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Stability Criteria and Choked Flow Condition for the One Dimensional Compressible Two-Fluid Model

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Abstract

Characteristic analyses are performed for the compressible one-dimensional two-fluid model to investigate the well-posedness of the governing differential equations and conditions for choked flow. The momentum flux parameters are introduced to consider the effect of void fraction profile and velocity profile across the flow area, which represent flow regime. It is shown that the compressible one-dimensional two-fluid model is well posed as an initial value problem with certain restrictions on the momentum flux parameters. The choked flow condition is also calculated for the one-dimensional two-fluid model with momentum flux parameters and is compared with that resulted from conventional model. It is suggested that the momentum flux parameters should be used for the one-dimensional two-fluid model.

1. Introduction

The well-posedness of the governing differential equation for the one-dimensional two-fluid model as an initial value problem is analyzed by the characteristic analysis (Lyczkowski, 1978;Ramshaw 1978; Stuhmiller, 1977; Jones and Prosperetti, 1985). From those studies, it is well recognized that the basic form the governing differential equations for the one-dimensional two-fluid model is ill-posed as an initial value problem. To resolve this intrinsic problem, there were lots of efforts to make the two-fluid model stable by considering the effect of virtual mass force (Lahey, 1980), phase-to-interface pressure difference for each flow regime (Hancox, 1980; Banerjee, 1980). However, the issue is still unsolved. Recently, Song (1998) proposed the use of momentum flux parameters to resolve this problem. The momentum flux parameters represents the flow morphology, that is, changes in void fraction profile and velocity profile across the flow area. They mathematically demonstrated that the incompressible one-dimensional two-fluid model is well-posed by incorporating the momentum flux parameters. Also by employing simplified flow structure, Song (1999) constructed a flow structure by using existing correlation C_0 and experimental velocity profile. Song (1999) calculated the momentum flux parameters for the whole range of flow regime. It is shown that the one-dimensional two-fluid model is well-posed for the simplified flow structure. As the previous researches were restricted to the incompressible one-dimensional two-fluid model, we will extend the argument to the compressible one-dimensional two-fluid model.

When we consider the compressible flow, we need to look at the choked flow condition. The choked flow is defined as the condition wherein the mass-flow rate becomes independent of the downstream conditions. As discussed by Trapp (1982), the path lines for a acoustic signal propagation are established from a characteristic analysis of the governing differential equations in the form of a system of the first-order, quasi-linear, partial differential equations. As the actual nozzle geometry of the critical flow is multi-dimensional, multi-dimensional analyses were sometimes employed to calculate the critical flow rate based on actual nozzle geometry (Rivard 1980; Minato 1988). However, the system analysis computer codes, such as, RELAP5/MOD3 (Ransom 1995) and CATHARE (Micaelli, 1988), are using one-dimensional two-fluid model. The multi-dimensional effect is hidden in the discharge coefficient, which depends on the nozzle geometry, such as, lenth and L/D. In Rivard (1980) and Minato (1988) the flow regime dependent void fraction profile and velocity profile were not accounted for. Therefore, we also would like to investigate the effect of momentum flux parameters on the two-phase critical flow.

2. The general form of one-dimensional two-fluid model

Here, we consider the compressible one-dimensional two-fluid model. The continuity equation is as follows

$$\partial(\rho_g\alpha_g)/\partial t + \partial(\rho_g\alpha_g u_g)/\partial z = \Gamma_g \qquad (1)$$
$$\partial(\rho_f\alpha_f)/\partial t + \partial(\rho_f\alpha_f u_f)/\partial z = \Gamma_f \qquad (2)$$

If we consider the equilibrium mass transfer between phases, the entropy equation is coupled with other equation. For the characteristic analysis, the following form of the entropy equation is used by Trapp(1982) by neglecting the non-differential terms.

$$\partial S_g/\partial t + u_g\partial S_g/\partial z - KS_g^*(\partial p/\partial t + u_g\partial p/\partial z) = 0 \qquad (3)$$
$$\partial S_f/\partial t + u_f\partial S_f/\partial z - KS_f^*(\partial p/\partial t + u_f\partial p/\partial z) = 0 \qquad (4)$$

K=1 corresponds to the thermal equilibrium case and K=0 corresponds to frozen flow case without mass exchange. Here, we assumed that the non-equilibrium mass transfer consists of the term proportional to the equilibrium mass transfer rate and term independent of derivatives. Here, we assumed that the bulk pressure of each phase is same and represent it as p. As the density is a function of pressure and entropy in general, the material derivative of density is written as

$$D\rho_k/Dt = (\partial\rho_k/\partial p)Dp/Dt + \partial\rho_k/\partial s_k\, Ds_k/Dt \qquad (5)$$

The adiabatic speed of sound for each phase is defined by $(\partial\rho_k/\partial p)_s = C_k^{-2}$ (6)

Then the continuity equation for each phase is presented as

$$\alpha_k C_k^{-2}(\partial p/\partial t + u_k\partial p/\partial z) + \alpha_k\rho_k\partial u_k/\partial z + \rho_k\partial\alpha_k/\partial t + \rho_k u_k\partial\alpha_k/\partial z = \Gamma_k - (\partial\rho_k/\partial s_k)Ds_k/Dt \quad (7)$$

The equation (3) or (4) can replace the last term in equation (7). In case of frozen flow, the characteristic roots of energy equation are independent from continuity and momentum equations.

The general form of the momentum equation is written as

$$\partial(\alpha_k\rho_k u_k)/\partial t + \partial(\alpha_k\rho_k C_{vk} u_k^2)/\partial z = -\alpha_k\partial p_k/\partial z - 4\alpha_{kw}\tau_{kw}/D + \partial[\alpha_k(\tau_{kzz} + \tau_{kzz}^T)]/\partial z$$
$$+ \alpha_k\rho_k g\cos\theta + (p_{ki} - p_k)\,\partial\alpha_k/\partial z + v_{ki}\Gamma_k + M_{ik} \qquad (8)$$

The generalized force term M_{ik} consists of transient forces such as Basset force and virtual mass force. C_{vk} is momentum flux parameter that accounts for the variation of velocity and void fraction over a cross-section. It contains information on the flow structure in the flow area normal to the main flow direction.

$$C_{vk} = \langle\alpha_k u_k^2\rangle / \langle\alpha_k\rangle \langle\langle u_k\rangle\rangle^2 \qquad (9)$$

Where, the quantity is area average and the quantity in the <<>> is void fraction weighted average (Ishii 1984).

3. Characteristic Analysis of Governing Differential Equations

The well-posedness of the governing differential equation as an initial value problem is analyzed by a characteristic analysis (Lyczkowski, 1978; Gidaspow, 1974; Stuhmiller, 1977; Jones and Prosperetti, 1985). Here, we look at the stability of two-fluid model by performing a characteristic analysis for the governing differential equations and derive the stability criteria. Let a vector $\mathbf{x} = (\alpha, u_g, u_f, p, s_g, s_f)$, then the above equation can be written as

$$[A]\ \partial \mathbf{x}/\partial t + [B]\ \partial \mathbf{x}/\partial z = [C] \qquad (10)$$

, where [A], [B], [C] are the coefficient matrices. The dependence of the solution on the prescribed initial data can be reduced to an investigation of the roots of equation

$$\text{Determinant}\ \{[A]\ \lambda - [B]\} = 0 \qquad (11)$$

, where we have introduced the characteristic curve λ. If we have real roots of λ for satisfying Determinant $\{[A]\lambda - [B]\}= 0$, then the set of differential equation is hyperbolic. If we have complex conjugate root of λ, then the set of differential equation becomes elliptic. In this case, the above set of equation becomes ill posed as an initial value problem.

Choked flow is defined as the condition wherein the mass-flow rate becomes independent of the downstream conditions. The path lines for a acoustic signal propagation are established from a characteristic analysis of the system of first-order, quasi-linear, partial differential equations. The eigen-value λ of the characteristic equation is related to the general Fourier component of the solution for the locally linear system. The real part of the root gives the velocity. The imaginary part of the root gives the rate of growth and decay of the signal along the path. For a hyperbolic system all the roots are real and non-zero. A choked flow condition exists when no information can propagate into the solution region from the exterior. Such a condition exists when

$$\lambda_j = 0 \text{ for some } j \quad \text{and} \quad \lambda_i \geq 0 \text{ for some } i \neq j \qquad (12)$$

Lahey (1980) considered the effect of virtual mass force term in generalized force M_{ik} on the stability of the governing differential equations. Hancox (1980) proposed to consider the phase-to-interface pressure difference for each flow regime to construct well-posed one-dimensional two-fluid model. Banerjee (1980) considered the phase-to-interface pressure difference for stratified flow. Incorporation of those terms in the one-dimensional two-fluid model resulted rather well-posed system. By neglecting the effect of those terms, we will focus on the role of momentum flux parameters on the stability and choked flow condition. Then, the momentum equation for compressible one-dimensional two-fluid model in equation (7) can be simplified as below

$$\partial(\alpha\rho_g u_g)/\partial t + \partial(\alpha\rho_g C_{vg} u_g^2)/\partial z = -\alpha\partial p/\partial z + M_{ig}^* \qquad (13)$$

$$\partial(\alpha_f\rho_g u_g)/\partial t + \partial(\alpha_f\rho_g C_{vg} u_g^2)/\partial z = -\alpha_f\partial p/\partial z - M_{if}^* \qquad (14)$$

where $\alpha_f = 1-\alpha$. M_{ig}^* and M_{if}^* represent non-derivative terms. It can be written as below

$$\alpha\rho_g\partial u_g/\partial t+\alpha\rho_g(2C_{vg}-1)u_g\partial u_g/\partial z+\rho_g(C_{vg}-1)u_g^2\partial\alpha/\partial z+\alpha\rho_g(C_{vg}-1)u_g^2\partial\rho_g/\partial z = -\alpha\partial p/\partial z - u_g\Gamma_g + M_{ig}^* \qquad (14)$$

$$(1-\alpha)\rho_f\partial u_f/\partial t+(1-\alpha)\rho_f(2C_{vf}-1)u_f\partial u_f/\partial z-\rho_f(C_{vf}-1)u_f^2\partial\alpha/\partial z+(1-\alpha)\rho_f(C_{vf}-1)u_f^2\partial\rho_f/\partial z = -(1-\alpha)\partial p/\partial z - u_f\Gamma_f + M_{if}^* \qquad (15)$$

4. Characteristic analysis for one-dimensional two-fluid model

The stability of the governing differential equations for general one-dimensional two-fluid model is determined from equation (7), (3), (4), (14), and (15). For simplicity let's consider the case of frozen flow. Then the characteristics of entropy equation are independent of other equations and determined as u_g and u_f. Also, we consider isothermal process, where the density is only function of pressure. Then, the system of first order differential equations to be considered becomes as follows

$$\alpha C_g^{-2}(\partial p/\partial t+u_g\partial p/\partial z)+\alpha\rho_g\partial u_g/\partial z+\rho_g\partial\alpha/\partial t+\rho_g u_g\partial\alpha/\partial z = \Gamma_g-(\partial\rho_g/\partial s_g)Ds_g/Dt \qquad (16)$$

$$(1-\alpha)C_f^{-2}(\partial p/\partial t+u_f\partial p/\partial z)+(1-\alpha)\rho_f\partial u_f/\partial z-\rho_f\partial\alpha/\partial t-\rho_f u_f\partial\alpha/\partial z = \Gamma_f-(\partial\rho_f/\partial s_f)Ds_f/Dt \qquad (17)$$

$$\alpha\rho_g\partial u_g/\partial t+\alpha\rho_g(2C_{vg}-1)u_g\partial u_g/\partial z+\rho_g(C_{vg}-1)u_g^2\partial\alpha/\partial z = -\alpha\rho_g(C_{vg}-1)u_g^2 C_g^{-2}\partial p/\partial z-\alpha\partial p/\partial z-u_g\Gamma_g+M_{ig}^* \qquad (18)$$

$$(1-\alpha)\rho_f\partial u_f/\partial t+(1-\alpha)\rho_f(2C_{vf}-1)u_f\partial u_f/\partial z-\rho_f(C_{vf}-1)u_f^2\partial\alpha/\partial z = -(1-\alpha)\rho_f(C_{vf}-1)u_f^2C_f^{-2}\partial p/\partial z-(1-\alpha)\partial p/\partial z-u_f\Gamma_f+M_{if}^* \qquad (19)$$

By neglecting non-derivative terms, the characteristic matrix $[A]\lambda - [B]$ becomes

$$\begin{matrix} \rho_g(\lambda-u_g) & -\alpha\rho_g & 0 & \alpha C_g^{-2}(\lambda-u_g) \\ -\rho_f(\lambda-u_f) & 0 & -(1-\alpha)\rho_f & (1-\alpha)C_f^{-2}(\lambda-u_f) \\ -\rho_g(C_{vg}-1)u_g^2 & \alpha\rho_g[\lambda-(2C_{vg}-1)u_g] & 0 & -\alpha[1+\rho_g(C_{vg}-1)u_g^2 C_g^{-2}] \\ \rho_f(C_{vf}-1)u_f^2 & 0 & (1-\alpha)\rho_f[\lambda-(2C_{vf}-1)u_f] & -(1-\alpha)[1+\rho_f(C_{vf}-1)u_f^2C_f^{-2}] \end{matrix} \qquad (20)$$

The stability of the governing differential equation and condition for choking depend on the root of λ for the equation below

$$\text{Determinant of } \{[A]\lambda-[B]\} = f(\lambda)=0 \qquad (21)$$

The conventional one-dimensional two-fluid models assumed that the value of momentum flux parameter is unity. They did not consider the effect of void fraction profile and velocity profile. Let $\alpha_f= 1-\alpha$. If we put C_{vg} and C_{vf} as 1, then the function $f(\lambda)$ becomes

$$f(\lambda)=\rho_f\alpha\rho_g\alpha_f[(\lambda-u_g)^2(\lambda-u_f)^2(\rho_f\alpha C_g^{-2}+\rho_g\alpha_f C_f^{-2})-\rho_g\alpha_f(\lambda-u_g)^2-\rho_f\alpha(\lambda-u_f)^2]=0 \qquad (22)$$

It is a fourth order polynomial in λ. It becomes positive when λ is a big positive or big negative number. As the function becomes negative when λ equals u_g or u_f, it always has two real roots satisfying $f(\lambda)=0$. These two characteristic velocities are related to the sound speed of two-phase mixture. The other two roots can be real or complex depending on the magnitude of relative velocity. As discussed by Lyczkowski (1978) the magnitude of relative velocity should be same order as that of homogeneous mixture sound speed to have 4 real roots unless relative velocity is zero. As that situation is quite rare in actual flow situation, the other two roots of equation (22) become complex. This indicates that the one-dimensional two-fluid model becomes easily ill-posed as an initial value problem.

Without performing detailed numerical calculation as those by Lyczkowski (1978), we can obtain similar conclusion in the following way. When the gas and liquid velocities are relatively small compared to the speed of sound of gas and liquid, equation (22) can be approximated by

$$f(\lambda) \cong - \rho_f\alpha\rho_g(1-\alpha)[\rho_g(1-\alpha)(\lambda-u_g)^2 + \rho_f\alpha(\lambda-u_f)^2] \qquad (23)$$

It is always negative. It means that equation (22) does not cross the x-axis when λ is at similar order of magnitude as that of gas or liquid velocity. In this case, $f(\lambda)=0$ has two real roots and two complex roots. Then one-dimensional two-fluid model becomes ill-posed. As the gas and liquid velocities are usually much smaller than the sound speed in practical cases, the ill-posedness is quite a problem for one-dimensional two-fluid model.

As discussed in the previous researches, it can be assumed that the value of the momentum flux parameters is very close to 1. Therefore, we can approximate the above matrix by the following matrix by neglecting the terms including $(C_{vg}-1)$ and $(C_{vf}-1)$.

$$\begin{matrix} \rho_g(\lambda-u_g) & -\alpha\rho_g & 0 & \alpha C_g^{-2}(\lambda-u_g) \\ -\rho_f(\lambda-u_f) & 0 & -(1-\alpha)\rho_f & (1-\alpha)C_f^{-2}(\lambda-u_f) \\ 0 & \alpha\rho_g(\lambda-C_{vg}u_g) & 0 & -\alpha \\ 0 & 0 & (1-\alpha)\rho_f(\lambda-C_{vf}u_f) & -(1-\alpha) \end{matrix} \qquad (24)$$

When we consider the effect of void fraction and velocity profiles by introducing C_{vf} and C_{vg}, we can obtain similar expression as that of equation (22) as below.

$$f(\lambda) = \rho_f\alpha\rho_g\alpha_f[(\lambda-C_{vg}u_g)(\lambda-u_g)(\lambda-C_{vf}u_f)(\lambda-u_f)(\rho_f\alpha C_g^{-2}+\rho_g\alpha_f C_f^{-2})$$
$$- \rho_g\alpha_f(\lambda-u_g)(\lambda-C_{vg}u_g)-\rho_f\alpha(\lambda-u_f)(\lambda-C_{vf}u_f)] = 0 \qquad (25)$$

As the coefficient of the λ^4 is positive, this function is also positive when λ is big positive or negative number. The behavior of this equation will be similar to that of equation (22), as the value of C_{vg} and C_{vf} is quite close to unity in the actual flow as discussed in the previous researches (Song 1998; Ishii 1984). However, the behavior of the equation (25) is very different from that of equation (22), when λ is close to gas or liquid velocity and it is much less than the speed of sound. In this case, equation (25) can be approximated as

$$f(\lambda) \cong -\rho_f\alpha\rho_g\alpha_f\{\alpha_f\rho_g[\lambda\lambda-\lambda(C_{vg}+1)u_g+C_{vg}u_gu_g]+\alpha\rho_f[\lambda\lambda-(C_{vf}+1)u_f\lambda+C_{vf}u_f u_f]\} = 0 \quad (26)$$

The exact form of the determinant for incompressible flow discussed in the previous research. The exact form for the incompressible flow is

$$f(\lambda) = -\rho_f\alpha\rho_g\alpha_f[\alpha_f\rho_g(\lambda\lambda-2\lambda C_{vg}u_g+C_{vg}u_g u_g)+\alpha\rho_f(\lambda\lambda-2C_{vf}u_f \lambda+C_{vf}u_f u_f)] = 0 \quad (27)$$

Equation (27) can two real roots, if momentum flux parameters C_{vg} and C_{vf} behave in a such way that following condition is met. Let $S = u_g/u_f$ and call it slip ratio and let $R= \alpha\rho_f/((1-\alpha)\rho_g)$ and call it modified density ratio. And assume that liquid velocity is positive. As discussed in Song(1998) equation (27) is satisfied when either of following inequality is met

$$C_{vf} \geq ¼[1/R+ 1]\, S^2/(S - 1), \quad S > 1 \qquad (28)$$

$$C_{vg} \geq 0.5(1+R) + 0.5[(1+R)^2-4(1+R)RC_{vf}1/S^2(S-1)]^{1/2} - RC_{vf}1/S \qquad (29)$$

$$C_{vg} \leq 0.5(1+R) - 0.5[(1+R)^2-4(1+R)RC_{vf}1/S^2(S-1)]^{1/2} - RC_{vf}1/S \qquad (30)$$

This feature is fundamentally different from that of equation (23). This enables equation $f(\lambda)=0$ to have 4 real roots more easily. Then, the generalized two fluid model becomes well-posed as a initial value problem in a broader range.

As the actual nozzle geometry of the critical flow is multi-dimensional, multi-dimensional analyses were sometimes employed to calculate the critical flow rate based on actual nozzle geometry (Rivard 1980; Minato 1988). However, the system analysis computer codes, such as, RELAP5/MOD3 (Ransom 1995) and CATHARE (Micaelli, 1988), are using one-dimensional two-fluid model. The multi-dimensional effect is hidden in the discharge coefficient, which depends on the nozzle geometry, such as, lenth and L/D. In Rivard (1980) and Minato (1988) the flow regime dependent void fraction profile and velocity profile were not accounted for. Therefore, we would like to investigate the effect of momentum flux parameters on the two-phase critical flow.

From equation (26) we can easily notice that the critical flow condition for two-phase flow is also affected by the momentum flux parameters. Song (1999) calculated values of momentum flux parameters which renders well-posed one-dimensional two fluid model for wide range of flow. When momentum flux parameters are not considered, the choked flow condition is determined from the equation below

$$f(\lambda)=\rho_f\alpha\rho_g\alpha_f[(\lambda-u_g)^2(\lambda-u_f)^2(\rho_f\alpha C_g^{-2}+\rho_g\alpha_f C_f^{-2}) - \rho_g\alpha_f(\lambda-u_g)^2-\rho_f\alpha(\lambda-u_f)^2] = 0 \quad (22)$$

When $\alpha=0$ or $\alpha=1$, the quantity in the bracket $f(\lambda)=0$ requires that

$$\lambda= u_f \; +/- \; C_f \quad \text{or} \quad \lambda= u_g \; +/- \; C_g \qquad (31)$$

Let's simplify the equation by introducing R and S. Then the above equation becomes

$$f(\lambda)=\rho_f\alpha\rho_g\alpha_f\, u_f^2\rho_g\alpha_f[u_f^2(\lambda^*-S)^2(\lambda^*-1)^2(RC_g^{*-2}+C_f^{*-2}) - (\lambda^*-S)^2-R(\lambda^*-1)^2] = 0 \quad (32)$$

Where $\lambda^*=\lambda/u_f$ and $C_g^* = C_g/u_f$, $C_f^* = C_f/u_f$. When the momentum flux parameters are considered, the choked flow condition is determined from the equation below

$$f(\lambda)=\rho_f\alpha\rho_g\alpha_f\, u_f^2\rho_g\alpha_f[u_f^2(\lambda^*-C_{vg}S)\,(\lambda^*-S)(\lambda^*-C_{vf})(\lambda^*-1)(RC_g^{*-2}+C_f^{*-2})$$
$$- \quad (\lambda^*-S)\,(\lambda^*-C_{vg}S)-R(\lambda^*-1)(\lambda^*-C_{vf})] = 0 \quad (33)$$

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5. Application of proposed arguments

For convenience, we can assume that the liquid velocity is 1m/s. At pressure of 1.17 Mpa, ρ_g=5.9795 kg/m^3, ρ_f=879.55 kg/m^3, C_g^*= 503.8 m/s, C_f^*= 1348.5 m/s. S can be determined from distribution parameter C_o. Then we can determine λ^* as a function of void fraction. According to Ishii, for bubbly and slug flow when the void fraction is less than 0.7, a lot of data for pipe flow was be correlated as

$$C_o = (1.2-0.2\sqrt{(\rho_g/\rho_f)})(1-e^{-18\alpha}) \quad (34)$$

By definition average slip ratio is determined as

$$S=u_g/u_f=(1-\alpha)\, C_o/(1- C_o\alpha) \quad (35)$$

The momentum flux parameters can be obtained from Song(1999) as a function of average void fraction for simplified flow for the whole range of flow regime. Then we can determine the roots of characteristic equations as a function of void fraction.

For the two-fluid model without momentum flux parameters, let's plot the terms in the bracket of equation (22), when liquid velocity is 1m/s at various void fractions.

$$f(x,\alpha)=[u_f^2(x-S(\alpha))^2(x-1)^2(R(\alpha)C_g^{*-2}+C_f^{*-2}) - (x-S(\alpha))^2-R(\alpha)(x-1)^2] \quad (36)$$

The points at which the function $f(x,\alpha)$ meets the x-axis are the roots of the characteristic equation. Fig. 1 shows that the function has only two real roots. The two real roots correspond to the sonic velocity of two-phase mixture. The sonic velocity is shown below as a function of void fraction.

For the two-fluid model with momentum flux parameter, the roots are determined from the following equation

$$g(x,\alpha)=[u_f^2(x-C_{vg}(\alpha)S(\alpha))(x-S(\alpha))(x-C_{vf}(\alpha))(x-1)(R(\alpha)C_g^{*-2}+C_f^{*-2})$$
$$- (x-C_{vg}(\alpha)S(\alpha))(x-S(\alpha) - R(\alpha)(x-C_{vf}(\alpha))(x-1)] \quad (37)$$

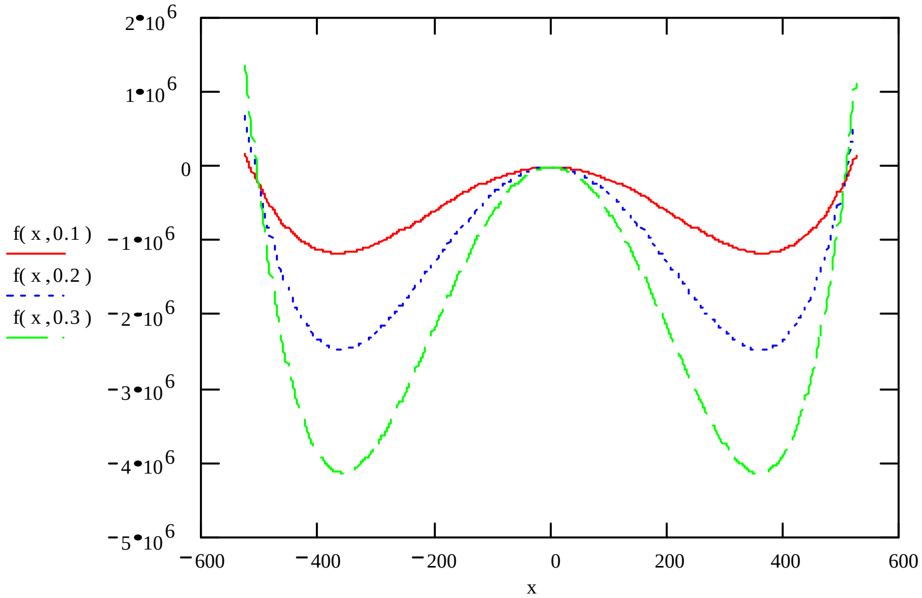

Fig. 1 Plot of function f(x, α)

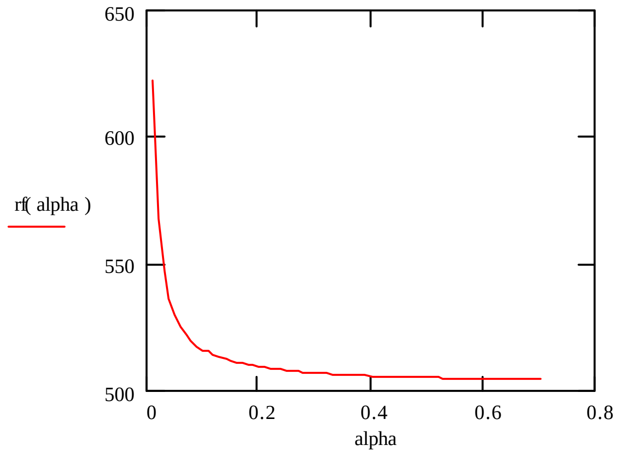

Fig. 2 Sonic velocity as a function of average void fraction

By taking values of momentum flux parameters from Song(1999), we can calculate above functions. Let' s plot the function. As the value of the momentum flux parameters is close to 1, the overall shape of the curve is same as equation (36). However, the behavior is very different when x is close to 1.

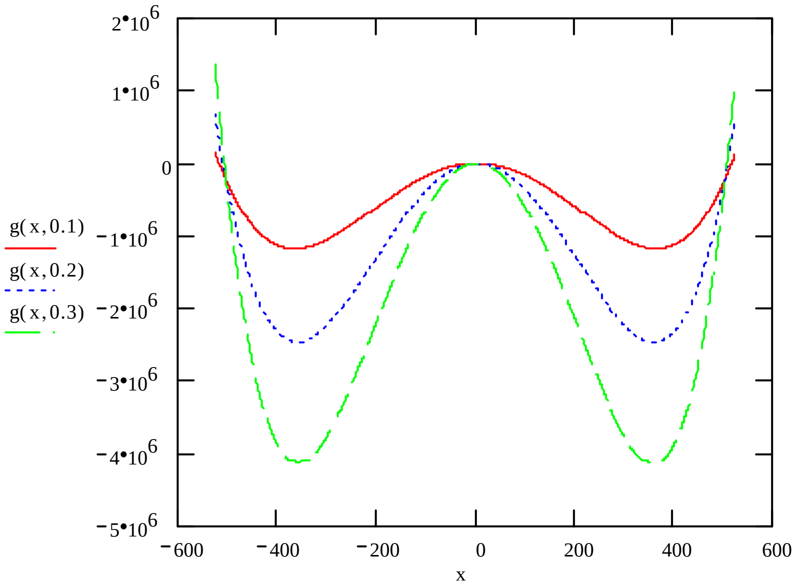

Fig. 3 Plot of function g(x, α)

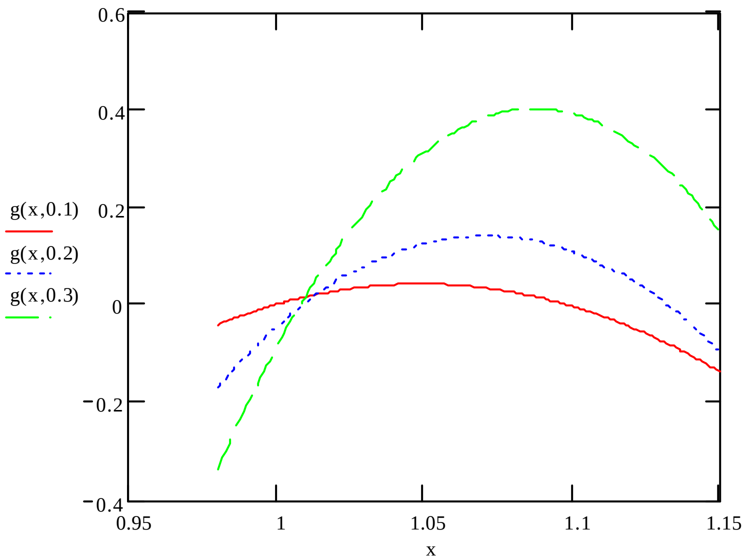

Fig. 4 Plot of function g(x, α)

It is shown that the function g(x, α) has two real roots near x=1.0 and two real roots near x=+/-500, that makes the one-dimensional two-fluid model hyperbolic. The small roots near liquid velocity correspond to propagation velocity of void fraction wave. The big roots near +/-500 correspond to the two-phase sonic velocity. Within the range of average void fraction of 0.2-0.7, the sonic velocities are calculated. It is shown that the sonic velocity is very close each other in Fig. 5. Since Song (1999) suggested momentum flux parameters for the whole range of void fraction, we can calculate choked condition for the whole range of void fraction.

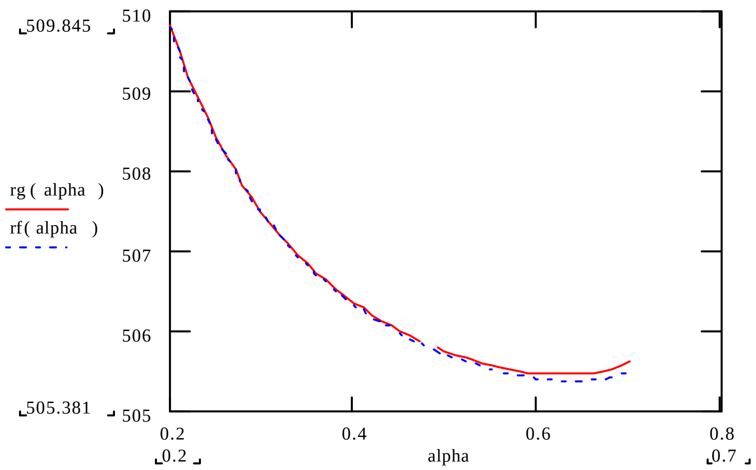

Fig. 5 Two-phase sonic velocity (rg: w/, rf: w/o momentum flux parameter)

6. Conclusions

The characteristic analyses showed that the compressible one-dimensional two-fluid model is well posed as an initial value problem with certain restrictions on the momentum flux parameters. The choked flow condition is determined. The sample calculation by using simplified flow structure at typical condition demonstrates the feasibility of proposed approach. Therefore, it is suggested that the momentum flux parameters should be used for one-dimensional two-fluid model.

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