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## Natural Ray Computer Tomography with Radon

### Transformation for Two-phase flow

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### Abstract

This paper describes the computer tomography using digital image technique under natural light to analyse two-phase flow. Recent progress in the digital image processing support to develop the present tomography in a low cost and with high resolution. Since complex two-phase structure needs three dimensional understanding of the interfacial deformation and spatial distribution, the tomography with high resolution in low cost is developed using the CC camera.

For back projection for the cross sectional image, Radon transformation is coded with appropriate filter algorithm. The test results for the various flow regime, the present algorithm successfully reconstructs cross sectional image as detail as representing ripples at the two-phase interface.

It was found that the present work is applicable to the air-water or air-steam two-phase flow and that it could be directly extend for the neutron tomography.

Key words: Digital, Image processing, Computer Tomography, Two-phase flow

### 1. Introduction

The complex interface structure in two-phase flow has been studied intensively due to its importance in the safety analysis of the nuclear power plant for the last decades. Although one dimensional analysis by the area averaging predicts successfully system dynamics during hypothetical

accident, the uncertainties lies in the two-phase modelling impose penalty to operation range and conservatism in design. Furthermore, in such a situation like UPI, the three dimensional behavior is very complex in analyzing and measuring the phenomena. The necessity of three dimensional recognition of two-phase flow is increasing fast. One of possible technique is tomography if its response is fast enough to real time application and if it works in the safe environment without giving operator radiation exposure. Recently, electromagnetic tomography is issued and studied intensively due to its partial fulfillment for the above requirements. Also, as an another possibility, natural ray computer tomography could play a role at least for the steam-water or air-water system where natural light could penetrate through the windows to the system.

As listed in the table 1, the natural ray tomography could reproduce image in a quite accurate way under 1% error. This remarkable accuracy is achieved only when it has a proper back projection algorithm.

Table 1 Comparison of the various tomography sensors.

Principles	accuracy	Tech rology related	<b>l</b> l o te	
Radiatio r	1%	O ptics	fast, Optical access req∎ired	
		Х-гау 6 дап па гау	slow, radiation confinement	
		positro i emissio i,	leveled particle, not on-line	
		∎rclear Wagretic resorance	Fast, expensive for large vessel	
l constic	3%	U firaso∎ic wave	sonic speed limitation, complex to use	
e Le c 1 r	i c 10 %	capacila i ce	fast, low cost	
ro perfies		Conductances	suitable for sin all or large vessels	
		lud rota roe		

However, the little poor accurate tomography, electric tomography, spreads wide in process industry nowadays as listed in Table 2. The major merits of the electric tomography is safe environment and low cost. But if the natural ray tomography presented in this paper is superior to the electric tomography in both factors. Because the natural ray has no health hazard effect and the CC camera and related equipments are easy to purchase in a relatively low price.

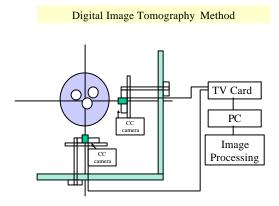
The present work is made to implement the tomography technique using natural ray. To do this, a total system for the tomography could be established in both hardware and software. Also, we could see its feasibility for the two-phase analysis. After succeeding in implementation, there is no difficulty in applying it to the radiation ray such as neuron or X-ray also.

Table 2. The utilization of electric tomography in the process engineering

a p p lica tions	sensor type	special features	D eve lo pe rs
prermatic corveyor	ca pa cita i ce	Heastrement of the solid mass flow in the fluidized bed(100frames/sec)	Edwards et al (1995)
Oil field pipeli <b>r</b> e	ca pa cita r ce	gas/void fraction (200frames/sec)	Xie et al (1995)
Fluidized bed	Gan na capacitance	Risers and standpipe (less then 1m in diameter), experimental rig with 0.15m in diameter for the gas bubble image	Bernard E1 al (1995) Halow (1995)
Tricked Bed	X - ra y ca pa cita∎ce	packing Morphology (0.6m in diameter) high speed fluid pulse (0.12 m in diameter)	Toye et al (1995) Reireck (1995)
Hydrocyclones	Resistance X-ray	Distribution and density in the Industrial separator 2014 M	William et al(1995) Hiller&Lin (1995)
Colloida l sispeisioi	Resistance magnetic resonance	The sedimentation of the debris in the tube of 1 mm diameter.	William (1995) Gibbs (1995)
Environmental large scale	Resistance	leakage detection in the Intelear Waster	Ram irez (1995) Daily (1995)
Wiker	Seis nic Resista∎ce	cross hole in age nulliplate in age	Elbring (1995)

### 2. Instrumentation for the natural light tomography

The equipments for the Normal light tomography are prepared in easy because they are all for the multimedia application. Recent popular usage of multimedia computer provides really a good environment for this technique. Many cheap and reliable image processing boards are exhibited in the market. CC camera or cam coder is easy to prepare. In this study, to demonstrate its full automation for the process control, a full set from image sensor to the computer data storage is prepared.



Figl. The conceptual drawing for the instrumentation of normal light CT

As an image sensor, both camcoder of SONY, and CC camera of Hitron system are tested. For develop the tomography sensor, small CC camera is better than the camcoder. As shown in Figl., the structure of the image sensor mounting and the connection to the personal computer are presented. The x-y alignment of the CC camera is tried here. Also, the image capture board which transforms the analogy signal into the digital form. Any board for the TV tuner for the personal computer could be usable. In this study, PC Vision Pro of Kasan Electronics (Korea company) is used because of its cheap and high resolution (1024\*768). As shown in Fig2.



Fig 2. the digital image captured from a TV program

### Digital Image processing

For the preprocessing of the captured image, digital image processing techniques such as the quantization and contrast equalization is necessary. The image is reconstructed by Radon transform. In this section, those processing techniques are described.

### 3.1 256 color quantization

The image file is treated under the frame of bitmap with 256 color for reduction of computing effort. At first, the mage of true color with 16,700,000 color should be quantized to the 256 colors using uniform quantization. Of course, best selection through sensitivity study is necessary among the uniform, adaptive color compression, popularity, median cut, etc.

### 3.2 image processing

The coexistence of bright part and dark part in a bubble produces big contrast skewness so that the contrast equalization could be made. Histogram analysis is used here to re-allocation of contrast.

### 3.3 Radon forward Transformation

The contrast in an image from light source depends on the scattering rate in the exponential reducing according to the effective length by the scattering tacks of incident ray:

$$I = I_0 \exp\left[-\int_L f(x, y) du\right] \tag{1}$$

where  $I_0$  is the intensity of the incident ray, L is the path of the ray, u is the distance along L. we could define the following term

$$g \doteq \ln\left(\frac{I_0}{I}\right) \tag{2}$$

The linear projection could be made by inserting Eq.(1) to Eq.(2):

$$g = g(s, \theta) = \int_{T} f(x, y) du - \infty \langle s(\infty, 0) \leq \theta \leq \pi$$
 (3)

where  $(s,\theta)$  represents the coordinate of light ray relative to the object. The image reconstruction problem is to determine f(x,y) from  $g(s,\theta)$ . In the cylindrical coordinate Eq.(3) could be represented as

$$g(s,\theta) = Rf = \int \int_{-\infty}^{\infty} f(x,y) \delta(x \cos \theta + y \sin \theta - s) dx dy, \quad -\infty \langle s \langle \theta, 0 \rangle \leq \theta \langle \pi \rangle$$
 (4)

where  $g(s,\theta)$  is the one dimensional projection of f(x,y) at an angle  $\theta$ .

The following mapping relations are useful for this transformation:

$$s = x \cos \theta + y \sin \theta \tag{5}$$

$$u = -x\sin\theta + y\cos\theta \tag{6}$$

Then Eq.(4) can be expressed

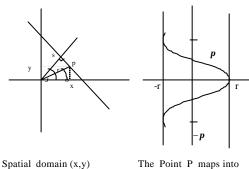
$$g(s,\theta) = \int_{-\infty}^{\infty} f(s \cos \theta - u \sin \theta, s \sin \theta + u \cos \theta) du, \tag{7}$$

Since the coordinate transformation between the cartesian coordinate and the polar coordinate is

$$x = r \cos \theta$$
,  $y = r \sin \theta$ , (8)

Eq.(5) could be represented in a simple form from the above relations:

$$S = r \cos(\theta - \theta)$$
. (9)



The Point P maps into sinusod in the (s, theta) plane

Fig. 3 The image transformation with the cylindrical coordinate

So a fixed point (r, D) could represent the locus of all the points in  $(s, \theta)$ 

### 3.4 Back projection of the Radon Transformation

Associated with Radon transformation is the back projection operator B, which is defined as

$$b(x,y) = Bg = \int_0^{\pi} g(x\cos\theta + y\sin\theta, \theta)d\theta$$
 (10)

In polar coordinates it can be written as

$$b(\mathbf{x}, \mathbf{y}) = b(\mathbf{r}, \mathbf{\Phi}) = \int_{0}^{\pi} g(\mathbf{r}\cos(\theta - \mathbf{\Phi}), \theta) d\theta$$
 (11)

Back projection represents the accumulation of the ray-sums of all the rays that pass through the point (x,y) or  $(r,\mathcal{D})$ . For example if there are only two point sources in case of PET)

$$g(s, \theta) = g_1(s)\delta(\theta - \theta_1) + g_2(s)\delta(\theta - \theta_2)$$
(12)

that is, if there are only two projection

$$b_{\theta}(r, \mathcal{O}) = g_1(s_1) + g_2(s_2) \tag{13}$$

where  $s_1 = r\cos(\theta_1 - \Phi)$ ,  $s_2 = r\cos(\theta_2 - \Phi)$ 

It can be shown that the back-projected Radon transformation is

$$f^{*}(x, y) = Bg = BRf(x, y)$$
(14)

where operator B is adjoint to R rather than inverse of R:

$$B = R^{+} \tag{15}$$

The object f(x,y) can be restored from  $f^*$  by a two-dimensional (inverse) filter whose frequency response is

$$|\xi| = \sqrt{\xi_1^2 + \xi_2^2}$$
 (16)

that is

$$f(x, y) = F_2^{-1} |\xi| F_2[Bg]$$
(17)

where F2 denotes the two-dimensional Fourier transformation operator. This filtering operations can be implemented approximately via FFT. However the unboundness of Bg prevent us from direly using FFT. The practical implementation could be made in the following form:

$$f(x,y) = \left(\frac{1}{2x^2}\right) \int_0^{\pi} \int_{-\infty}^{\infty} \frac{\left[\frac{\partial g}{\partial s}(s,\theta)\right]}{x\cos\theta + y\sin\theta - s} \, ds d\theta \tag{18}$$

in the polar coordinate:

$$f(\mathbf{r}, \mathbf{\Phi}) = \left(\frac{1}{2\mathbf{x}^2}\right) \int_0^{\mathbf{x}} \int_{-\infty}^{\infty} \frac{\left[\frac{\partial g}{\partial s}(s, \theta)\right]}{\mathbf{r}\cos(\theta - \mathbf{\Phi}) - s} \, ds d\theta \tag{19}$$

The inverse Radon Transformation is obtained in two steps/ First, each projection  $g(s,\theta)$  is filtered by a one-dimensional filter whose frequency response is  $|\xi|$ . The resulted g function is back projected by the back projection operator. To produce better image, the convolution back projection method adopts the differentiation and Hilbert transformation in stead of one-dimensional filter.

Defining derivative operator as

$$\Delta \Phi = \frac{\partial \Phi}{\partial s} \tag{20}$$

From the definition of the Hilbert Transform:

$$H\mathcal{O} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\mathcal{O}(t)}{s-t} dt \tag{21}$$

The final back projection could be made using the Hilbert transformation

$$f(x,y) = \frac{1}{2\pi} BH\Delta g \tag{22}$$

Also, Filter back projection method replace it with the sequential operations of fourier transformation, filter, inverse fourier transformation

$$f(x, y) = BHg \tag{23}$$

where

$$g^{\dagger} = Hg = \int_{-\infty}^{\infty} |\xi| G(\xi, \theta) e^{i2\pi \xi t} ds \tag{24}$$

$$g^* = F_1^{-1} |\xi| [F_1 g]$$
 (25)

$$f(x, y) = BF1^{-1}[|\xi|F_1g]$$
(26)

### 4. Results and Discussions

In this stage, we need the further development of the transformation of the x-y alignment to the cylindrical center focused signal mapping. Therefore, the experimental data from the present facility could not be directly applicable to the developed Radon Projection. Therefore, the CT image generated artificially to generates the ray intensity numerically by calculating the ray intensity along the straight line crossing the center points. Using these values, the present Radon algorithm is evaluated whether it could back project its original CT image or not. For the two-phase flow analysis, three representative flow regimes are chosen as the bubbly flow with bubbles of different size and antisymmetric distribution, the stratified flow, and annular flow.

### 4.1 Bubbly flow with different bubble size and irregular position

The tomography image of the irregular distribution of the small bubbles is hardly producible using normal optimization technique. Figure 4 shows the results of the transformation, the first figure is the original CT image to be evaluated which generates the numerical ray intensity on the surface of the objects. The second figure shows the results after radon transformation using the produced signal (or measured). It is in vague with high background noise. The third figure shows the back projection of the second figure. Comparing to the original figure, the first one, the outer tube could not be recognizable. Also, it is very difficult to identify three bubbles boundary. However, in the last figure, the three bubbles boundary and the outer tube images are identified well when filter, forth figure, is applied. The filtered radon transformation is shown in fifth figure. From this results, the Radon transformation could be usable for the accurate analysis of the two-phase flow with proper filter.

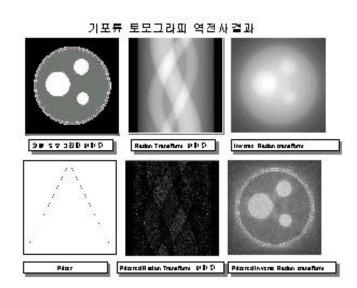


Fig 4 The back projection results of the tomography image using Radon Transformation

### 4.2 The stratified flow

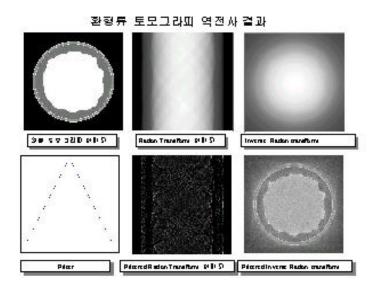
For the wavy stratified flow, the present radon transformation is evaluated as shown in the Fig.5. The the presentation order of thumbnail figures in the Figure is the same as the Fig.4. In this case, as expected, direct application of the Radon transformation could not generate identifiable image. The application of filter to the projection, shows the small ripple on the surface of the stratified flow as shown in the last thumbnail.

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Fig5. The tomography image of the stratified flow processed by the Radon Transformation

### 4.3 The annular flow

Thin film identification is quite difficult to identify in the normal back projection because, it is difficult in identify compare with the mist flow.



Figs. The tomography image of the annular flow processed by the Radon Transformation.

In terms of signal, the signal is quite similar to each other. As shown in Fig. 6, the thumbnails shows the results also. The filtered Radon transformation generates very accurate image even representing the surface ripples as expected.

### 5. conclusions

In the present paper, a normal light ray computer tomography using the digital image processing technique is proposed. This technique provides no harmful environment and is relatively cheap and easy to prepare. Also, the key technique for the back projection to get the CT image is developed using the Radon transformation with a filter. The CT images for bubbly flow, stratified flow and annular flow show the developed algorithm is applicable to the two-phase flow analysis. The remained issue to be resolved is how to map the x-y oriented ray intensity to the conical ray.

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