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Review of Higher Order Wilks' Method to Identify Code-based Maximum Parameter Value

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1. Introduction

Wilks' method [1][2][3] has been used worldwide mainly to estimate the maximum parameter output from the limited number of code simulations. One biggest reason why it is gaining popularity might be resorted to the fact that the method itself has a sound statistical framework to provide or demonstrate the conventional regulatory limit of 95%/95% which corresponds to the statistical statement that is given in Appendix K to the 10 CFR 50 as an alternative to the evaluation method.

In Korea, the Wilks' approach has been used for best estimate plus uncertainty evaluation models such as KREM or KINS-REM, where the models credit the 3rd largest code output from the 124 code simulation outputs based on the one-sided 3rd order formula. This practice has been preferred to the simplest 1st order one-sided approach, where only 59 code simulations are required for the maximum output to satisfy the 95%/95% criterion. For example, this kind of situation may come up when a safety analyst wants to estimate the peak cladding temperature (PCT) during an accident analysis, or when a core analyst wants to estimate the maximum power coefficient including the uncertainty for the code calculations.

In this paper, a general p-th order one-sided Wilks' formula is discussed for clarification purposes followed by numerical validation tests for the above practice.

2. Wilks' One-Sided Method and Its Meanings

A detailed set of Wilks formulas [4][5][6] can be found in a few papers. Among the formulas, the p-th order one-sided formula is as follow:

$$1-\sum_{k=n-p+1}^{n} {}_{n}C_{k}\alpha^{k}(1-\alpha)^{n-k} \geq \beta \qquad (1)$$

where α and β denote the tolerance limit and the confidence level, respectively. And n represents the total number of code simulations to satisfy the criteria of α and β.

For example, in Equation (1), if p is 3 (3rd order) and α , β are both the 0.95 to mean 95%, the resultant minimum number of code simulations (n) that is necessary to satisfy the criteria is 124. As mentioned, it is a present practice to select the 3rd largest code output from the 124 code simulation outputs both in KREM or KINS-REM.

3. Numerical Validation and Discussion

Numerical experiments, for example, as below were performed to check the validity of using a higher order p-th Wilks' formula and selecting p-th largest code output to satisfy the percentile/confidence level criterion of 95%/95%:

- An unknown code output parameter x (like the PCT) is assumed to follow the uniform random distribution (between 0.0 ~ 1.0) as a trial distribution. (Note that the Wilks' approach corresponds to a distribution-free approach.)
- A set of n , for example 124, code simulations is assumed to constitute the trial code output distribution. And the corresponding n code output values are generated using a uniform pseudo-random generator.
- Then, a total of one million sets of n code runs are simulated to investigate the validity of selecting the p-th largest output for the p-th order against selecting the 1st largest output for the simplest 1st order.

Table 1 lists minimum number of code simulations from the 1st order to 5th order for the 95%/95% criterion, which were used as test cases.

Table 2 compares numerically experimented confidence values for the five orders in Table 1 with the analytic confidence values that are calculated from Equation (1). As discussed in the previous section, the differences between the numerically experimented values and the analytic values all agree within a 0.03% difference. Therefore, it is judged that the approach of crediting the p-th largest output for the p-th order is exactly the same approach of crediting the 1st largest output for the 1st order. Thus, it seems that only 59 code simulations, in lieu of the 124 or other number of code simulations, are enough to satisfy the 95%/95% criterion.

It should be noted that, however, for example for the 95%/95% criterion and for the 1st order, the statistical statement which is related to Equation (1) means that the maximum value will be located above the 95% percentile along the PDF curve with a 95% confidence level. That is, we can find that the maximum will be above the 95% percentile with 95% chance but the exact location of the maximum between the 95% percentile and 100% percentile can never be known, while the maximum can be known only based on another probabilistic statement. Figure 1 shows the expected distributions of the p-th largest values for the p-th order methods, where an outstanding behavior is that the p-th largest value's maximum likelihood location shifts from the side of 100% percentile to that of the 95% percentile as the number of order increases.

Table 1. Minimum Number of Code Simulations for p-th Order Wilks' One-Sided Formula

Order of Wilks Formula (p value)	Minimum Number of Code Simulations (n)
1	59
2	93
3	124
4	153
5	181

Table 2. Experimental Vs. Analytic Confidence Levels for Higher Order Wilks' Formulas

p, n	Experimental Confidence (%)	Analytic Confidence (%)	Diff. (%)
1, 59	95.1374	95.1505	0.01
2, 93	94.9686	95.0024	0.03
3, 124	95.0158	95.0470	0.03
4, 153	95.0726	95.0555	0.02
5, 181	95.0941	95.0837	0.01

* Note: The confidence levels in the above Table are for selecting the p-th largest value among the p-th order corresponding number of code simulations.

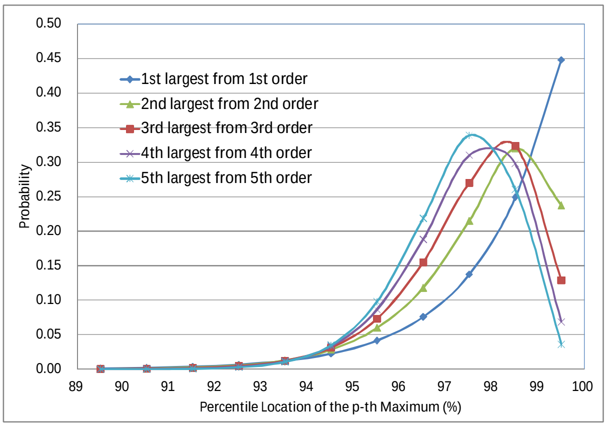

Figure 1. Distributions of p-th Largest Outputs from p-th Order Methods

4. Conclusions

Effectiveness was investigated for a present practice of using the p-th largest code output for the p-th order Wilks' method instead of the simplest 1st order approach.

The conclusion is that the p-th largest code output for the p-th order method, which uses a larger number of code simulations, can be used to generate the p-th largest values distributed closer to the side of 95% percentile than the 1st order method. On the other hand, the statistical criterion of the 95%/95% is confirmed to be met for all p-th largest code output for the p-th order method.

Thus, it is recognized that a higher order method can be clearly advantageous over the 1st order, specifically from designers' point of view to demonstrate a higher safety margin for a parameter of interest. While it should be noted that this higher order method could be used to provide a maybe less conservative limit, and thus the regulator has to be aware that the resultant p-th value from the p-th order is lower than the 1st value from the 1st order with a stronger chance.

For the approach itself, one another disadvantage of the p-th order compared to the 1st order is that it requires a much larger number of code simulations but the other parameters than the p-th largest values are not fully utilized. Further research would be necessary in the near future to find a more effective way in utilizing the information from all the code outputs.

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