Proceedings of the Korean Nuclear Society Autumn Meeting
Seoul, Korea, October, 1999

A Spring Back Calculation Model for the Sensitivity Analysis of Tube Design Parameters of Helical Steam Generator

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Abstract

The spring back phenomena occurring in the coiling process of a steam generator tube induces the dimensional inaccuracy and makes the coiling procedure difficult. In this research, an analytical model was developed to evaluate the amount of the spring back for SMART steam generator tubes. The model was developed on the basis of beam theory and elastic-perfectly plastic material property. This model was extended to consider the effect of plastic hardening and the effect of the tensile force on the spring back phenomena. Parametric studies were performed for various design variables of steam generator tubes in order to minimize the spring back in the design stage. A sensitivity analysis has shown that the low yield strength, the high elastic modulus, the small helix diameter, and the large tube diameter result in a small amount of the spring back. The amount of the spring back can be controlled by the selection of adequate design values in the basic design stage and reduced to an allowable limit by the application of the tensile force to the tube during the coiling process.

1. Introduction

The SMART steam generator tube bundle is manufactured by the group coiling of titanium tubes on the central shell[1]. Tubes of subsequent coils are arranged over the previous coiled tubes. After the coiling of the tubes, the tube bundle is subjected to heat treatment to eliminate residual stress[2]. The process of coiling is a very important technological operation, which determines the quality and the lifetime of the steam generator tube.

The coiling of the titanium tube has some difficult problems. Titanium has limited ductility and elongation at room temperatures and the titanium tube has a tendency to neck down in some areas when bent. The nonuniform mechanical and metallurgical properties in the welded region make it difficult to predict coiling performance. It has been reported that tubes smaller than 3 inches in diameter can be satisfactorily coiled without heating[3]. When the tube wall thickness is relatively thin, preheating the tubes is not necessary.

The straight tubes are coiled by winding around a rigid mandrel. In all cases the residual stresses remaining after the elastic-plastic deformation has taken place during the production process tend to further deform the coiled tube. For physical reasons, the coiled tube geometry must be conserved within very close limits. Therefore, any spring back behavior of the coiled tube must be controlled.

In this study, a simple analytic equation was derived on the basis of beam theory to evaluate the amount of the spring back during the coiling of the heat transfer tube of once-through helical steam generator. In order to minimize the absolute amount of the spring back in the design stage, a parametric study has been carried out for various design variables.

2. Modelling of Coiling

2.1 Spring back model

The spring back analysis model was derived based on the following assumptions. The dimension of the tube cross-section is negligible compared to the length of the tube. The shear deformation of the tube is negligible compared to the bending deformation. The constitutive behavior of the material is considered linear elastic-perfectly plastic or linear hardening. Also, it has been assumed that yielding is determined by the normal stress component in the axial direction of the tube. The helix angle of the coiled tube is assumed to be zero.

As shown in Fig. 1, consider a circular ring of annular cross-section. If the bending moment M is released, a circle of radius R_o becomes an arc of radius R_f

Strain at $r=r_o$ when the tube is subjected to pure bending moment M is expressed as follows:

$$\varepsilon|_{y=r_o} = \frac{r_o}{R_o} \tag{1}$$

When the moment M is removed, the tube shows elastic behavior. The amount of stress relieved can be expressed as follows:

$$\sigma_\theta^{unloading}(y) = -\frac{My}{\frac{\pi}{4}(r_o^4 - r_i^4)} \tag{2}$$

The amount of relieved strain at $r=r_o$ during the unloading of the bending moment.

$$\varepsilon|_{y=r_o} = \frac{4 r_o M}{E\pi (r_o^4 - r_i^4)} \tag{3}$$

Residual strain at $r=r_o$ after unloading

$$\varepsilon|_{y=r_o} = \frac{r_o}{R_f} \tag{4}$$

Strain can be easily obtained from the stress in a one dimensional model. The residual strain equals to the strain of the tube subjected to the bending moment M minus the relieved strain during the unloading of the bending moment. From the equation (1), (3) and (4), a relation between the radius of curvature before and after the unloading of the bending moment can be expressed as follows:

$$R_f = \frac{R_o}{1 - \dfrac{4R_o M}{E\pi\,(r_o^4 - r_i^4)}} \qquad (5)$$

2.2 Elastic-perfectly plastic and pure bending

In this model, the material property of the tube is assumed to be elastic-perfectly plastic. The axial stress distribution is shown in Fig.2 when the tube is subjected to pure bending moment. Where r_i and r_o are the inner and outer radius of the tube, respectively. σ_Y is the yield strength. Maximum stress occurs at $y=r_o$ and α means the elastic range. When α equals one or larger than one, there is no residual plastic deformation after unloading of the bending moment. In order to coil the tube, α should be less than one. The stress distribution can be expressed as follows:

$$\sigma_\theta(y) = \sigma_Y \frac{y}{\alpha r_o} \qquad y \le \alpha r_o \qquad (6\text{-a})$$

$$\sigma_\theta(y = r_o) = \sigma_Y \qquad y \ge \alpha r_o \qquad (6\text{-b})$$

The bending moment necessary to coil the tube can be expressed as follows:

$$M = M_o - M_i \qquad (7)$$

Where M_i and M_o are the bending moments necessary to bend a circular beam of radius r_o and r_i respectively. The bending moment is obtained as below.

$$M_o = \sigma_Y r_o^3 \left\{ \frac{4}{3}\cos^3\beta - \frac{1}{8\alpha}\sin 4\beta + \frac{1}{2\alpha}\beta \right\} \qquad (8)$$

$$M_i = \begin{cases} \dfrac{\pi \sigma_Y r_i^4}{4\alpha r_o} & if \quad \alpha r_o \ge r_i \\ \sigma_Y r_i^3 \left\{ \dfrac{4}{3}\cos^3\gamma - \dfrac{1}{8\alpha}\left(\dfrac{r_i}{r_o}\right)\sin 4\gamma + \dfrac{1}{2\alpha}\left(\dfrac{r_i}{r_o}\right)\gamma \right\} & if \quad \alpha r_o \le r_i \end{cases} \qquad (9)$$

where

$$\beta = \sin^{-1}\alpha$$

$$\gamma = \sin^{-1}\left(\alpha \frac{r_o}{r_i}\right)$$

Depending on the boundary of the plastic region as shown in Fig.3, M_i should be divided into two cases. If the thickness of the tube is much smaller than the radius of the tube, it is not necessary to consider two cases independently. The radius of the curvature after coiling of the tube can be obtained by substituting the moment of equation (7) into equation (5).

2.3 Linear hardening and pure bending

Most of the structural material shows the hardening behavior with the progress of the plastic deformation. The effect of plastic hardening on the amount of the spring back can be evaluated in this model by implementing the linear hardening behavior to the previous model.

The stress distribution can be written as follows:

$$s_q(y) = s_Y \frac{y}{ar_o} \qquad y \le ar_o \tag{10}$$

$$s_q(y) = s_Y + E_T\left(\frac{y}{R} - \frac{s_Y}{E}\right) \qquad y \ge ar_o \tag{11}$$

Similiar to the previous elastic-perfectly model, the moment necessary to bend the tube can be expressed as follows:

$$M_o = s_Y r_o^3 \left[\frac{4}{3}\left(1-\frac{E_T}{E}\right)\cos^3 b + \frac{1}{8a}(4b - \sin 4b) + \frac{r_o E_T}{8Rs_Y}(2p + \sin 4b - 4b)\right] \tag{12}$$

$$M_i = \begin{cases} \dfrac{p s_Y r_i^4}{4ar_o} & if \quad ar_o \ge r_i \\ s_Y r_i^3\left[\dfrac{4}{3}\left(1-\dfrac{E_T}{E}\right)\cos^3 g + \dfrac{1}{8a}\left(\dfrac{r_i}{r_o}\right)(4g - \sin 4g) + \dfrac{r_i E_T}{8Rs_Y}(2p + \sin 4g - 4g)\right] & if \quad ar_o \le r_i \end{cases} \tag{13}$$

where

$$b = \sin^{-1} a$$

$$g = \sin^{-1}\left(a\frac{r_o}{r_i}\right)$$

The radius of the curvature after the unloading of the bending moment can be obtained by the same procedure as the elastic-perfectly plastic model.

2.4 Bending Moment and Tensile Force

Tensile force has a strong influence on the amount of the spring back in the coiling process of the tube. The material property of the tube was assumed to be elastic-perfectly plastic in this model. The bending moment and the tensile force are applied to the tube. The application of the tensile force shifts the neutral axis. The stress distribution is schematically shown in Fig. 4 when the tube carries the bending moment M and the tensile force F. The stress distribution can be expressed as follows:

$$s_q(y) = -s_Y \qquad -r_o \le y \le -(a+x)r_o \tag{14-a}$$

$$s_q(y) = -s_Y + s_Y \frac{y + (a+x)r_o}{ar_o} \qquad -(a+x)r_o \le y \le (a-x)r_o \tag{14-b}$$

$$s_q(y) = s_Y \qquad (a-x)r_o \le y \le r_o \tag{14-c}$$

The force equilibrium along the tube axis can be expressed as follows:

$$F_o|_{M+F} - F_i|_{M+F} = F \tag{15}$$

where

$$F_o\,|_{M+F} = \frac{1}{2} s_Y r_o^2 \left[\frac{x}{a} \left\{ b_1 + b_2 + \frac{1}{2} (\sin 2b_1 + \sin 2b_2) \right\} - \left\{ b_2 - b_{12} + \frac{1}{2} (\sin 2b_2 - \sin 2b_1) \right\} \right.$$

$$\left. - \frac{2}{3a} \left(\cos^3 b_2 - \cos^3 b_1 \right) \right] \qquad (16)$$

$$F_i\,|_{M+F} = \frac{1}{2} s_Y r_i^2 \left[\frac{x}{a} \left\{ g_1 + g_2 + \frac{1}{2} (\sin 2g_1 + \sin 2g_2) \right\} - \left\{ g_2 - g_1 + \frac{1}{2} (\sin 2g_2 - \sin 2g_1) \right\} \right.$$

$$\left. - \frac{2}{3a} \left(\frac{r_i}{r_o} \right) \left(\cos^3 g_2 - \cos^3 g_1 \right) \right] \qquad (17)$$

$$b_1 = \sin^{-1}(a + x)$$

$$b_2 = \sin^{-1}(a - x)$$

$$g_1 = \sin^{-1}\left[\frac{r_o}{r_i} (a + x) \right]$$

$$g_2 = \sin^{-1}\left[\frac{r_o}{r_i} (a - x) \right]$$

Subscript $M+F$ denotes that the tube carries tensile force and bending moment. The shift of the neutral axis ξ can be obtained by inserting the equation (16) and the equation (17) into equation (15). The bending moment necessary to bend the tube can be calculated as follows:

$$M\,|_{M+F} = M_o\,|_{M+F} - M_i\,|_{M+F} \qquad (18)$$

where

$$M_o\,|_{M+F} = \frac{1}{2} s_Y r_o^3 \left[\frac{2}{3} \left(1 + \frac{x}{a} \right) \cos^3 b_1 + \frac{2}{3} \left(1 - \frac{x}{a} \right) \cos^3 b_2 - \frac{1}{16a} (\sin 4b_1 + \sin 4b_2) + \frac{1}{4a} (b_1 + b_2) \right] \qquad (19)$$

$$M_i\,|_{M+F} = \frac{1}{2} s_Y r_i^3 \left[\frac{2}{3} \left(1 + \frac{x}{a} \right) \cos^3 g_1 + \frac{2}{3} \left(1 - \frac{x}{a} \right) \cos^3 g_2 - \frac{1}{16a} \left(\frac{r_i}{r_o} \right) (\sin 4g_1 + \sin 4g_2) + \frac{1}{4a} \left(\frac{r_i}{r_o} \right) (g_1 + g_2) \right]$$

$$(20)$$

For the simplicity of the model, it has been assumed that the tensile force is released first and then the bending moment is released. The stress distribution after the release of tensile force can be summarized as follows:

$$s_q(y) = -s_Y \qquad -r_o \le y \le -(a + x - d) r_o \qquad (21\text{-a})$$

$$s_q(y) = -s_Y + (1 + h) s_Y \frac{y + (a + x - d) r_o}{(2a - d) r_o} \qquad -(a + x - d) r_o \le y \le (a - x) r_o \qquad (21\text{-b})$$

$$s_q(y) = h s_Y \qquad (a - x) r_o \le y \le r_o \qquad (21\text{-c})$$

where

$$d = 2 - a(1 + h)$$

Force equilibrium in the direction of tube axis should become zero since the tensile loading was released.

$$F_o|_M - F_i|_M = 0 \tag{22}$$

where

$$F_o|_M = \frac{1}{2} s_Y r_o^2 \left[\left\{ \frac{p}{2}(h-1) + b_1' - h b_2 + \sin 2b_1' - h \sin 2b_2 \right\} - \frac{2(1+h)}{3(2a-d)} \left(\cos^3 b_2 - \cos^3 b_1' \right) \right.$$
$$\left. + \left\{ \frac{(a+x-d)(1+h)}{2a-d} - 1 \right\} \left\{ b_1' + b_2 + \frac{1}{2} \left(\sin 2b_1' + \sin 2b_2 \right) \right\} \right] \tag{23}$$

$$F_i|_M = \frac{1}{2} s_Y r_i^2 \left[\left\{ \frac{p}{2}(h-1) + g_1' - h g_2 + \sin 2g_1' - h \sin 2g_2 \right\} - \frac{2(1+h)}{3(2a-d)} \frac{r_i}{r_o} \left(\cos^3 g_2 - \cos^3 g_1' \right) \right.$$
$$\left. + \left\{ \frac{(a+x-d)(1+h)}{2a-d} - 1 \right\} \left\{ g_1' + g_2 + \frac{1}{2} \left(\sin 2g_1' + \sin 2g_2 \right) \right\} \right] \tag{24}$$

$$b_1' = \sin^{-1}(a+x-d)$$

$$b_2 = \sin^{-1}(a-x)$$

$$g_1' = \sin^{-1}\left[\frac{r_o}{r_i}(a+x-d) \right]$$

$$g_2 = \sin^{-1}\left[\frac{r_o}{r_i}(a-x) \right]$$

The residual moment after the unloading of the tensile force can be expressed as follows:

$$M|_M = M_o|_M - M_i|_M \tag{25}$$

where

$$M_o|_M = s_Y r_o^3 \left[\frac{1}{3} \left(\cos^3 b + \cos^3 b_1' \right) - \frac{1}{3} \left\{ \frac{(a+x-d)(1+h)}{2a-d} - 1 \right\} \left(\cos^3 b_2 - \cos^3 b_1' \right) \right.$$
$$\left. - \frac{(1+h)}{32(2a-d)} \left\{ \sin 4b_2 + \sin 4b_1' \right\} - 4(b_2 - b_1') \right] \tag{26}$$

$$M_i|_M = s_Y r_i^3 \left[\frac{1}{3} \left(\cos^3 g_2 + \cos^3 g_1' \right) - \frac{1}{3} \left\{ \frac{(a+x-d)(1+h)}{2a-d} - 1 \right\} \left(\cos^3 g_2 - \cos^3 g_1' \right) \right.$$
$$\left. - \frac{(1+h)}{32(2a-d)} \frac{r_i}{r_o} \left\{ \sin 4g_2 + \sin 4g_1' \right\} - 4(g_2 - g_1') \right] \tag{27}$$

The radius of curvature after unloading of the moment can be obtained by inserting the equation (25) into the equation (5).

These equations can be expressed more simply by adding the thin tube assumption. Force equilibrium along the tube axis can be expressed as follows:

$$F = s_Y r t \{ p - 2\sin^{-1}(1-2x) \} \tag{28}$$

When the bending moment and the tensile force are applied to the tube, the bending moment necessary to bend the tube can be expressed as follows:

$$M\,|_{M+F}=2s_Y r^2 t\left\{(\cos b_1+\cos b_2)+\frac{x}{a}(\cos b_1-\cos b_2)+\frac{1}{2a}(b_1+b_2)-\frac{1}{4a}(\sin 2b_1+\sin 2b_2)\right\} \quad (29)$$

Force equilibrium in the direction of tube axis should become zero after the release of the tensile force.

$$\frac{p}{2}(h-1)+(\frac{x}{a}+1)b_1'+(\frac{x}{a}-h)b_2+\frac{1}{a}(\cos b_1'-\cos b_2)=0 \quad (30)$$

The residual moment after unloading of tensile force can be expressed as follows:

$$M\,|_{M}=2s_Y r^2 t\left\{(\cos b_1'+h\cos b_2)+\frac{x}{a}(\cos b_1'-\cos b_2)+\frac{1}{2a}(b_1'+b_2)-\frac{1}{4a}(\sin 2b_1'+\sin 2b_2)\right\} \quad (31)$$

3. Sensitivity Analysis For Tube Design Parameters

By using the derived model and the equations, parametric study has been carried out for various design variables of the steam generator heat transfer tubes.

3.1 Effect of Material Properties on the Spring Back

The SMART steam generator was designed using titanium alloy tubes. The yield strength of the titanium tube is 350MPa and the elastic modulus is 117GPa. The radius of mandrel was assumed to be 100mm and the tube inner diameter and the outer diameter are 9mm and 12mm respectively. The result of the parametric study for yield strength and the elastic modulus are shown in Fig. 6. The absolute value of the spring back has been increased as the yield strength increases. That is, the amount of spring back is large for high yield strength material although high yield strength is recommended in the view point of strength. The amount of the spring back was smaller for the higher value of elastic modulus. However, the elastic modulus is not a controllable design variable after the selection of the material.

The effect of plastic hardening on the amount of spring back was investigated utilizing the elastic-linear plastic material properties. As shown in Fig. 7, the amount of spring back increased as the plastic tangent stiffness increases. If the plastic tangent stiffness E_T becomes zero, the model becomes elastic-perfectly plastic model. If the plastic tangent stiffness approaches to E, it becomes linear elastic.

3.2 Effect of Geometric Parameters on the Spring Back

The radius of the curvature at the final configuration was calculated for various radius of curvature of the mandrel. The radius of curvature in the SMART steam generator varies from 100mm in the inner most layer to 356mm in the outer most layer. When other design variables are fixed, the amount of spring back was larger for a larger value of mandrel radius as shown in Fig.8. The effect of the tube diameter on the amount of the spring back has been investigated. As the tube diameter increases, the amount of the spring back has been decreased. If the mandrel radius is fixed, a large plastic deformation is expected for the large tube diameter.

The amount of spring back was calculated for the various thickness of tubes with fixed outer diameter. The tube thickness has little influence on the amount of the spring back compared to other geometric design variables. Reduction of tube thickness can diminish only a small amount of spring back. The tube thickness is not a controllable variable in the view point of the spring back since it is determined by the strength criteria.

3.3 Effect of the Tensile Force on the Spring Back

The application of the tensile force in the process of coiling is an effective method to reduce the amount of the spring back. The effect of the tensile force on the amount of the spring back was studied in Fig. 9 where the shift of neutral axis ξ denotes the amplitude of tensile force applied to the tube indirectly. As the tensile force increases, the spring back has decreased. The amplitude of the tensile force to be applied in the coiling process should be determined considering all the other design variables.

4. Conclusion

An analytical model was developed to evaluate the amount of spring back for SMART steam generator tubes in this study. The model was developed on the basis of beam theory and elastic-perfectly plastic material property. Then, this model was extended to consider the effect of plastic hardening and the effect of tensile force on the amount of the spring back. Parametric studies were performed for the spring back of helical tubes in order to minimize the spring back during the coiling process. A sensitivity analysis has shown that low yield strength, high elastic modulus, small helix diameter, and large tube diameter induce small amount of spring back. The amount of spring back can be controlled by the selection of an adequate design value in the basic design stage. It can be reduced to the allowable limit by the application of tensile force to the tube in the coiling stage.

Acknowledgement : This work has been performed under the nuclear research & development program sponsored by MOST.

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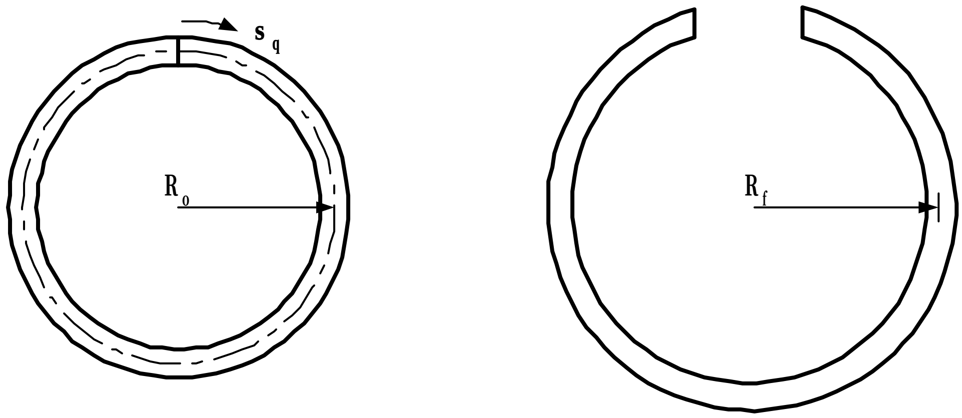

Fig. 1 Curvature change after the release of bending moment

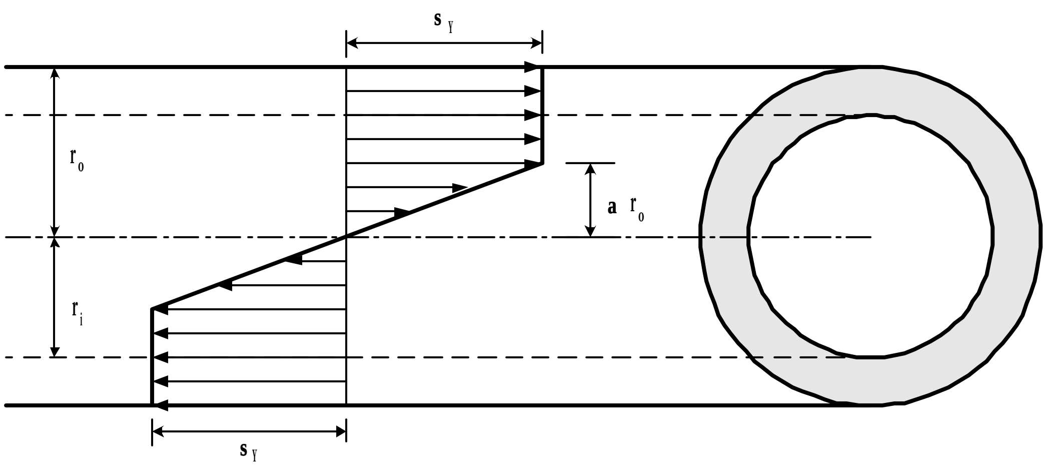

Fig. 2 Stress distribution of the tube subjected to pure bending

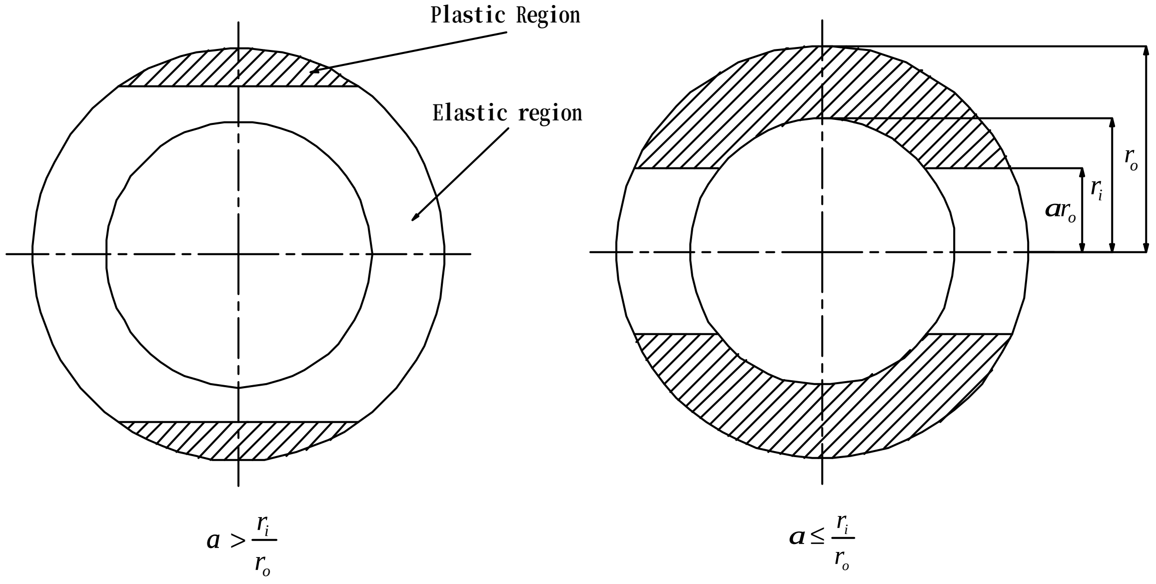

Fig.3 Boundary of plastic deformation

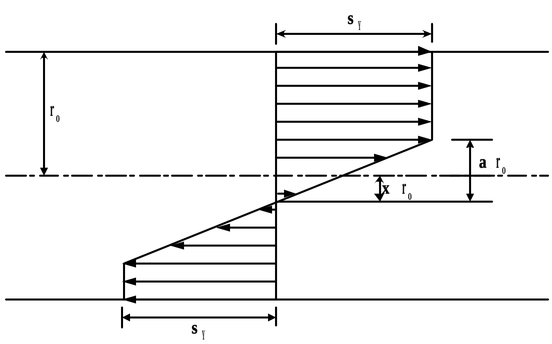

Fig. 4 Stress distribution of the tube subjected to bending and tensile force

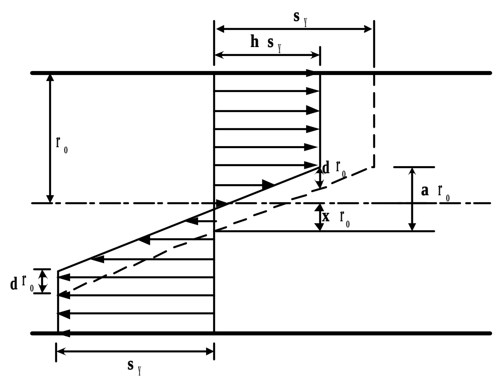

Fig.5 Stress distribution of the tube after release of tensile force.

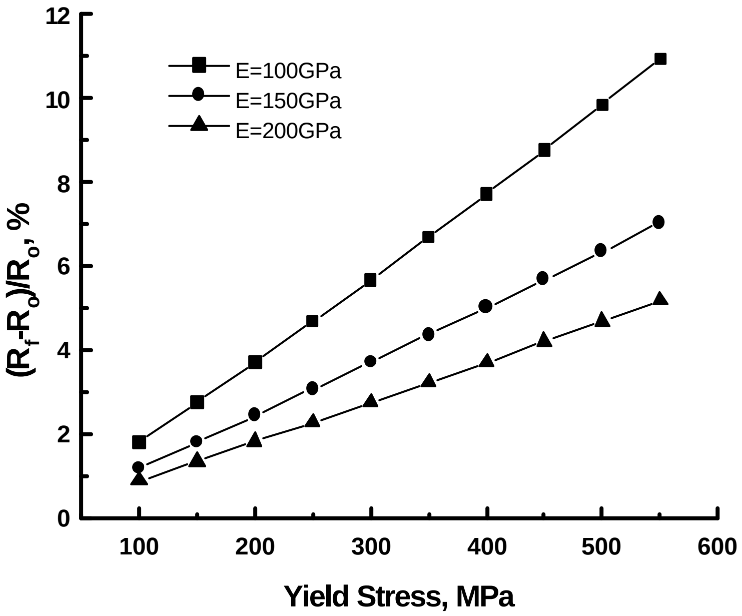

Fig. 6 Effect of yield strength on the amount of spring back

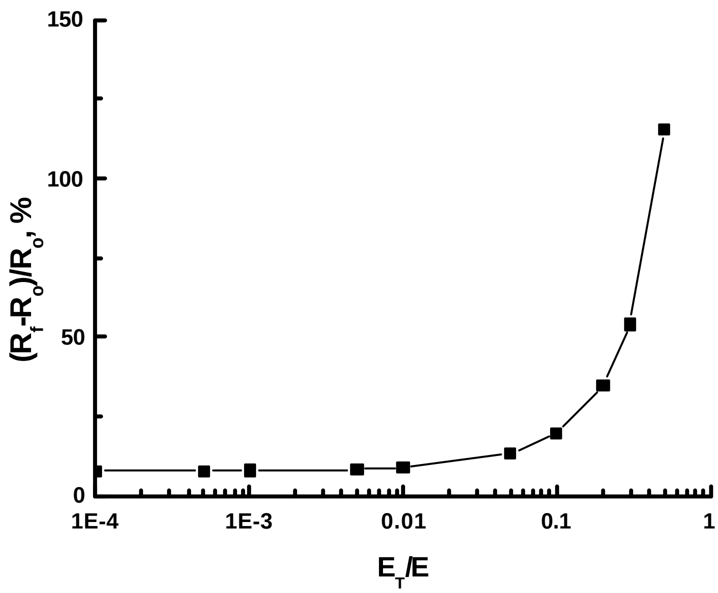

Fig. 7 Effect of tangent stiffness on the amount of spring back

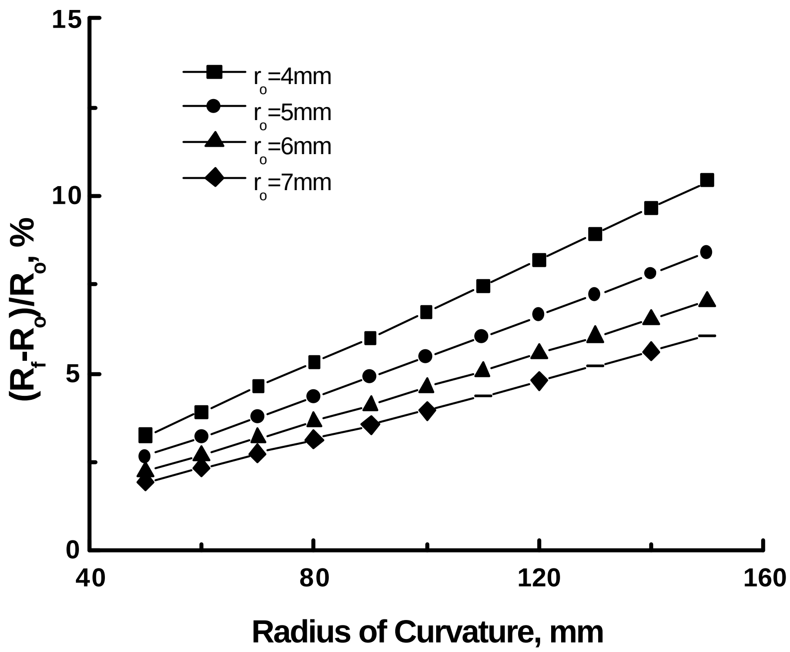

Fig. 8 Effect of curvature on the amount of spring back

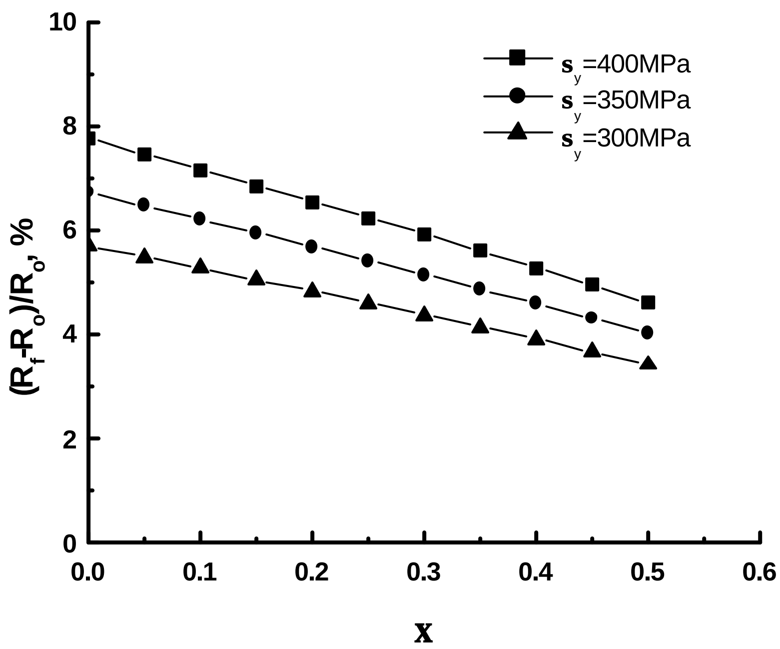

Fig. 9 Effect of tensile force on the amount of spring back