

Monte Carlo Neutron Transport using Advanced Exponential Transform

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Abstract

The difficulty of neutron transport in radiation shielding is to find the fair degree of performance. The radiation shielding is the major role in the calculation of health physics, medical physics, and nuclear facilities. Several Variance Reduction Techniques are compared. The Survival Biasing (SB) and Russian Roulette (RR) can improve the time saving and simple calculations. The newly introduced exponential transform function, Local Importance Function Transform (LIFT), increases the Figure of Merit factor (FOM). This means time saving is developed. These methods are tested for the spherical fuel in the High Temperature Gas-cooled Reactor (HTGR).

1. Introduction

Oxide Uranium fuel has been studied for the nuclear power plants many years. The alternative geometry of HTGR fuel is suggesting for the new type reactor economic factor and safety consideration. This paper is focusing on the spherical geometry transport calculation using some Monte Carlo methods. This can be applied for the calculation of health physics, medical physics, and nuclear facility calculations.

2. Spherical Fuel

The HTGR fuel particles have been studied for the radiation stability and high thermal effect. The fuel is an enriched uranium carbide center and a buffer layer of pyrolytic graphite. The buffer is used to stop fission fragments, to give a space of fission gas, and to limit swelling. The silicon carbide barrier coating exists for the particle strength and for the fission products migration stopping. There are 2 kinds of barriers in the fuels. The inner one is for fission product migration. The outer one is to protect SiC coating and to hold the overall micro sphere through radiation enhanced volume changes. The early developed HTGR was based on 93 wt% enriched

Uranium. Otherwise, the recent design is 19.9 wt% enriched fuel. The Pebble Bed reactor was developed by Hochtemperatur-Reaktorbau GmbH [HRB] in the Germany. The commercial scale is 1120 MWe power reactor. The coated fuel is similar to HTGR fuel, which is contained in 6 cm diameter graphite sphere fuel.

3. Survival Biasing (SB) and Russian Roulette

Survival Biasing (SB) and Russian Roulette (RR) can make the difficulties to find a fair degree of experience, or to employ a significant amount of trial and error in order to find an accurate and efficient output. These kinds of jobs are very difficult and costly tasks.

4. Local Importance Function Transform (LIFT)

The simple exponential transform is used for the simple variance-reduction method. The exponentially varying in space is,

$$w(r, \Omega) = \text{EXP}(-\alpha r)$$

where the α is the user defined biasing amount and direction parameter. \mathbf{r} is localized as a vector toward the region of interest. The adjustments to the weight of the particle are necessary to keep a fair game. The discrete case mode or asymptotic solution to adjoint problem with isotropic scattering,

$$w(r, \Omega) = s_0 \text{EXP}(-\alpha r) / \{4\pi \alpha (1 - \alpha)\}$$

where α is defined by,

$$1 = \int s_0 / \{4\pi \alpha (1 - \alpha)\} d\Omega$$

A new variance reduction technique, Local Importance Function Transform (LIFT) dates back to the beginning years of Monte Carlo. It has been realized that to transport particles through several mean free paths of material in a reasonable amount of time, one must change the rules of the simulation. This rule change is based on a priori information about the solution like the knowing that a neutron population decays exponentially in space for a deep-penetration, fixed-source problem. Kahn first described the exponential transform in 1950. This transform adjusts the distance to collision so that particle traveling toward the region of interest suffer fewer collisions than particles traveling away from the region of interest. Later, in 1958, Goertzel and Kalos described the potential of the zero-variance problem in connection with the exponential transform. During the several decades, variance reduction methods began to include angular biasing, weight windows, and the use of analytic or deterministic approximations to the importance function to guide these biasing techniques. The idea of

combining the biasing of the transport process with biasing of the collision process by using a factorized analytical expression for the importance function with a common biasing parameter originated with Dwivedi, Gupta, and Gupta and Dwivedi in the early 1980s. Several papers involving one or more of the concepts of zero variance, angular biasing, the exponential transform, weight windows, and analytic or numerical approximations of the importance function have emerged since then. The LIFT method is similar to the conventional exponential transform, but the addition of energy and angular biasing. This is similar to the method of Dwivedi and Gupta.

The philosophy behind the LIFT method resembles that articulated by Lux and Koblinger, who presented a generalized derivation of zero-variance schemes for transport problems, including multiplying systems. They also presented several ideas for approximating a zero-variance simulation, including methods based on the exponential transform and on the exponential transform with angular biasing suggested by Dwivedi and Gupta.

Lux and Koblinger extended the latter theory suggesting a biasing parameter, which is local in space and energy like left method. They suggested a self-learning Monte Carlo scheme for approximating the adjoint solution rather than using a deterministic solution.

$$\phi(r, \Omega) = \phi_0 \int_{s_0}^s \beta(r-r') \exp[-\rho(r-r')] \exp[-\rho\Omega] d\Omega$$

$$\beta = \left\{ \int \exp(-\rho(r-r')) dr \right\} \times \left\{ \int_{s_0}^s \exp(-\rho\Omega) d\Omega \right\}^{-1}$$

linearly anisotropic factor

$$b(\Omega) = 1 + 3\mu \left\{ \frac{(\tau - s_0)}{|\rho|^2} \right\} \rho \Omega$$

the biasing parameter

$$\rho = \tau \lambda$$

the spatial component of the solution to be an exponential and the unspecified angular component leaves

$$\phi(r, \Omega) = f(\Omega) \exp\{\rho(r - r_0)\}$$

$$\int f(\Omega) d\Omega = 1$$

the probabilistic distribution function for the distance to collision is,

$$P(s) = \tau \exp(-\tau s)$$

where

$$\epsilon = \epsilon - \Omega$$

5. Figure of Merit (FOM) factor

The variance is the method of performance measurement in the conventional statistics. The other method for the amount of CPU time and Error is the relative Figure of Merit (FOM) factor. This is also called Efficiency. The FOM is,

$$\text{FOM} = 1/\epsilon^2 T$$

where

$$\epsilon = \sqrt{\text{variance}}$$

the T does not include the time required for obtaining or processing the deterministic adjoint calculations. Inclusion of these times would make the FOM a function of the number of Monte Carlo histories. Because the relative error is proportional to $1/\sqrt{N}$ and CPU time is proportional to N. A constant value for FOM increases the reliability of the answer. Also, a larger value of FOM indicates a more efficient calculations. In other words a larger value of FOM means that the calculations require less computer time for the same value of precision.

6. Quality Factor(QF)

The quality factor is defined,

$$\text{QF} = \text{FOM}/(\text{FOM})_{\text{Ana}}$$

$(\text{FOM})_{\text{Ana}}$ is Analog Survival Biasing Monte Carlo calculation. QF is called Benefit Factor, too.

Also, we can find,

$$\text{QF} = T_{\text{Ana}}/T$$

Therefore, the larger QF is the less time consuming calculation. If Benefit Factor = 2 for a Monte Carlo method, then the method being studied required half the computer time required to solve a given problem with a given precision, as compared to Monte Carlo with Analog Survival Biasing.

7. Experiment

The SUN workstation is used for the calculation. The Fig.1 and Fig.2 show the HTGR and pebble-bed reactor fuel each. The HTGR fuel is simplified as Fig.3. The material's characteristics is modified. The scatter cross section is 0.2 for simplification. Until the radius 3 Cm of total radius 12 Cm, there is a source region. The source generates 100,000 neutrons. Every generated neutron has 1.0 MeV.

8. Result and Discussion

In the Table1 and Table2, Russian Roulette, Survival Biasing, and LIFT method's results are shown. Fig.4 shows the Monte Carlo Flow Chart using Local Importance Function Transform (LIFT). Fig.5 and Fig.6 show the FOM factor and QF in the Russian Roulette and Survival Biasing simulations. The survival biasing is a non-analog processing calculation. The comparisons of the quality factors are on Fig.7. The LIFT has higher value in the comparisons. This means that the calculation using LIFT is the faster than Russian Roulette and Survival Biasing. This kind of variance reduction is helpful for the large radiation particles calculation. The time saving is accomplished. The enhanced neutron transport method is constructed. This is applicable for the radiation shielding and Health/Medical Physics calculations.

9. References

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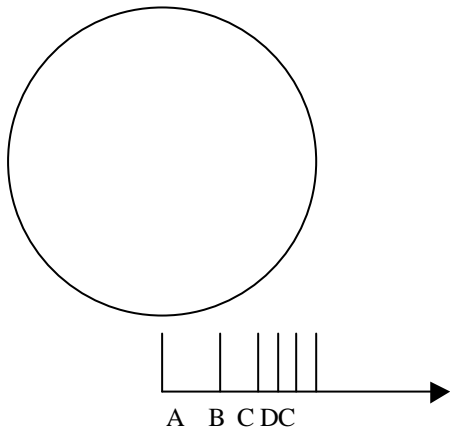


Fig.1 Fissile spherical HTGR fuel
(A : UC_2 , B : SiC, C : PyC, D : Buffer)

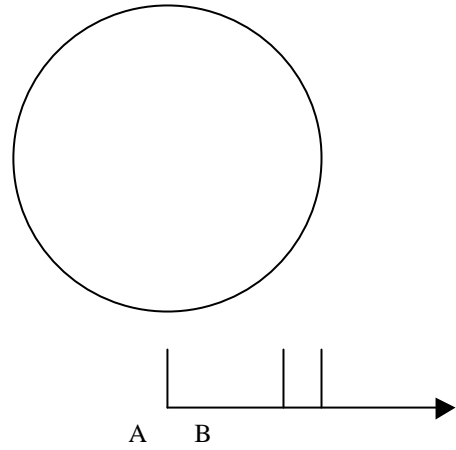


Fig.2 The pebble-bed reactor fuel
(A : Graphite Matrix Coated Particles,
B : Graphite Shell)

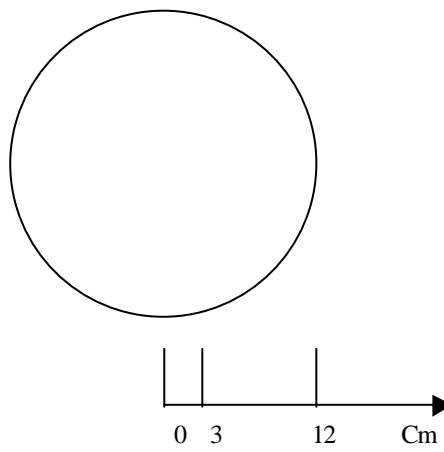
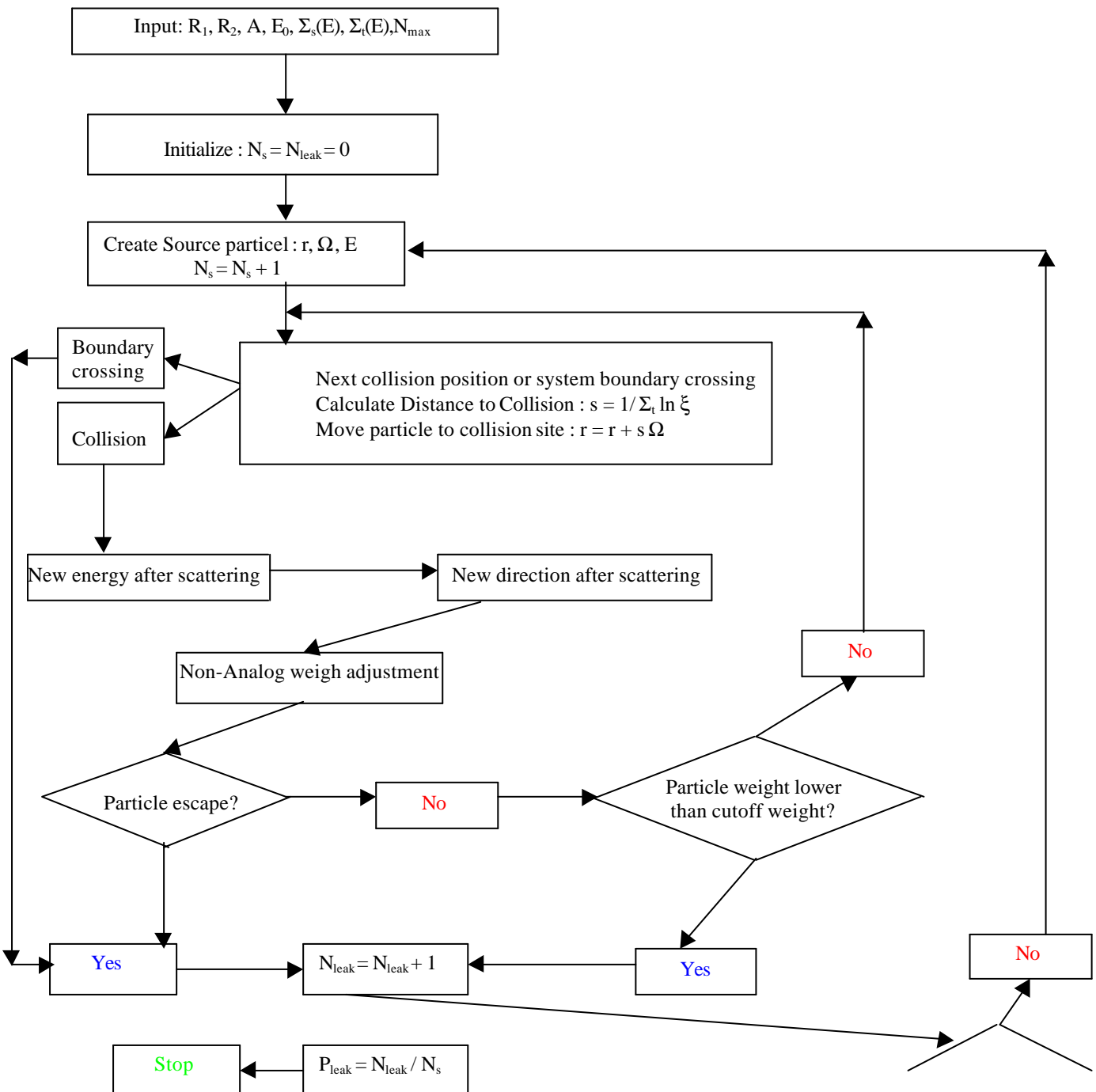


Fig.3 Simplified spherical HTGR fuel

Monte Carlo Flow Chart using Local Importance Function Transform (LIFT) (Spherical Fuel)



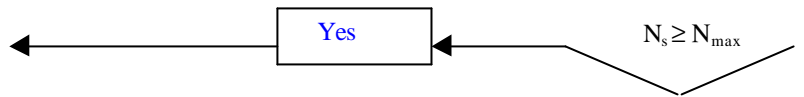


Fig.4 Monte Carlo Flow Chart using Local Importance Function Transform (LIFT)

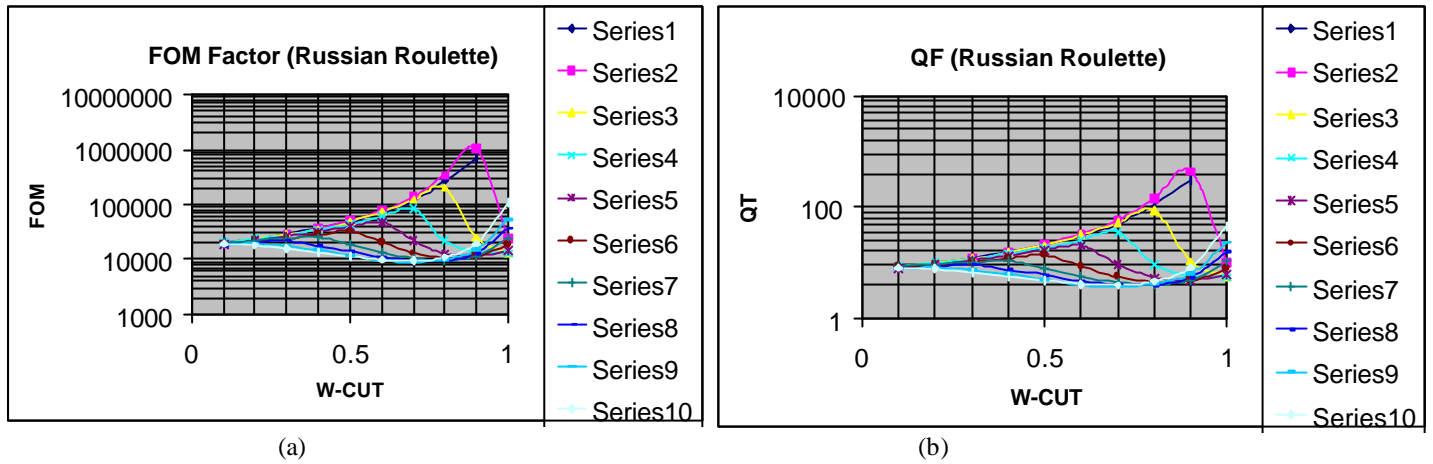


Fig.5 (a) FOM Factor, (b) QF (Series 1 : WA=1.0, Series 2 : WA = 0.9, Series 3 : WA = 0.8, Series 4 = 0.7, Series 5 = 0.6 , Series 6 = 0.5, Series 7 = 0.4, Series 8 = 0.3, Series 9 = 0.2, Series 10 = 0.1)

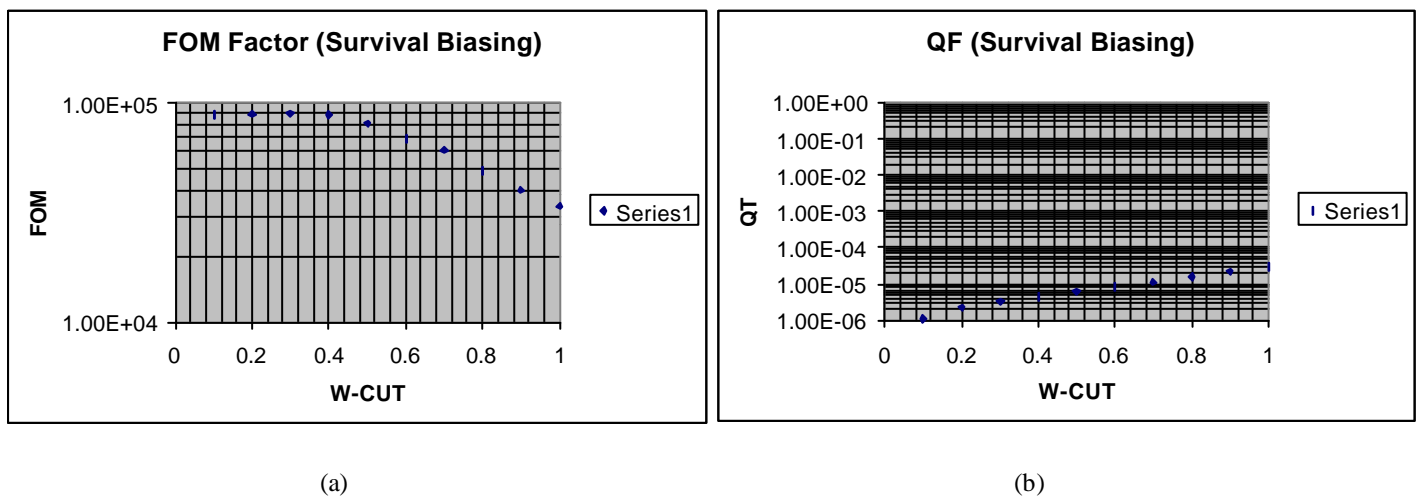


Fig.6 (a) FOM Factor, (b) QF

WA	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
RR	46.2	22.9	9.3	10.8	13.8	20.4	35.6	86.5	442.0	297.0

Table1 The Maximum QF (Russian Roulette)

SB	8.8E-06 (Wcut=0.6)
LIFT	430.5

Table2 The Maximum QF (Survival Biasing and LIFT)

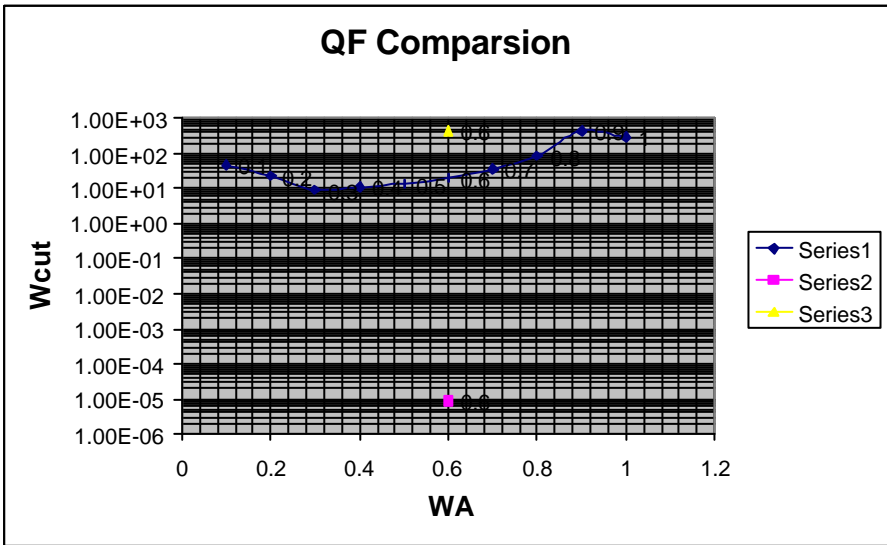


Fig.7 The comparisons of the QF (Series 1 : Russian Roulette, Series2 : Survival Biasing (Wcut = 0.6), Series3 : LIFT)