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Analysis of Gamma Resonance Absorption in ^{14}N by Using the Radiative Direct Capture Reaction Theory

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Abstract

The 9.17 MeV gamma resonance absorption by ^{14}N can be used to detect nitrogenous explosives because of the narrowness of the resonance, its large integrated cross section, and its uniqueness to nitrogen. The inverse reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ at E_p=1.75 MeV is advantageous to obtain the corresponding gamma ray. In this work, the radiative capture reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ and the inverse reaction $^{14}\mathrm{N}(\gamma,p)^{13}\mathrm{C}$ have been theoretically analyzed on the basis of the direct reaction theory for the radiative capture reaction.

I Introduction

The photonuclear resonance reactions involving ^{14}N, especially the $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ reaction, have been of interest in various fields in nuclear physics[1–3]. From the theoretical point of view, ^{14}N is the heaviest stable odd-neutron, odd-proton nuclide. The $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ process is one of the CNO (carbon-nitrogen-oxygen) cycle of nucleosynthesis, because the reaction is useful to study the evolution of stars in astrophysics. And many experimental studies have been made.

There are many kinds of characteristics in the radiative proton capture reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$, especially 9.17 MeV radiation from the reaction. The 9.17 MeV radiation is distinguished by the narrowness of the resonance, its large integrated cross section, and its uniqueness to nitrogen−all of which result in high detection sensitivity and insensitivity to backgrounds. Moreover, the 9.17 MeV gamma-rays are very penetrating in ordinary matter, making it difficult to shield the explosives from detection. Recently, Kim *et al.* have also verified the transmission characteristics of the resonant absorption[4].

For these reasons, the 9.17 MeV radiation from the reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ at E_p=1.75 MeV can be used to detect nitrogenous explosives. Recently, a laboratory prototype system has been developed by Morgado *et al.* [5, 6] for the experimental evaluation of an explosives detection technique based on the nuclear resonance absorption of 9.17 MeV gamma-rays in ^{14}N.

In spite of the usefulness, there is no theoretical interpretation about the strong 9.17 MeV radiation from the reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ or the 9.17 MeV gamma-ray absorption resonance in ^{14}N. The purpose of this work is to theoretically analyze the radiative capture reaction ^{13}C$(p,\gamma)^{14}$N and the inverse reaction ^{14}N$(\gamma,p)^{13}$C. The analysis is on the basis of the direct reaction theory for the radiative capture reaction, which is based on a microscopic description, with the first order of perturbation theory. The theory has been quite successful in explaining radiative capture reactions, such as ^{3}He$(\alpha,\gamma)^{7}$Be[7] and ^{7}Be$(p,\gamma)^{8}$B[8], at low energy. In the work, the scattering wave function of ^{13}C by proton and the bound state wave functions of ^{14}N are constructed by the empirical model, phenomenological Woods-Saxon potential model, and orthogonality condition.

The concept of the explosives detection system is shown in Section II.1. Evaluation of the cross section for the reaction ^{13}C$(p,\gamma)^{14}$N has been done using the single level Breit-Wigner formula with given resonance parameters[2, 9]. With the evaluated cross section for the reaction ^{13}C$(p,\gamma)^{14}$N, the cross section for the photoemission reaction ^{14}N$(\gamma,p)^{13}$C is calculated by using the reciprocity theorem. They are shown in Section II.2. The formalism of the direct reaction theory for the radiative capture reaction and the calculated results with the theory are presented in Section II.3. Finally, Section III concludes the paper.

II Gamma Resonance Absorption in ^{14}N

II.1 Explosives Detection System Based on Gamma Resonance Absorption in ^{14}N

The basis of the technique for the explosives detection system is the existence of a narrow energy state in the nucleus of ^{14}N that results in a strong resonance in the photonuclear reaction cross section for ^{14}N$(\gamma,p)^{13}$C at 9.17 MeV[1, 2, 10]. Gamma rays are absorbed by nitrogen, followed by the prompt emission of a proton and a ^{13}C nucleus. The transition rate from the ground state of ^{14}N is unusually large, and as a result, 9.17 MeV gamma rays are strongly absorbed, resulting in a strong indication of the presence of nitrogen. Furthermore, the 9.17 MeV γ-rays are very penetrating in ordinary matter, making it difficult to shield the explosives from detection.

From the technical point of view, there are two sources for production of gamma rays : the bremsstrahlung beam from an electron accelerator and the inverse resonance reaction with the beam from a proton accelerator. The former required the development of γ-ray detectors capable of operating in the intense backgrounds of a pulsed electron accelerator. Attempts to model and measure bremsstrahlung beams indicated the practical difficulties of this approach. The latter one, the proton source, is technically advantageous in many ways. The proton source can be easily controlled because of its charge. Although the energy of the incident proton is low, the Q-value for the reaction ^{13}C$(p,\gamma)^{14}$N is high. As a result, high energy γ-rays can be obtained from the relatively low energy proton beam. Therefore, the inverse reaction ^{13}C$(p,\gamma)^{14}$N is used to produce the corresponding gamma-rays at the resonance energy. A schematic diagram of the explosives detection system is shown in Figure 1.

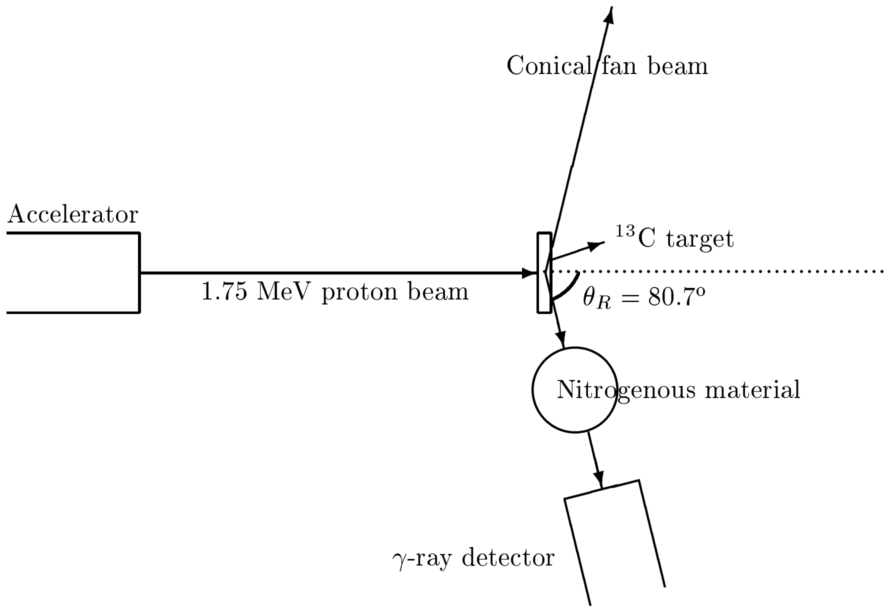

Figure 1: Resonance-absorption-based detection concept.

In the radiative capture of particles having fixed energy and direction of motion, the energy of the radiation varies with the angle of emission but has a definite value at a given angle. The energy of incident particles may be fixed by selecting a resonant capture process or, if the capture is non-resonant, by using a mono-energetic beam of particles and a thin target. In an early experiment[10] the source of radiation was the reaction $^{13}C(p,\gamma)^{14}N$ at the resonant energy E_p=1.75 MeV corresponding to the level at 9.17 MeV in ^{14}N. This radiation was then used to study the inverse absorption process $^{14}N + \gamma \longrightarrow ^{14}N^*$ by varying the energy (*i.e.*, angle of emission) of the radiation and observing the change in transmission through a nitrogen absorber. Since an amount of energy $E^2/2Mc^2$ is given to the recoiling nucleus in both the emission and the absorption process, it is necessary to increase the energy of the emitted radiation by an amount $E^2/2Mc^2$ to produce resonance, where E is the energy of the radiation and M the mass of the emitting (or absorbing) nucleus. Since the energy of the emitted radiation varies as $E(v/c)\cos\theta_R$, where v is the velocity of radiating nucleus, the resonant angle θ_R is obtained from the condition

$$\cos\theta_R = (E/2Mc^2)/(v/c) = p_\gamma/p, \tag{1}$$

where p_γ and p are the momenta of the γ-ray and the radiating nucleus, respectively.

II.2 Breit-Wigner Single Resonance Formula

Consider a radiative proton capture reaction $A(p,\gamma)B$. The fact that the radiative capture resonances are found experimentally to have the symmetrical Breit-Wigner form[11] leads one to the conclusion that there is little or no interference between levels. The basic reason for this lack of interference is the large number of exit channels. Each of the many possible gamma transitions from the capturing level to a lower level represents a distinct exit channel. For processes involving a large number of exit channels, the R-matrix multilevel formula[11] reduces to the single level Breit-Wigner form.

In the work, the γ-ray yield from the radiative capture reaction as a function of proton energy is described by the single level Breit-Wigner form of the cross section[11, 12],

$$\sigma(p,\gamma) = \frac{\pi \lambda\!\!\!{}^{-}{}_p^2 \, g \, ,_p \, ,_\gamma}{(E_p - E_R)^2 + \left(\frac{\Gamma_t}{2}\right)^2}, \tag{2}$$

where g is the statistical factor equal to $(2J+1)/(2J_A+1)(2J_p+1)$, in which J is the total angular momentum of the compound nucleus, J_A and J_p are the spins of the initial particles, $,_\gamma$ is the partial width of direct γ-ray transition to the ground state, $,_t$ and $,_p$ are the total resonance width and the partial decay width by proton, respectively, $\lambda\!\!\!{}^{-}{}_p$ is $1/2\pi$ times the wavelength of the proton, and E_R is the proton resonance energy.

The photodisintegration cross section of the reaction $B(\gamma,p)A$ as a function of photon energy can be obtained from the cross section of the reaction $A(p,\gamma)B$ by the reciprocity theorem[11–13],

$$\sigma(\gamma,p) = \frac{(2J_A+1)(2J_p+1)\; p_p^2}{(2J_B+1)(2J_\gamma+1)\; p_\gamma^2}\; \sigma(p,\gamma), \tag{3}$$

where p_p and p_γ are momenta of proton and photon, respectively.

Apply the single level Breit-Wigner formula to the ^{14}N system. The width of the 9.17 MeV level in ^{14}N was measured several times using various techniques[1, 10, 14]. In a recent compilation[9] of spectroscopic data, the total width of the level, $,_t$, and the partial width $,_\gamma$ are quoted as 122 eV and 6.3 eV, respectively. Therefore, $,_p$ is used with 115.7 eV. The experimental cross sections for the $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ and the $^{14}\mathrm{N}(\gamma,p)^{13}\mathrm{C}$ reactions produced from the resonance parameters using the single level Breit-Wigner formula and the reciprocity theorem are shown in Figure 2 and Figure 3, respectively.

II.3 Direct Reaction Theory for the Radiative Capture Reaction

In the first order perturbation theory, the radiative direct capture cross section of the reaction $A(a,\gamma)B$ can be written in terms of the transition amplitudes as[7, 8]

$$\frac{\mathrm{d}\sigma(a,\gamma)}{\mathrm{d}\Omega_\gamma} = 2\Big(\frac{e^2}{\hbar c}\Big)\Big(\frac{mc^2}{\hbar c}\Big)\Big(\frac{k_\gamma}{k_a}\Big)^3 \frac{1}{2I_A+1} \sum_{M_B M_A \sigma} |T_{M_B M_A \sigma}|^2, \tag{4}$$

where k_a and k_γ denote the wave numbers of the incident channel and emitted γ-rays, respectively, and m is the reduced mass of the incident channel. $I_A(M_A)$ stands for the spin (its projection) of nucleus A and σ for the polarization of the radiation, *i.e.*, σ=±1. In the long

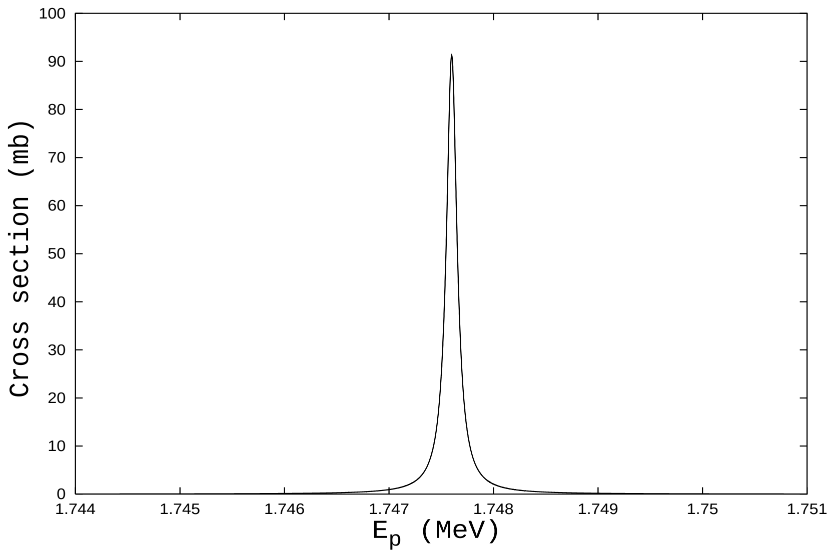

Figure 2: The radiative proton capture cross section for the reaction ^{13}C$(p, \gamma)^{14}$N evaluated from the corresponding resonance parameters using the single level Breit-Wigner formula.

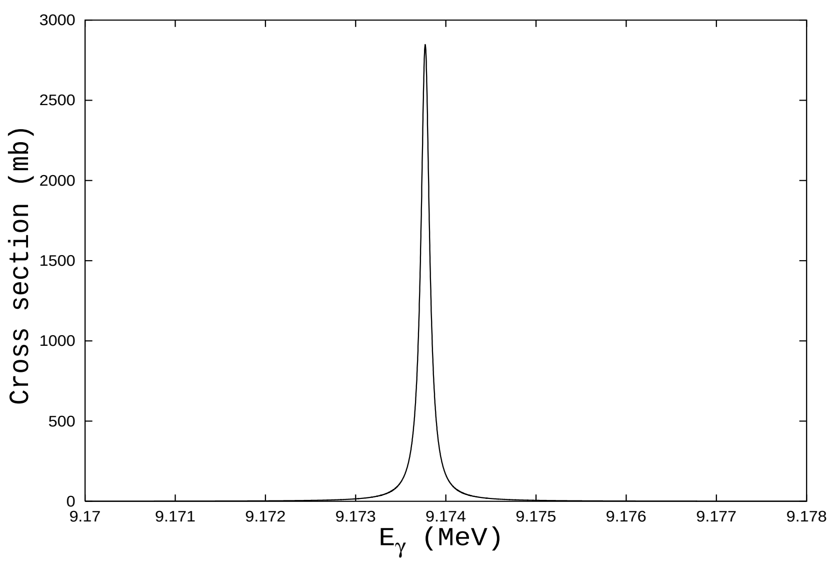

Figure 3: The photoabsorption cross section for the reaction ^{14}N$(\gamma, p)^{13}$C calculated from the radiative capture cross section for the reaction ^{13}C$(p, \gamma)^{14}$N using the reciprocity theorem.

wavelength limit, the transition amplitude can be expanded in terms of d functions with the electro-magnetic multipole λ as

$$T_{M_B M_A \sigma}(\theta) = \sum_\lambda T^\lambda_{M_B M_A \sigma} d^\lambda_{M_A - M_B \sigma}(\theta), \tag{5}$$

where $\boldsymbol{k_a}$ is taken to be parallel to the z axis and θ is the polar angle of $\boldsymbol{k_\gamma}$.

The electromagnetic multipole transition amplitude $T_{M_B M_A \sigma}$ can be further decomposed in terms of partial waves $l_a j_a$:

$$T^\lambda_{M_B M_A \sigma} = \sum_{l_a j_a} \left[\beta^{E\lambda}_{l_a j_a} + (-)^{\delta(\sigma+1)} \beta^{M\lambda}_{l_a j_a}\right] \langle\, l_a 0 I_a M_a | j_a M_A \,\rangle \langle\, \lambda M_a - M_B I_B M_B | j_a M_A \,\rangle\,. \tag{6}$$

For the $E1$, $M1$, and $E2$ transitions, which play an important role in the low-energy capture process, the β's are

$$\beta^{E\lambda}_{l_a j_a} = i^{l_a} C(E\lambda) \hat{l}_a \hat{l}_b \hat{I}_B \langle\, l_b 0 \lambda 0 | l_a 0 \,\rangle \times W(\lambda l_b j_a I_a; l_a l_B) I^\lambda_{l_b I_B, l_a j_a}, \tag{7}$$

where l_b is the orbital angular momentum of $A + a$ two clusters in the nucleus B, and

$$C(E1) = im\Big(\frac{Z_a}{m_a} - \frac{Z_A}{m_A}\Big), \tag{8}$$

$$C(E2) = \frac{k_\gamma}{12} m^2 \Big(\frac{Z_a}{m_a^2} + \frac{Z_A}{m_A^2}\Big), \tag{9}$$

and

$$\begin{aligned} \beta^{M\lambda}_{l_a j_a} &= i^{l_a} \hat{l}_a \hat{l}_b \hat{I}_B \delta_{l_a l_b} \Big[\frac{m\hbar c}{2m_p c^2} \Big(\frac{Z_a}{m_a^2} + \frac{Z_A}{m_A^2}\Big) [l_a(l_a+1)]^{1/2} W(1 l_a j_a I_A; l_a I_B) \\ &+ \frac{\hbar c}{2m_p c^2} \mu_A (-)^{I_B - j_a} \hat{I}_A W(1 I_A j_a l_a; I_A I_B)\Big] I^0_{l_b I_B, l_a j_a}, \end{aligned} \tag{10}$$

where μ_A and Z_A are the magnetic moment and charge of A, respectively. $\hat{j}$ denotes $\sqrt{2j+1}$ and $W(1 I_A j_a l_a; I_A I_B)$ is the Racah coefficient. The overlap integral I^λ is

$$I^\lambda_{l_b I_B, l_a j_a} = \int \mathrm{d}r U_{l_b I_B}(r) r^\lambda \chi_{l_a j_a}(r), \tag{11}$$

where $\chi_{l_a j_a}$ is the radial distorted wave function and $U_{l_b I_B}$ the radial part of the bound state wave functions used by Tamura[15]. Numerical evaluation of the radial integral is necessary to obtain the cross sections after a model is adopted to describe the relative motion between A and a.

The differential cross section for the radiative capture of a nucleon can be obtained from Eq. (4) using the reciprocity theorem, Eq. (3). The integrated total cross sections $\sigma(\gamma, a)$ and $\sigma(a, \gamma)$ are calculated by integrating the differential cross sections over their solid angles.

The direct reaction theory for the radiative capture reaction is applied to the radiative proton capture reaction ^{13}C$(p,\gamma)^{14}$N at E$_p$=1.7476±0.0009 MeV and to the 9.172±0.002 MeV gamma resonance absorption in the reaction ^{14}N$(\gamma,p)^{13}$C.

First, it is assumed that the particles in the target nucleus behave like independent particles in the Hartree-Fock (HF) mean field real potential. Furthermore, the ground state of ^{14}N can be assumed to be a single proton configuration on the ^{13}C core with central and spin-orbit nuclear interactions[8]. Therefore, the ground state wave function of ^{14}N was generated under the condition that the experimental binding energy be reproduced in a single proton potential well.

The radiative proton capture reaction ^{13}C$(p,\gamma)^{14}$N is understood by the following. A free proton with angular momentum and intrinsic spin of $\frac{1}{2}^+$ will be captured by the ^{13}C core nucleus which is in the ground state, thus leading to the excited state of the ^{14}N nucleus. The ground state of the ^{13}C nucleus of $\frac{1}{2}^-$ can be understood of the shell structure of $1s_{1/2}^2$ and $1p_{3/2}^4$ for protons, $1s_{1/2}^2$, $1p_{3/2}^4$, and $1p_{1/2}^1$ for neutrons. The excited nucleus will be de-excited by emitting gamma-ray, thus leading to the ground state with 1^+ or any other excited state. The shell structure of ^{14}N in the ground state is assumed to be $1s_{1/2}^2$, $1p_{3/2}^4$, and $1p_{1/2}^1$ for both protons and neutrons. Our interest is in the electromagnetic transition from the excited state of 9.17 MeV of 2^+ to the ground state because of the strong and very sharp resonance of the transition.

For the nucleus, a phenomenological local potential of the Woods-Saxon form is assumed and the corresponding parameters are determined from the experimental separation energies of a single proton and neutron. The radius parameter r_o for the nucleus is chosen to be 1.25 fm in all cases and the diffuseness parameter a is fixed as 0.53 fm. The depth parameters U_o for the HF mean field potentials are taken to be -51.756 MeV for proton and -51.331 MeV for neutron, respectively, and the depth for the spin-orbit potential is -5.5 MeV. The calculated single-particle energy level scheme for the ^{14}N calculated in the self-consistent manner[16, 17], on the basis of the shell model, is shown in Figure 4.

The potentials for excited particles are taken to be the usual optical potentials [18], which are a sum of a real potential with a Woods-Saxon shape, an imaginary absorption potential peaked near the nuclear surface, a spin-orbit term of the Thomas type, and a Coulomb potential for protons :

$$-U_p(r) = \frac{V_o}{1+e^x} \; + \; \frac{4iW_o e^x}{(1+e^x)^2} \; + \; V_{s.o.}\left[\frac{\hbar}{m_\pi c}\right]^2 \frac{1}{ar}\frac{e^x}{(1+e^x)^2}\,\boldsymbol{\ell}\cdot\boldsymbol{\sigma} \; - \; C(r) \qquad (12)$$

where $C(r)$ is the Coulomb potential and $x{=}(r-R)/a$ and $R{=}r_o(A-1)^{\frac{1}{3}}$ for a target nucleus of mass number A. r_o and a are the radius and the diffuseness parameters, respectively. The scattering waves are constructed by fitting the peak position of the 2^+ resonance at E_γ=9.172 MeV.

The real and the imaginary potential depth for the particle are adjusted to reproduce the experimental peak position and the width of the resonance in the cross section, respectively.

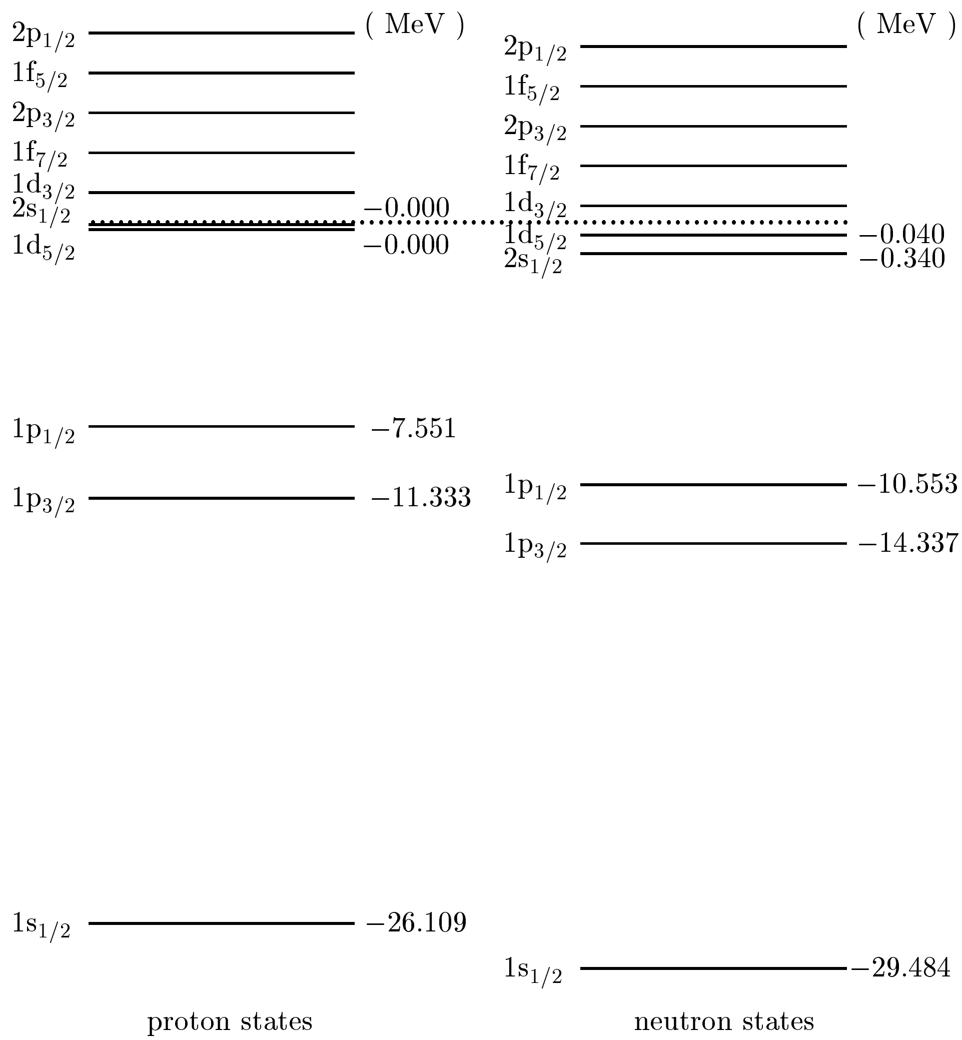

Figure 4: Single-particle energy levels in ^{14}N for protons and neutrons.

The diffuseness parameter is sensitive to the position and the strength of the resonance, simultaneously. The smaller the diffuseness parameter, the larger the strength of the cross section and the lower the peak position value in energy. The smaller the radius parameter, the higher the peak position value and the broader the width of the resonance. In the calculation, the depth of the real potential, V_o, is taken to be 72.182 MeV. The depth of the absorption potential, W_o, is chosen to be 10 eV to represent the large magnitude in the resonance region. The radius and the diffuseness parameters for the real potential are taken to be 1.17 fm and 0.15 fm, respectively. The depth and the diffuseness parameters for spin-orbit potential are chosen to be 5.5 MeV and 0.5 fm, respectively.

Figure 5 shows the calculated cross section for the radiative proton capture reaction by ^{13}C, $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$, with respect to the energy of the incident proton. The calculated result peaks at the energy of 1.7476 MeV and is in good agreement with the experimental one. Furthermore, the theoretical energy-integrated cross section over the resonance, 17.474 μb·MeV, is almost same as the experimental one of 17.495 μb·MeV. However, the theoretical width obtained from the radiative direct capture model is broader than the experimental one. The incident proton beam consists of particles having all allowed values of angular momentum (l=0,1,2,3,...) relative to the target nuclei. In order to describe the narrow resonance peak in the capture reaction cross section, the particles in the discrete bound states must be captured. According to the calculation, the main contribution in the strong resonance originated from the capture of the particle having d-wave. There is also contribution from the particle having s-wave. However, the contribution is small and negligible.

Figure 6 shows the calculated cross section for the photoproton emission by the absorption of photon by ^{14}N, $^{14}\mathrm{N}(\gamma,p)^{13}\mathrm{C}$. The photoproton reaction cross section is obtained from the inverse reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ using the reciprocity theorem. The corresponding excitation function peaks at the energy of 9.172 MeV, as shown in Figure 6, and is in good agreement with the experimental data.

III Conclusions

The experimental cross section for the reaction $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$ is reproduced from resonance parameters using the single level Breit-Wigner formula because of the absence of the corresponding excitation function. The cross section for the photoabsorption reaction $^{14}\mathrm{N}(\gamma,p)^{13}\mathrm{C}$, which is the inverse reaction of the $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$, is calculated using the reciprocity theorem.

The proton capture reaction by ^{13}C, $^{13}\mathrm{C}(p,\gamma)^{14}\mathrm{N}$, is theoretically analyzed on the basis of the direct reaction theory for the radiative capture reaction. Similarly, the cross section for the inverse reaction $^{14}\mathrm{N}(\gamma,p)^{13}\mathrm{C}$ is calculated with the reciprocity theorem. The narrow and strong resonance peaks in the experimental cross sections are well described with the radiative direct capture model. In the reaction, the capture of particles having d-wave, which are in barely bound states, by the ^{13}C core nucleus contribute to the production of the strong and sharp resonance peaks. It seems to be natural because the narrow resonance peak can be represented

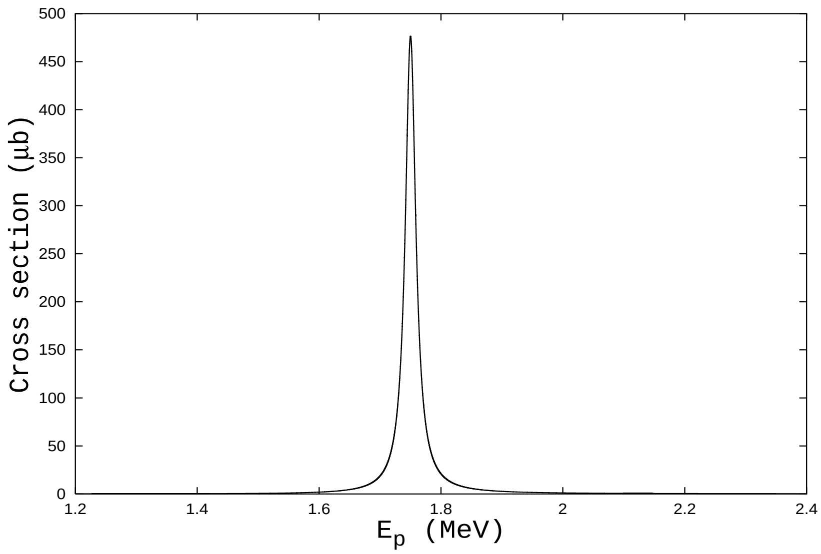

Figure 5: Theoretical cross section for the reaction ^{13}C$(p,\gamma)^{14}$N using the radiative direct capture reaction theory.

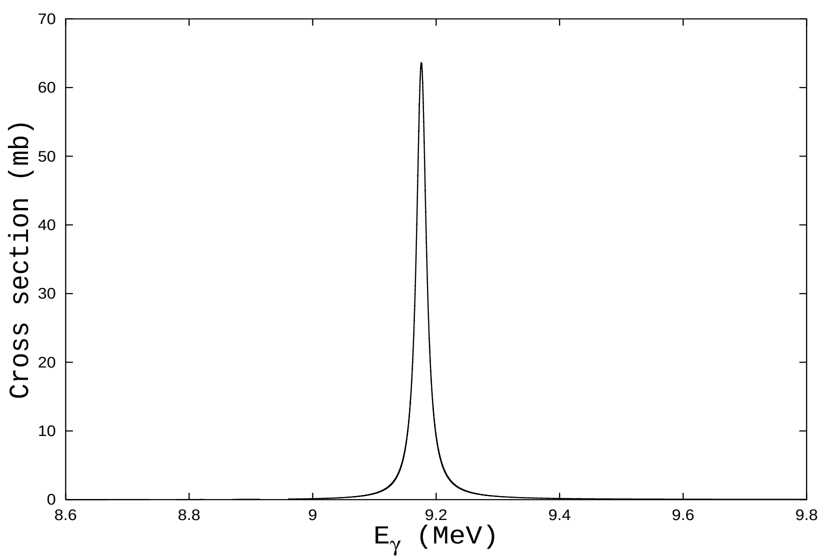

Figure 6: Theoretical cross section for the reaction ^{14}N$(\gamma,p)^{13}$C based on the radiative direct capture reaction theory.

with the discrete bound state characteristics. There is also contribution from the capture of particles with s-wave, even though the contribution is so small and negligible. However, the widths of the resonance in the theoretical cross sections are wider than the experimental ones, though the integrals of the cross sections over the resonance regions are in good agreement with the experimental ones. Including some effects which must be considered in the analysis may improve the situation. However, in conclusion, the radiative direct capture reaction theory gives results in good overall agreement with the somewhat uncertain experimental data.

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