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Theoretical Prediction of MHD Pressure Drop of Sodium Flow under Transverse Magnetic Field

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Abstract

Magnetic Field has much effects on hydraulic pressure drop of fluid with the high electrical conductivity. In the present study, solution on MHD pressure drop is sought theoretically for the uniform current density model with simplified physical geometry. Using the MHD equation in the rectangular duct of the sodium liquid flow under transverse magnetic field, electrical potential is sought in terms of the duct geometry and the electrical parameters of liquid metal and duct material. By the product of induced current inside liquid metal and transverse magnetic field, the pressure gradient is found as a function of the duct size and the electrical conductivity of liquid metal. As a result, pressure drop is theoretically predicted according to flow velocity and magnetic flux density. A experiment is prepared for the examination of the theory.

I. Introduction

Liquid metal sodium with the high electrical conductivity is used as a coolant of Liquid Metal Reactors of KALIMER kinds.[1] In the KALIMER, the pressure drop of the liquid sodium flow can be strongly affected by the external magnetic field. Various works have been carried out for the circulation systems in use of Li or NaK.[2-5] Practically, pressure drop is predicted to be caused by the duct system of non-magnetic stainless steel in addition to liquid sodium itself. Generally, pressure drop has negative aspects in the hydraulic system. Electrically conducting fluid can experience pressure drop by the magnetic field induced by the device such as EM flowmeter or EM pump in the Liquid Metal Reactor. Then, pressure drop by magnetic field becomes the function of duct geometry and electrical parameters. In the present study, theoretical calculations are worked out with uniform current model on the rectangular flow section. It is analyzed under reasonable assumption that the pressure drop by wall friction is negligible compared to that by MHD action in the strong magnetic field. In this kinds of flowing with Hartmann number ≫1,[5] viscous term at the Navier-Stokes equation is negligibly small in comparison with electromagnetic force term and thus pressure drop(∇p) becomes $\nabla p = J \times B$ for inviscid flow where J is uniform current density under D. C. magnetic flux density(B).

II. Formulation

Fig.1 for the present experimental device shows the cross-section of the rectangular duct section with transverse uniform magnetic field and Fig.2 is an illustrative diagram for the analysis based on uniform current density model. Along the upper wall current and voltage are given as follows.[4]

$$I(x) = Jx = \frac{Ix}{a} \tag{1}$$

$$V(x) = V_0 - \frac{1}{2} IR_e (\frac{x}{a})^2 \tag{2}$$

In the sodium flowing, voltage is obtained from Ohm's law.

$$V(x) = u(x)Bb - IR_i \tag{3}$$

Combining Eqs. (2) and (3) gives velocity distribution.

$$u(x) = u_0 - k_u (x/a)^2 \tag{4}$$

where $u_0 = (V_0 + IR_i)/Bb$ and $k_u = IR_e/2Bb$

And average velocity over flow channel is given by integration from 0 to a.

$$U = u_0 - k_u/3 \tag{5}$$

Therefore,

$$UBb = (V_0 + IR_i - IR_e/6) \tag{6}$$

Along the side walls, applying Kirhichoff's law shows following relation.

$$V_0 = IR_e/2 + IR_w \tag{7}$$

By eliminating V_0 from Eqs. (6) and (7), current density is obtained as follows.

$$J = \frac{UBb}{a(R_w + R_e/3 + Ri)} \tag{8}$$

Therefore, pressure gradient is calculated by Lorentz's product.[2-5]

$$-dp/dz = JB = K_p \sigma_f UB^2 \tag{9}$$

$$K_p = C/(1 + a/3b + C) \tag{10}$$

$$(\text{Ref. } K_p = C'/(C' + 1) \text{ in circular geometry})$$

where $C = \sigma_w t_w / \sigma_f a$ and $C' = \sigma_w (R_0^2 - R_i^2)/\sigma_f (R_0^2 + R_i^2)$. In Eq. (10) pressure gradient is known to be proportional to velocity and squared magnetic flux density. Practically, In the Miyazaki's NaK blowdown experiment[2-5], k_p has the experimental value of 0.0819 and 0.04 each in the rectangular (aspect ratio = 0.451) and circular geometry(inner radius = 22.65 mm and outer radius = 24.3 mm) where the range of the operating magnetic flux density is 0.3 - 1.75T and that of velocity is 2-15 m/sec. He also shows good agreement with the theoretical prediction.

III. Theoretical Results and Discussion

In the Fig.3, pressure coefficient (Kp) is plotted as a parameter of duct thickness according to operating temperature. The value of the coefficient is increased at the larger duct thickness. As a result, relatively thin duct geometry is required for the reduction of the efficient pressure drop from the Fig.3. Fig.4-Fig.6 shows the theoretical prediction over the our operating range of the fluid velocity with 0-4 m/sec and the magnetic flux density with 0.2-0.48 T. Fig.4 shows pressure gradient (dp/dz) according to the magnetic fields against mean flow velocity(U) in the rectangular channel of the sodium temperature with 200 ℃. And Fig.5 shows pressure gradient versus the magnetic field (B) with the mean flow velocity as a parameter at the same temperature. In the Fig.4 and Fig.5, it is shown that pressure gradient increases in proportion to flow velocity and squared magnetic field. Fig.6 shows pressure gradient on the electromagnetic force density ($\sigma_f UB^2$) at the different temperature. Because electrical conductivity is smaller at higher temperature, electromagnetic force density becomes smaller at higher temperature. But on the contrary pressure gradient is shown to be larger at higher temperature in the Fig.6. Because electrical conductivity of duct material also becomes smaller at the high temperature and so Joule's dissipation in the duct is reduced, it is thought that most of the electromagnetic force can be transferred to pressure drop without much loss. In fact this large pressure drop can be applied to the flow control of the liquid metal flow.

IV. Conclusion and future plan

The pressure drop in rectangular channel with uniform magnetic field has been found as a function of duct geometry and magnetohydrodynamic parameters using the uniform current model. Theoretical prediction has been carried out for the some variables and it is shown that pressure drop is given as the value of the developed electromagnetic force multiplied by the coefficient, Kp ($-\Delta p = \int k_p \sigma_f UB^2 dx$. In the near future, experimental characterization are going to be worked out and comparative analyses will be accompanied. While, pressure drop by the magnetic field which is negative aspect in the hydraulic system can be reciprocally applied to control the flow where liquid metal is adopted.

- Nomenclature

J : Current Density
I : Total current per unit length (=Ja)
V_0 : Half of inter-upper wall voltage
B : Magnetic flux density
p : Pressure
u : Fluid velocity
U : Mean velocity
a : Half length of upper wall
b : Half length of side wall

t_w : Thickness of duct wall
σ_f : Electrical conductivity of the sodium
σ_w : Electrical conductivity of the duct material
R_e : Resistance of upper wall $(= a/\sigma_w t_w)$
R_w : Resistance of side wall $(= b/\sigma_w t_w)$
R_i : Resistance of fluid $(= b/\sigma_f a)$

V. Acknowledgements

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VI. References

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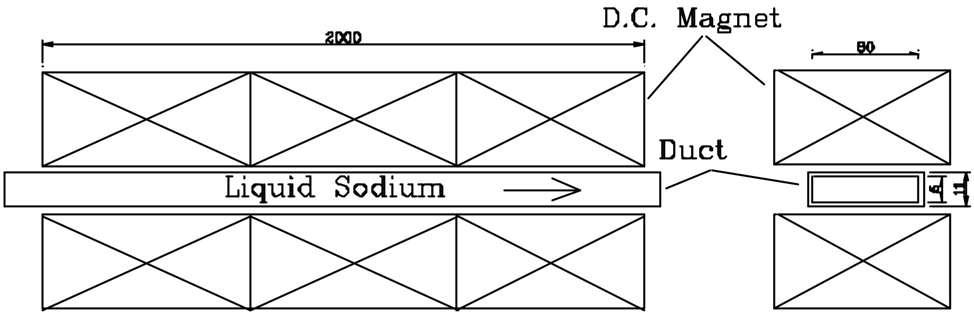

Fig 1. Cross-sectional View of Test Section

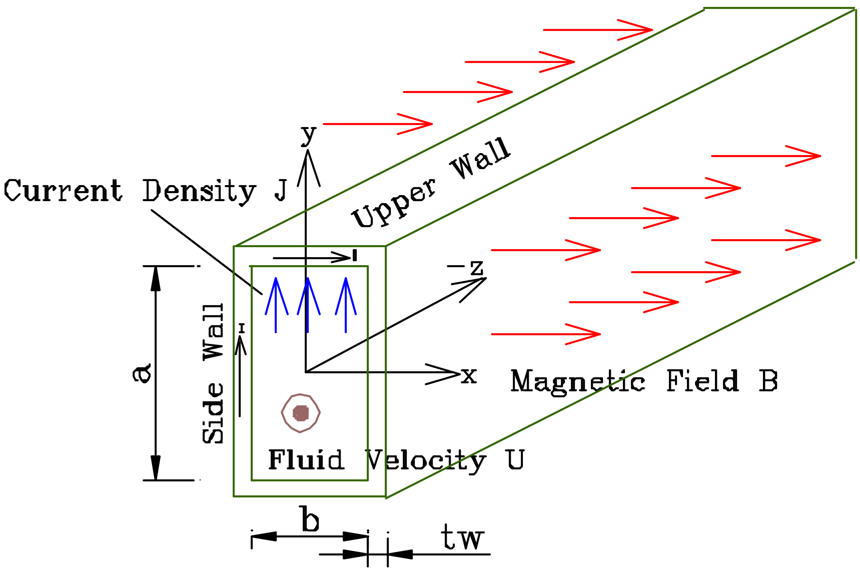

Fig 2. Illustrative Diagram for Theoretical Analysis

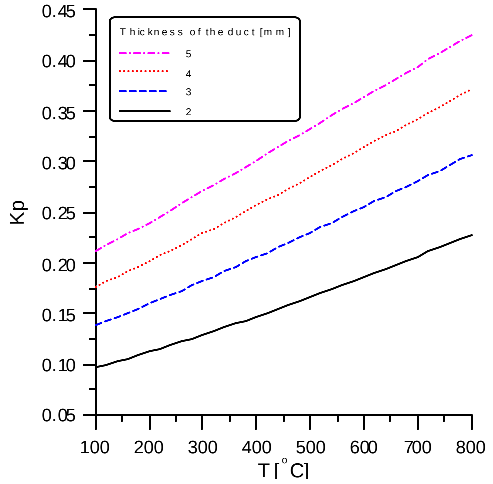

Fig. 3 Pressure Coefficient (Kp) vs Temperature according to Duct Thickness

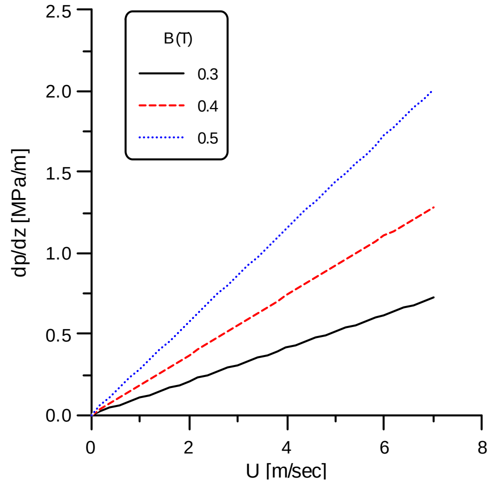

Fig. 4 Pressure Gradient vs Flow Velocity in Uniform Magnetic Field

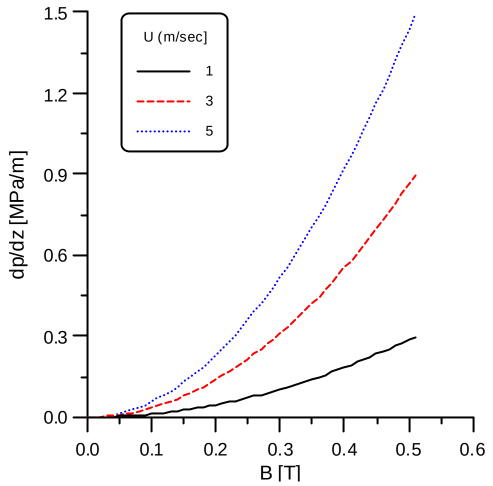

Fig. 5 Pressure Gradient vs Magnetic Flux Density according to Flow Velocity

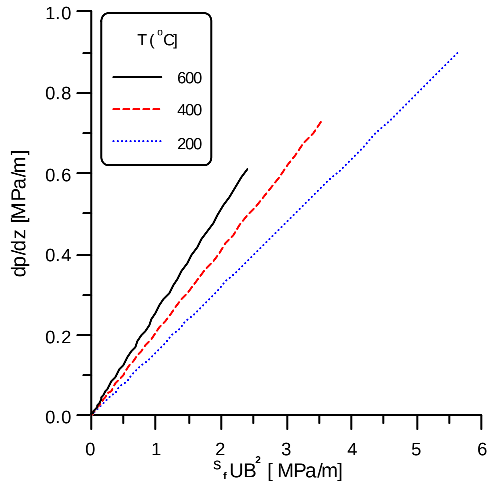

Fig. 6 Pressure Gradient vs Electromagnetic Force Density according to Temperature