis (OUA) is performed based on these power shapes, they have a role to reduce thermal margins. To improve thermal margin, it is necessary to develop a new method providing more accurate axial power distribution than those of FFM. For it seemed not easy to find any dramatic ways in deterministic approach to reduce power reconstruction error, we introduced stochastic method, the alternating conditional expectation (ACE) algorithm<sup>[2,3,4]</sup>, to attain an optimal correlation between each plane power and five detector powers which converted from detector signals.

The ACE method is a generalized regression algorithm that yields an optimal relationship between a dependent variable,  $y$ , and multiple independent variables,  $\{x_i, i = 1, \dots, N\}$ . The objective of the ACE algorithm is to find optimal transformations  $\theta(y)$  and  $\{\phi_i(x_i), i = 1, \dots, N\}$  that maximize the statistical correlation between  $\theta(y)$  and  $\sum_{i=1}^N \phi_i(x_i)$ . Generally, this object is achieved by treating each value of the transformed dependent variable  $\theta(y)$  as the expectation of several realizations of the sum of transformed independent variables  $\sum_{i=1}^N \phi_i(x_i)$ . Once the optimal transformations are found by iteration, one can determine the coefficients of functional form for the transformed dependent and independent variables through the simple regression analysis.

### II. DESCRIPTION OF DATA SET

In YGN unit 3&4 reactor, the 45 in-core Rhodium detector assemblies with axially five detectors at 10%, 30%,

50%, 70% and 90% of active core height are distributed to radial direction of core. Because the signal intensity, i.e., electric current, of Rhodium detector is proportional to neutron flux or power level around the detector, COLSS converts these detector signals into powers at detector position. Then, COLSS reconstructs 20 or 40 node axial power shapes based on these values and evaluate DNBR and LHR limit. We would like to reproduce these core axial power shapes by new method the ACE algorithm applied, using in-core detector information.

To apply a stochastic method on this reconstruction problem, enough simulation or measurement data sets must be known with regard to normalized plane power and five detector powers for the various core situations. Because it is impossible to measure core axial power shapes exactly for all cases of core conditions, we used simulated data sets. First of all, a number of ROCS calculations are performed for various core conditions. The independent variables are given as; Core power level ( $50\% \leq P \leq 100\%$ ), the depth of insertion ( $0\text{cm} \leq H_{CEA} \leq 381\text{cm}$ ) of control element assembly (CEA), and core average burnup ( $0\text{MWD/MTU} \leq B \leq 18\text{MWD/MTU}$ ). From ROCS results, we generated  $\sim 3500$  reference data sets of one-dimensional normalized axial power shapes having same node size for each cycle and subtracted axial five level detector powers. The entire  $\sim 21000$  data sets of six cycles, having 20 node axial powers and normalized five detector powers, covering cycles of YGN unit 3 & 4 were used to get the ACE transformations. We also performed two cases of Xenon oscillation simulation, and a total of 298 data sets generated by simulation, excluded in the ACE transformations, were used to test the validation and accuracy of new stochastic method.

Fig. 1 shows an example of relationship between node power  $y$  and each detector powers,  $\{D_n, n=1, \dots, 5\}$ . All data points plotted in figure 1, representing 3492 data sets, are generated for YGN unit 3 cycle 2. Fig. 1 shows that it is very difficult to find initial trial functions due to the diffuse and complex nature of the relationships between  $y$  and  $D_n$ . We can find these characteristics in every axial plane for each cycle.

### III. DETERMINATION OF AXIAL POWER SHAPES THROUGH ACE ALGORITHM

To estimate a maximal correlation between plane node power and five detector powers by stochastic method, we adopted the ACE algorithm<sup>[3,4]</sup>

#### III.A. The Alternating Conditional Expectation Algorithm

For multivariate regression problem with a set of data  $\{(y_i, D_{1i}, D_{2i}, \dots, D_{5i}), i=1, \dots, N\}$ , the optimal transformations of multivariate ACE algorithm is readily derived from bivariate optimal transformations as following:

$$\phi_n(D_n) = E \left[ \theta(y) - \sum_{d \neq n}^5 \phi_d(D_d) \right], n = 1, \dots, 5, \quad \text{and} \quad \theta(y) = \frac{E \left[ \sum_{d=1}^5 \phi_d(D_d) \right]}{\left\| E \left[ \sum_{d=1}^5 \phi_d(D_d) \right] \right\|} = \frac{E \left[ \sum_{d=1}^5 \phi_d(D_d) \right]}{E \left[ \sum_{d=1}^5 \phi_d^2(D_d) \right]}, \quad (1)$$

where the optimal transformations,  $\theta(y)$  and  $\phi_1(D_1), \dots, \phi_5(D_5)$ , are mean zero functions. These transformations are coupled each other and solved by iterative procedure to minimize the square error of regression,

$$e^2 = \frac{1}{N} \sum_j \left[ \theta(y) - \sum_{d=1}^5 \phi_d(D_d) \right]^2. \quad (2)$$

The  $i$ 'th element of optimal transformation,  $\theta(y)$ , means a conditional expectation at  $y_i$  and is determined by evaluating a expectation about  $\phi_1(D_1), \dots, \phi_5(D_5)$  with the neighboring values in the interval  $[i-M, i+M]$  for a given  $M$ . In this study,  $M$  is decided by  $\omega$  which is a user defined windowing factor. Kim and Lee<sup>[3,4]</sup> have derived heuristically the ACE algorithm from equations (1) for a data smoothing being performed through weighted averaging process. The overall square error is summed over the entire data sets following:

$$e^2 = \sum_{i,j=1}^N W_{ij} \left[ \theta(y_i) - \sum_{d=1}^5 \phi_d(D_{dj}) \right]^2, \quad (3)$$

where  $W_{ij} = W(y_i, x_{1j}, \dots, x_{5j})$  is a weight assigned to the  $j$ 'th neighbor of point  $i$  and is selected so that  $0 < W_{ij} < 1$  in the interval  $[i-M, i+M]$  and  $W_{ij} = 0$  outside the interval, together with the normalization  $\sum_j W_{ij} = 1$ . Setting the partial derivatives of Eq. (3) with respect to  $\theta(y)$  and  $\phi_d(D_d)$ , respectively, to zero yields, one can derive following equations for the  $i$ 'th data point:

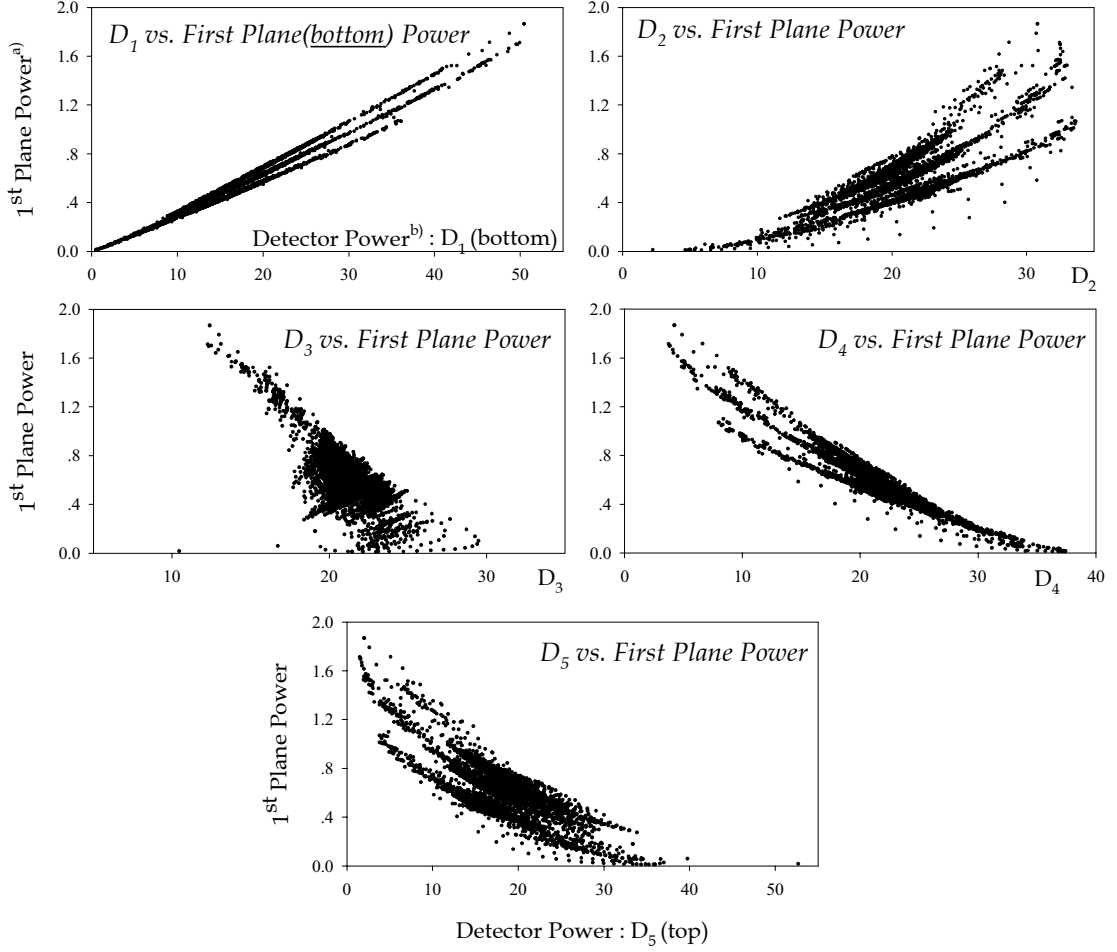


Fig. 1 . The first plane power data for a total of 3,492 data points of YGN unit 3 cycle 2:  
a) 20 node powers are normalized with core height, b) 5 detector powers are normalized with 100.

$$\theta(y_i) = \frac{\sum_{j=i-M}^{i+M} W_{ij} \sum_{d=1}^5 \phi_d(D_{dj})}{\frac{1}{N} \sum_i \left[ \sum_{j=i-M}^{i+M} W_{ij} \sum_{d=1}^5 \phi_{dd}(D_{dj}) \right]^2}, \quad \text{and} \quad \phi_d(D_{\bar{d}}) = \sum_{j=i-M}^{i+M} W_{ij} \left[ \theta(y_j) - \sum_{q \neq d}^5 \phi_q(D_{dj}) \right], \quad (4)$$

where  $E(\theta^2(y)) = 1$ . Equation (4) is the final form of the ACE algorithm used in this paper.

### III. B. Implementation of Alternating Conditional Expectation Algorithm

Data smoothing operation plays a key role in the ACE algorithm. We adopted a first-order locally weighted regression method that is known to give a good accuracy. Let a first-order linear regression model,  $\xi(x_j) = ax_j + b$ , be an optimal function for given local data interval  $[i - M, i + M]$  centered round the  $i$ 'th point. Then, the fitting coefficients  $a$  and  $b$  can be found by minimizing the weighted square error at the  $i$ 'th point:

$$e_i^2 = \sum_j W_{ij} [\phi(x_j) - \xi(x_j)]^2 = \sum_j W_{ij} [\phi(x_j) - ax_j - b]^2, \quad (5)$$

giving

$$a = \frac{\text{Cov}(x\phi)_i}{\text{Var}(x)_i} \quad \text{and} \quad b = E(\phi)_i - aE(x)_i, \quad (6)$$

where

$\text{Cov}(x\phi)_i = E(x\phi)_i - E(x)_i E(\phi)_i =$  weighted covariance of  $x$  and  $\phi$ ,

$\text{Var}(x)_i = E(x^2)_i - E(x)_i^2 =$  weighted variance of  $x$ ,

and

$$E(u)_i = \sum_{j=i-M}^{j=i+M} W_{ij} u_j = \text{weighed expectation of } u; u = x \text{ or } \phi.$$

Finally, from Eq. (6), the data smoothing operation is represented for  $i$ 'th data point as following;

$$E[\phi(x)|i] = S[\phi(x)|i] = E(\phi)_i + \frac{Cov(x\phi)_i}{Var(x)_i} (x_i - E(\phi)_i). \quad (7)$$

In equation (7), the function  $\phi$  means  $\{\phi_d(D_d), d=1, \dots, 5\}$  or  $\theta(y)$  and variable  $x$  means  $D_d$  or  $y$ . The overall flow of heuristic ACE algorithm is represented in Fig. 2.

### III. C. Determination of Fitting Coefficients of Polynomial Function for transformations

After the optimal transformations,  $\theta(y)$  and  $\{\phi_d(D_d), d=1, \dots, 5\}$ , were solved, we can construct a simple regression model to obtain the approximated polynomial functions for each transformation, based on the least square model. And we developed a computer code, TACE, to perform these works.

Fig. 3 shows an example of final regression result with regard to first plane power and five detector powers, obtained through the TACE code. In Fig. 3, one can see that the obtained six transformations have very simple form to be represented by liner polynomial functions. For considering an extrapolation at the both boundaries of whole detector range, we divided all transformations into three regions. First and third region are represented by a linear function,  $f(x) = ax + b$ , and second region is fitted by up to 9<sup>th</sup> order polynomial function,  $\{\sum a_k x^k, k=1, \dots, N, N \leq 9\}$ , for  $\{\phi_d(D_d), d=1, \dots, 5\}$ . But for the second region of transformation  $\theta(y)$ , a more simple function,  $f(x) = ax^2 + bx + c$ , is used to get a simple inverse form. The range of first and third region is decided by the variance,  $\sigma^2$ , of the difference between transformation and its analytical solution. If evaluated  $\sigma^2$  is less than the criteria for initial trial range, TACE code repeats the evaluation process for larger data sets than trial. When the variance  $\sigma^2$  is greater than the criteria, TACE stops the evaluation and takes just previous data position as the beginning or ending point of that region because of data sets being sorted in an ascending order. On the other hand, we selected the best fitting polynomial order  $N$  as the one giving the minimum value of the variance. If the variances are all within the criteria, for simple calculation, we select one which order is the lowest. All data points within the calculated range are used when the variances are calculated. As for results, TACE code computes up to 76 polynomial coefficients for each axial plane for a given  $\sim 3500$  data sets of a cycle.

Fig. 3 also shows how to determine each node power and where to use the obtained polynomial functions. For the given detector powers, axial power shapes are calculated as:

- 1) compute analytic solution of transformation,  $\phi_d(D_d)$ , for each  $D_d$  at current plane and sum those five values,
- 2) invert the transform equation  $y = \theta^{-1}\left(\sum_{d=1}^5 \phi_d(D_d)\right)$  to get plane power  $y$ 
  - a) if the current plane is top, then normalize axial power shape with core height and exit
- 3) go to next plane and repeat 1) ~ 2) steps.

### III. D. Fourier Fitting Method (FFM)

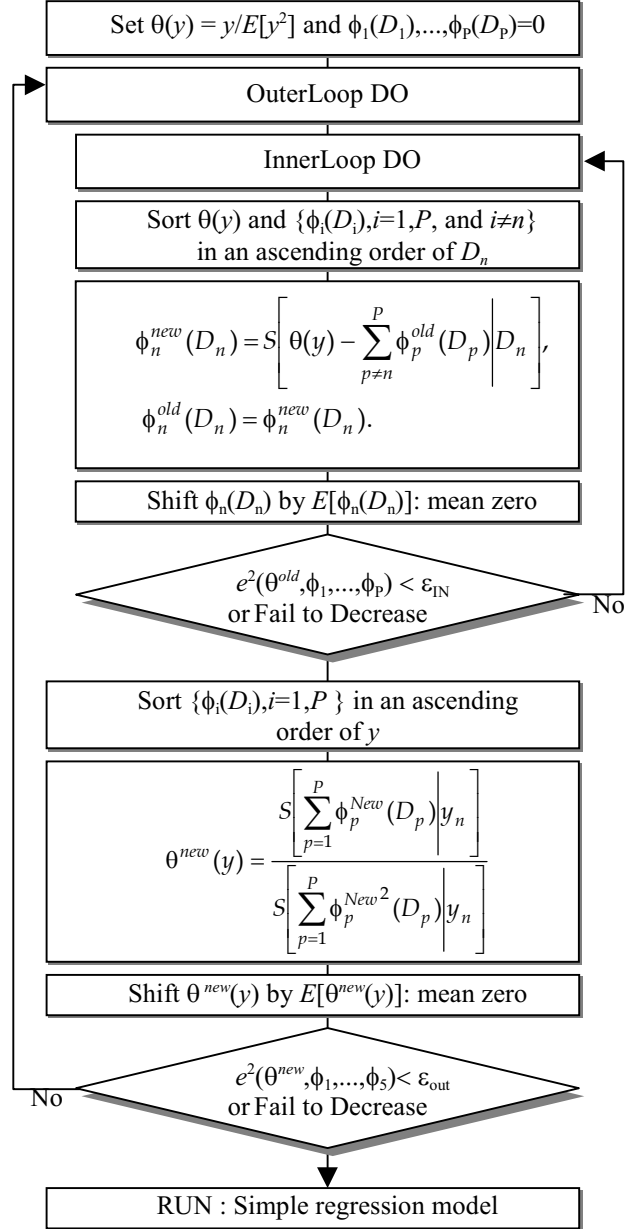


Fig. 2. A flow diagram of the ACE algorithm

To predict axial 20(up to 40) plane power shapes from five detector powers, ABB-CE applies FFM in COLSS. The Fourier fitting function consisting of sine and cosine functions gives the power shape as

$$P(z) = f(n, a_n, B_C, \text{COS}Q_n z, \text{SIN}Q_n z), \quad (8)$$

where

$$\begin{aligned} n &= \text{number of mode, } n=1, \dots, 5, & a_n &= \text{Fourier coefficient for mode } n, \\ Q_n &= \text{functional coefficients} = f(B_C, n), & z &= \text{axial elevation in fraction of core height,} \end{aligned}$$

and

$$B_C = \text{fitting parameter.}$$

The Fourier coefficients,  $a_n(n=1, \dots, 5)$ , are computed by matching the integrals over each detector to the actual powers,  $PD$ , at the detector:

$$PD_m = \int_{z_i^m}^{z_h^m} P(z) dz, \quad (m = 1, \dots, 5), \quad (9)$$

where  $Z_i^m$  and  $Z_h^m$  are bottom and top height of  $m$ 'th detector in fraction of core height, respectively. Since five values are known, we can obtain readily the Fourier coefficients by inversion of matrix which results from the combination of equation (9) at each five detector levels. With the appropriate boundary conditions, one can calculate axial 20(or 40) plane power shapes. In general, they are selected to minimize both axial peak error ( $\Delta F_z$ ) and rms error resulted from comparing to ROCS results, but in this study, we selected them minimizing rms error only. For comparison, the same data sets used in ACE transformations were also utilized for obtaining the Fourier coefficients and fitting parameters.

#### IV. NUMERICAL RESULTS

To test the validity and accuracy of the developed axial power reconstruction method, we reevaluated ~21,000 data sets used in ACE algorithm. These data sets were drawn out from ROCS three-dimensional calculations for various core states. On the other hand, ~300 data sets were taken as another testing samples. These data sets were from the results of xenon-oscillation simulation performed at 50% and 80% power level of BOC of YGN Unit 3 cycle 2.

In this paper, we use a windowing factor  $\omega = 0.25$ , and a convergence criterion of  $1.0E-05$  both for inner and outer iterations. The CPU time consumed to obtain the transformations of single plane with 3492 data sets is ~20min on an HP9000/735 machine.

To compare two axial power reconstruction methods, we selected six quantities such as maximum and average error of RMS,  $\Delta F_z$  and  $\Delta ASI$ . These factors are the main parameters determining the LHR and DNBR margin as well as deciding the accuracy of reconstructed power shapes.

##### IV.A. Reevaluation of Data Sets

We reevaluate total 21,204 data sets using the analytic functions, which was developed by the ACE transformations and simple regression method. We arrange the reevaluation results according to core burnup and summarize them in table I and II. Table I and II show detail comparisons of parameters for the case of YGN unit 3 cycle 2 and overall comparison of six parameters of two axial power reconstruction

Table I

. The Comparison of new method and FFM for the data sets of YGN unit 3 cycle 2

BOC(1164 data sets)						
Power Level	Avg. RMS (%)	Max. RMS (%)	Avg. $\Delta F_z$ (%)	Max. $\Delta F_z$ (%)	Avg. $\Delta ASI$ ( $\times 10^{-3}$ )	Max. $\Delta ASI$ ( $\times 10^{-3}$ )
100(%)	0.98 <sup>a)</sup> 2.26 <sup>b)</sup>	10.95 9.05	0.42 2.20	1.55 5.66	0.195 0.280	1.111 1.064
90(%)	0.86 2.09	4.81 7.73	0.36 1.70	1.11 5.27	0.175 0.289	0.741 0.688
80(%)	0.81 1.93	1.53 2.87	0.54 1.55	1.72 4.57	0.310 0.302	1.145 0.908
70(%)	0.93 2.01	3.62 4.76	0.64 1.84	3.07 4.95	0.229 0.240	1.018 0.715
50(%)	0.90 1.89	1.48 2.25	0.48 1.39	1.63 4.57	0.323 0.296	0.995 0.679
MOC(1164 data sets)						
100(%)	2.13 4.67	73.63 49.63	0.35 3.68	1.64 6.27	0.192 0.312	1.03 1.014
90(%)	0.98 3.67	16.99 38.88	0.38 4.03	1.49 6.03	0.191 0.341	1.075 1.013
80(%)	0.83 2.66	3.02 6.73	0.64 3.82	2.37 5.71	0.311 0.333	1.394 1.139
70(%)	0.92 2.93	2.04 13.68	0.68 3.02	2.36 6.25	0.243 0.288	1.056 1.099
50(%)	0.99 2.44	1.73 2.9	0.62 3.00	2.43 6.21	0.292 0.292	0.999 0.591
EOC(1164 data sets)						
100(%)	1.12 3.77	4.00 18.32	1.00 3.82	2.58 6.28	0.278 0.271	0.847 1.026
90(%)	0.97 3.42	2.69 8.19	0.86 3.99	2.01 5.68	0.260 0.382	0.657 1.094
80(%)	1.04 3.36	1.98 9.56	0.80 3.98	1.98 5.79	0.335 0.420	0.930 1.463
70(%)	1.06 3.10	1.98 5.53	0.56 3.36	2.41 5.42	0.277 0.238	0.882 0.922
50(%)	1.10 3.04	2.46 3.65	0.76 3.52	2.84 6.37	0.296 0.299	1.073 0.639
Xenon oscillation simulation (129 data sets) at BOC						
Case I <sup>c)</sup>	1.22 2.10	1.94 2.95	0.46 1.82	1.53 3.07	0.287 0.375	1.451 1.018
Case II <sup>d)</sup>	0.76 2.16	1.82 3.58	0.43 2.47	0.81 3.97	0.198 0.335	1.262 1.025

<sup>a)</sup> New method

<sup>b)</sup> FFM

methods for all cases, respectively. From Table I and II, one can see that the average quantities of new method are always less than those of FFM. The average rms errors of new method vary from 0.55% and 1.52%, while FFM from 0.77% to 3.54%. By comparison case by case, new method's the average rms and average  $\Delta F_z$  error is just  $\sim 1/3$  and  $\sim 1/5$  of that of FFM, respectively. In spite of the fact that there are some cases including YGN3C2 MOC where FFM's maximum error is lower, we can find that the new method has generally lower error distributions than FFM has. If rms is defined by absolute differences between calculated TACE power and ROCS power shape, then in all cases the maximum rms errors of new method are computed about  $\sim 1/3$  of those of FFM. For example, for the data set where the maximum relative rms error goes up to 73.6%, the new method's rms error which based on absolute difference is 3.94%, while that of FFM is reduced from 49.93% to 8.93%. This means that the axial shapes of new method are more similar to ROCS results. We especially note the

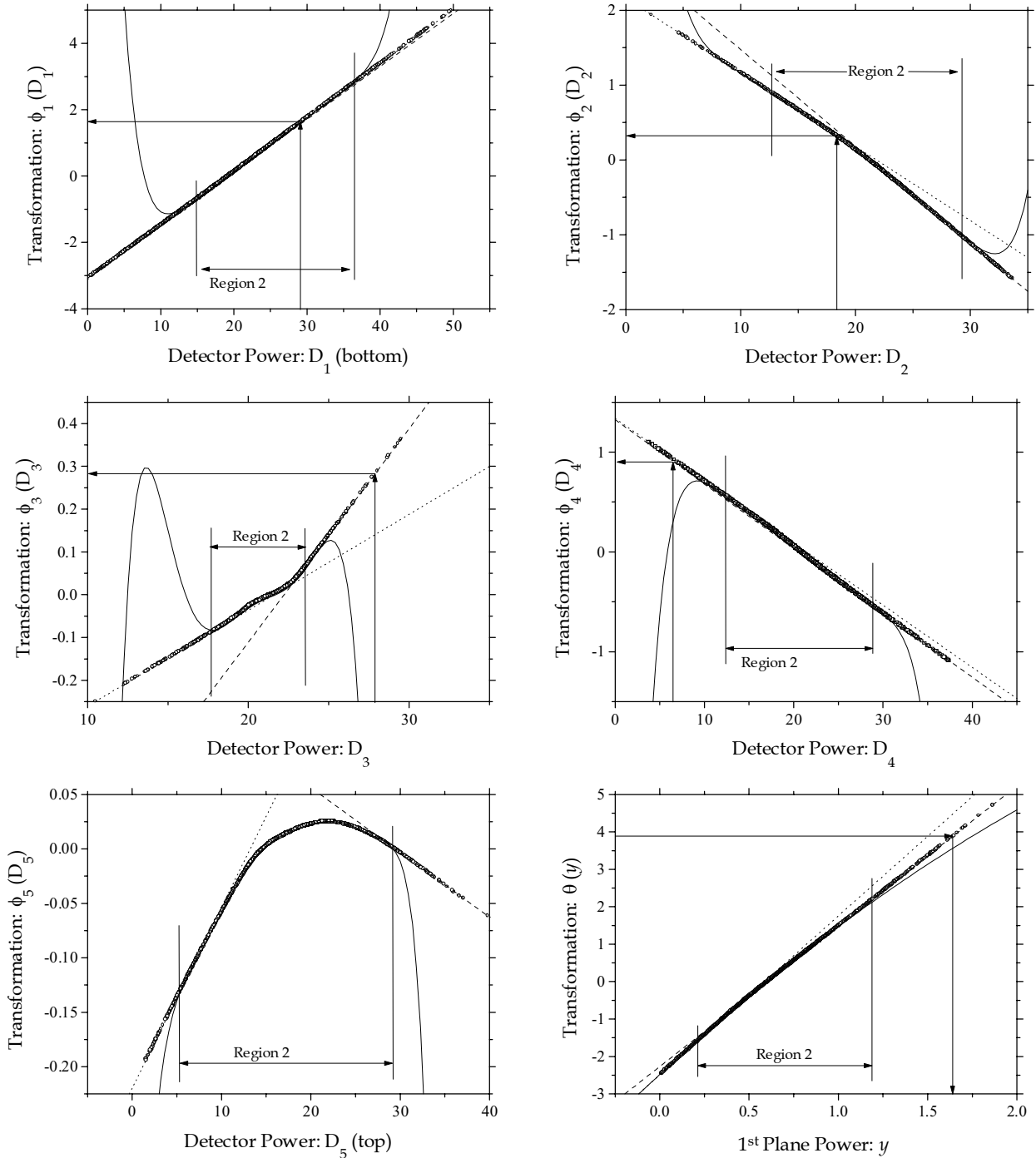


Fig. 3. Final transformations of first plane power,  $\gamma$ , and five detector powers ( $D_k, k=1,5$ ) and its regressed polynomial functions in the case of YGN unit3 Cycle 2

results of case I and II. Because these data sets were not used in the ACE transformations, they are good samples to test the accuracy of developed analytic functions. In Table I, we can observe those facts mentioned above. The new method's average rms,  $\Delta F_z$ , and  $\Delta ASI$  error for case II are 0.76%, 0.43%, and 0.00198, while those of FFM are 2.16%, 2.47%, and 0.00335, respectively. Another evidence that proves the superior of new method is in Fig. 4. It shows the error histogram of two methods for all data points of YGN unit 3 cycle 2, where 69,840 data points (= 3,492 data sets times 20 axial powers) are arranged according to its error value. The number of data points representing < 1% error is 53,085 in new method, while 21,374 in FFM.

## V. CONCLUSITONS

It is important to improve the accuracy of axial power reconstruction method, because the accuracy is very strongly related with thermal margin. In deterministic methods such as Fourier fitting method or other functional fitting method, there are two ways on that improvement. First is to expand fitting function's order. Second is to use more accurate fitting functions than present one. But existing deterministic method can not expand its order of fitting functions because there are no more information except restricted five detector values. And for the diffuse and complex nature of the relationships between power shape and detector powers as shown in Fig. 1, it is more difficult to find optimal fitting functions. In this paper, we developed a new axial power reconstruction method based on the ACE algorithm and simple regression method. New method has two steps for building an axial power shape: first, we calculate each plane power using detector powers. Second, we normalize them with axial core height. It is a different point with deterministic methods that directly reconstruct axial power shape from given detector signals.

From Table I to III, where the reevaluation results for total 21,204 data sets are summarized, we conclude that the accuracy of new method is better than that of FFM with boundary conditions minimizing rms error and that the new method's power profiles are more similar to ROCS profiles even at the data sets maximum errors occurred. The new method's average rms errors are in the range [0.5% < avg. rms < 1.6%] and calculated ~1/3 of that of FFM. The maximum values of average  $\Delta F_z$  and  $\Delta ASI$  error of new method are 1.06% and 0.00295, while FFM 3.73% and 0.0045, respectively.

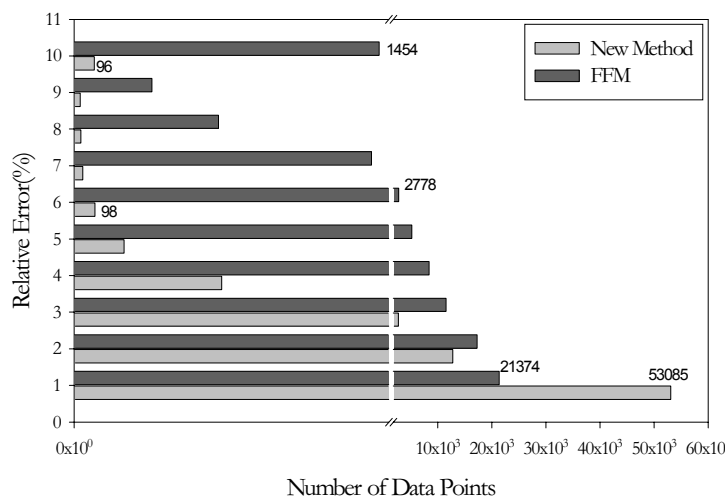


Fig. 4. The Comparison of Relative Error distributions

Table III

The Comparison of average RMS and  $\Delta F_z$  Error at Each Cycle for the data set at which Max.  $\Delta ASI$  occurred.

	FFM		New method	
	rms (%)	$\Delta F_z$ (%)	rms (%)	$\Delta F_z$ (%)
YGN3 C1	1.26	0.90	1.66	0.06
YGN3 C2	2.13	3.29	1.46	1.68
YGN3 C3	2.13	4.03	1.33	0.38
YGN3 C4	1.80	0.08	1.26	1.33
YGN4 C3	2.16	4.04	1.06	0.12
YGN4 C4	1.88	0.24	1.11	1.35
CASE I	2.38	2.75	1.94	1.53
CASE II	2.45	3.41	1.76	0.37

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Table II  
The Overall Comparison of Six Parameters of Two Methods along with burnup rate

	Total number of Data Sets	Fourier Fitting Method						New Method					
		Avg. rms (%)	Max. rms (%)	Avg. $\Delta F_z$ (%)	Max. $\Delta F_z$ (%)	Avg. $\Delta ASI$ ( $\times 10^{-2}$ )	Max. $\Delta ASI$ ( $\times 10^{-2}$ )	Avg. rms (%)	Max. rms (%)	Avg. $\Delta F_z$ (%)	Max. $\Delta F_z$ (%)	Avg. $\Delta ASI$ ( $\times 10^{-2}$ )	Max. $\Delta ASI$ ( $\times 10^{-2}$ )
YGN3C1 BOC	1164	0.77	1.57	0.45	1.29	0.225	0.810	0.57	1.44	0.39	2.12	0.202	1.241
YGN3C1 MOC	1164	3.28	27.77	3.46	6.16	0.450	1.443	1.51	23.56	1.28	<b>4.57</b>	0.302	1.363
YGN3C1 EOC	1164	3.36	15.96	3.72	5.99	0.420	<b>1.567</b>	1.55	27.76	0.72	3.18	0.371	<b>1.645</b>
YGN3C2 BOC	1164	2.07	9.05	1.79	5.66	0.280	1.064	0.92	11.24	0.47	3.07	0.242	1.126
YGN3C2 MOC	1164	3.45	<b>49.63</b>	3.52	6.27	0.313	1.139	1.39	<b>76.34</b>	0.51	2.41	0.241	1.415
YGN3C2 EOC	1164	3.40	18.32	3.73	6.37	0.308	1.463	1.06	6.82	0.83	2.82	0.297	1.127
YGN3C3 BOC	1163	1.74	2.76	1.25	4.80	0.244	0.729	0.60	1.43	0.32	1.59	0.186	0.827
YGN3C3 MOC	1163	2.51	13.30	3.30	5.84	0.243	0.810	0.74	8.31	0.69	2.67	0.202	1.045
YGN3C3 EOC	1163	3.19	24.79	3.41	5.58	0.252	1.057	0.85	5.11	0.67	2.27	0.249	0.957
YGN3C4 BOC	1163	1.77	2.91	1.17	4.05	0.296	0.910	0.64	1.45	0.35	2.13	0.196	0.990
YGN3C4 MOC	1163	2.44	16.01	3.16	6.12	0.273	1.022	0.83	10.44	0.67	3.30	0.236	1.273
YGN3C4 EOC	1163	3.54	39.41	3.66	<b>6.58</b>	0.311	1.114	0.97	34.88	0.64	2.65	0.220	0.993
YGN4C3 BOC	1164	1.72	2.74	1.21	4.70	0.232	0.698	0.55	1.40	0.32	1.54	0.187	0.720
YGN4C3 MOC	1164	2.16	9.72	2.94	5.67	0.220	0.798	0.67	5.55	0.60	2.91	0.197	1.018
YGN4C3 EOC	1164	3.18	24.57	3.42	5.63	0.251	1.053	0.71	4.43	0.53	1.93	0.232	0.956
YGN4C4 BOC	1164	1.78	2.84	1.19	4.20	0.245	0.875	0.61	1.97	0.29	1.89	0.174	0.884
YGN4C4 MOC	1164	2.21	3.54	2.69	5.93	0.242	0.928	0.67	1.61	0.52	3.24	0.189	1.049
YGN4C4 EOC	1164	3.51	31.17	3.71	6.54	0.297	1.122	0.79	20.15	0.50	2.42	0.191	0.877
CASE I	129	2.10	2.95	1.82	3.07	0.375	1.018	1.23	1.88	0.47	1.57	0.289	1.419
CASE II	129	2.16	3.58	2.47	3.97	0.335	1.025	0.85	1.79	0.46	0.83	0.201	1.229
Avg. or Max. over all cases	21204	2.29	49.63	2.37	6.58	0.264	1.567	0.81	76.34	0.51	5.47	0.204	1.449

1. Case I) & II) : for data sets drawn from the simulation results of Xe-oscillation at BOC of YGN unit 3 cycle 2 at 50% and 80% power level, respectively.
2. Reference: Axial power shapes of ROCS code for given core condition