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Generation of the Analytic Solutions of the Multigroup Discrete Ordinates Transport Equation in Slab Geometry by Using Infinite Medium Green's Function

Ser Gi Hong and Nam Zin Cho

Korea Advanced Institute of Science and Technology
Department of Nuclear Engineering
373-1 Kusong-dong, Yusong-gu
Taejon, Korea 305-701

Abstract

Analytic solutions of the multigroup discrete ordinates transport equation with linearly anisotropic scattering in slab geometry are obtained by using infinite medium Green's function (IMGF) and Placzek's lemma. In this approach, the infinite medium Green's function is derived analytically by using the spectral analysis for the multigroup discrete ordinates transport equation and its transposed equation, and this infinite medium solution is related to the finite medium solution by Placzek's lemma. The resulting equation leads to an exact relation that represents the outgoing angular fluxes in terms of the incoming angular fluxes and the interior inhomogeneous sources for each slab. In multi-slab problems, the slabs are coupled through the interface angular fluxes. Since all derivations are performed analytically, the method gives exact solution with no truncation error. After the interface angular fluxes are calculated, the continuous distribution of the angular flux (i.e., analytic solution) in each slab are calculated straightforwardly with IMGF. Therefore, in our method, the number of meshes that is equal to the number of the homogeneous slabs is sufficient.

I. Introduction

Since the neutron transport equation cannot be solved analytically even in slab geometry, much attention has been given to the problem of obtaining accurate numerical methods of the transport equation. The discrete ordinates approximation of the angular variable and the multigroup approximation of the energy variable are the most direct approach for simplifying the complexities of the transport equation. Recently, some authors have devised exact solution methods with no truncation error for solving the multigroup discrete ordinates problems in slab geometry. First, Barros and Larsen[1],[2] have developed the spectral Green's function method (SGF) method where an exact relation between cell-edge and cell-average angular fluxes is derived by using a spectral analysis (i.e., obtaining eigenfunctions). Second is a direct method by using the Laplace transform[3] and its inverse transform.

In this paper, a new method that gives analytic solutions (with no spatial truncation error) of the slab geometry multigroup discrete ordinates transport equation with linearly anisotropic scattering is presented. This is an extension of the one group method[4] by the authors. The method is based on the infinite medium Green's function and Placzek's lemma[5].

The IMGF is derived analytically by using the spectral analysis for the multigroup discrete ordinates transport equation and its transposed equation. The infinite medium solution is related to the finite medium solution through the Placzek's lemma. This procedure gives an exact relation that represents the outgoing angular fluxes in terms of the incoming angular fluxes and the interior inhomogeneous source for each slab. In multi-slab problems, the slabs are simply coupled through the interface angular fluxes. Therefore, the interface angular fluxes are calculated by using an iteration method. After these interface angular fluxes are obtained, the continuous distribution of the angular flux in each slab are calculated straightforwardly with IMGF.

Our method is new in the area of the discrete ordinates transport methods. In case of the continuous angle, Case[6] originally obtained IMGF of the one group transport equation with singular eigenfunctions while the evaluation of IMGF is difficult and numerical results were not readily available. Second, the Case's IMGF has been used in the F_N [7]and C_N [8]methods that gives very accurate solutions. In case of multigroup problems, the IMGF is obtained by using the Fourier transform[8] and its inversion rather than Case's methodology. A similar approach by Ganapol[9],[10] was recently performed to obtain highly accurate solution (analytic benchmark solution) of the one group transport equation. However, to our knowledge, IMGF of the slab geometry multigroup discrete ordinates transport equation has not been derived and used to solve multi-slab problems. Therefore, the key feature of our approach is the analytical derivation of IMGF of the multigroup slab geometry discrete ordinates transport equation with linearly anisotropic scattering and its effective use in generating the analytic solutions for multigroup mult-slab problems.

II. Theory and Methodology

II.1. Spectral Analysis

The multigroup slab discrete ordinates transport problems with linearly anisotropic scattering is described by

$$\mu_m \frac{d\vec{\psi}_m(x)}{dx} + \Sigma\vec{\psi}_m(x) = \Sigma_{\mathbf{s0}}\vec{\phi}(x) + 3\mu_m\Sigma_{\mathbf{s1}}\vec{\phi}_1(x) + \vec{q}_m(x), \tag{1}$$

where the vector $\vec{\psi}_m$ has components $\psi_{g,m}$ which are the angular fluxes for each energy group g and direction m, the vector $\vec{\phi}$ has components ϕ_g which are the scalar flux for each energy group g and the vector $\vec{\phi}_1$ has components $\phi_{1,g}$ which are the net current for each energy group g. In Eq.(1), the scattering matrix ($\Sigma_{\mathbf{s0}}$) that represents the isotropic component of scattering is defined by

$$\Sigma_{\mathbf{s0}} = \begin{pmatrix} \sigma_{0,1\to1} & \sigma_{0,2\to1} & \sigma_{0,3\to1} & \cdot & \cdot \\ \sigma_{0,1\to2} & \sigma_{0,2\to2} & \sigma_{0,3\to2} & \cdot & \cdot \\ \sigma_{0,1\to3} & \sigma_{0,2\to3} & \sigma_{0,3\to3} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{pmatrix}, \tag{2}$$

and the scattering matrix ($\Sigma_{\mathbf{s1}}$) that represents the linearly anisotropic component of scattering can be similarly given and the diagonal matrix Σ has components that are the total macroscopic cross section for each energy group. To obtain the eigenfunctions, the homogeneous equation of Eq.(1) is considered :

$$\mu_m \frac{d\vec{\psi}_m(x)}{dx} + \Sigma\vec{\psi}_m(x) = \Sigma_{\mathbf{s0}}\vec{\phi}(x) + 3\mu_m\Sigma_{\mathbf{s1}}\vec{\phi}_1(x). \tag{3}$$

We seek solutions of the following form[11],[2] :

$$\vec{\psi}_m(x) = e^{-x/\nu}\vec{\phi}_\nu(\mu_m). \tag{4}$$

Substituting Eq.(4) into Eq.(3) gives

$$(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)\vec{\phi}_\nu(\mu_m) = \Sigma_{\mathbf{s0}}\vec{N}_0(\nu) + 3\mu_m\Sigma_{\mathbf{s1}}\vec{N}_1(\nu), \tag{5}$$

where $\vec{N}_0(\nu) = \sum_{m=1}^{N} w_m\vec{\phi}_\nu(\mu_m)$ and $\vec{N}_1(\nu) = \sum_{m=1}^{N} w_m\mu_m\vec{\phi}_\nu(\mu_m)$.

After some algebraic procedures with Eq.(5), the following eigenvectors are obtained :

$$\begin{aligned}\vec{N}_0(\nu) &= \sum_{m=1}^{N} w_m(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)^{-1}\Sigma_{\mathbf{s0}}\vec{N}_0(\nu)\\ &\quad + \sum_{m=1}^{N} w_m 3\mu_m(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)^{-1}\Sigma_{\mathbf{s1}}\nu(\Sigma - \Sigma_{\mathbf{s0}})\vec{N}_0(\nu),\\ \vec{N}_1(\nu) &= \nu(\Sigma - \Sigma_{\mathbf{s0}})\vec{N}_0(\nu).\end{aligned} \tag{6}$$

The eigenvalue (ν) is determined by the following characteristic equation :

$$\begin{aligned} det[\mathbf{I} - &\sum_{m=1}^{N} w_m(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)^{-1}\Sigma_{\mathbf{s0}}\\ &- 3\nu\sum_{m=1}^{N} w_m\mu_m(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)^{-1}\Sigma_{\mathbf{s1}}(\Sigma - \Sigma_{\mathbf{s0}})] = 0,\end{aligned} \tag{7}$$

where *det* means the determinant of a matrix. Eq.(7) is a polynomial of $G \times N$'th degree and its roots (ν) are the eigenvalues of the G group discrete ordinates problem (i.e., Eq.(3)). The roots are symmetrically distributed around the origin due to the symmetry of the Gauss-Legendre quadrature set. In practical problems, the eigenvalues are all simple and real (in eigenvalue problems having fission source, complex eigenvalues can occur).

Since the orthogonality of the eigenvectors was efficiently used in analytically deriving the IMGF of the one group problems, a similar property is devised in the multigroup problems. Unfortunately, the orthogonality does not exist in case of the multigroup problem. However, the bi-orthogonality[12] between the eigenvector of the forward problem and the eigenvector of the transposed problem has been found in the contiunuous angle case. In this paper, the bi-orthogonality is derived in a similar fashion for the discrete ordinates transport problem. The spectral analysis of the transposed equation of Eq.(3) can be performed similarly as the forward case. The result is given by

$$(-\frac{\mu_m}{\nu}\mathbf{I} + \Sigma)\vec{\phi}^*_\nu(\mu_m) = \Sigma^{\mathbf{T}}_{\mathbf{s0}}\vec{N}^*_0(\nu) + 3\mu_m\Sigma^{\mathbf{T}}_{\mathbf{s1}}\vec{N}^*_1(\nu), \tag{8}$$

Using Eq.(5) and Eq.(8) gives the following bi-orthogonality :

$$\sum_{m=1}^{N} w_m \mu_m < \vec{\phi}^*_{\nu_\beta}(\mu_m), \vec{\phi}_{\nu_\alpha}(\mu_m) >= M_{\nu_\alpha} \delta_{\alpha\beta}, \tag{9}$$

where $\delta_{\alpha\beta}$ is the Kronecker's delta, $< \cdot >$ means the dot product.

II.2. Infinite Medium Green's Function (IMGF)

An infinite homogeneous medium having a unit source at origin, emitting in the direction of μ_p and with a particular energy of g_p is considered. The solution of this infinite medium problem is called the infinite medium Green's function $\vec{G}^{g_p}(0, \mu_p; x, \mu_m)$. The problem is described as follows :

$$\begin{aligned}(\mu_m \mathbf{I}\frac{d}{dx} + \Sigma)\vec{G}^{g_p}(0, \mu_p; x, \mu_m) &= \Sigma_{\mathbf{s0}} \sum_{n=1}^{N} w_n \vec{G}^{g_p}(0, \mu_p; x, \mu_n) \\ &+ 3\mu_m \Sigma_{\mathbf{s1}} \sum_{n=1}^{N} w_n \mu_n \vec{G}^{g_p}(0, \mu_p; x, \mu_n) + \delta(\mu_m - \mu_p)\delta(x)\vec{\delta}_{g_p},\end{aligned} \tag{10}$$

where the vector $\vec{\delta}_{g_p}$ has only one non-zero component that is unity in the g_p'th component, and the $\delta(\mu_m - \mu_p)$ is defined to satisfy $\sum_{m=1}^{N} w_m \delta(\mu_m - \mu_p) = 1$, and the vector Green's function is defined as

$$\vec{G}^{g_p}(0, \mu_p; x, \mu_m) = \begin{pmatrix} G^{g_p \to 1}(0, \mu_p; x, \mu_m) \\ G^{g_p \to 2}(0, \mu_p; x, \mu_m) \\ G^{g_p \to 3}(0, \mu_p; x, \mu_m) \\ \cdot \\ \cdot \end{pmatrix} \tag{11}$$

In Eq.(11), $G^{g_p \to g}(0, \mu_p; x, \mu_m)$ represents the angular flux at x for direction μ_m of energy group g due to the unit source at origin for direction μ_p of energy group g_p.

While the solution must satisfy the homogeneous equation for position $x \neq 0$, it must also satisfy the following conditions. The first condition is the finite condition at infinity. The second condition that is used to determine the expansion coefficients is the jump condition at the source position. This condition is derived by integrating Eq.(10) over an infinitesimal interval around the source position :

$$\vec{G}^{g_p}(0, \mu_p; 0^+, \mu_m) - \vec{G}^{g_p}(0, \mu_p; 0^-, \mu_m) = \frac{\delta(\mu_m - \mu_p)}{\mu_m} \vec{\delta}_{g_p}. \tag{12}$$

With these conditions and the bi-orthogonality of the eigenvectors, we obtain the following IMGF with the unit source at $x = x_0$:

$$\vec{G}^{g_p}(x_0, \mu_p; x, \mu_m) = \begin{cases} \displaystyle\sum_{\alpha=1}^{NG/2} \frac{[\vec{\phi}^*_{\nu_\alpha}(\mu_p)]_{g_p}}{M_{\nu_\alpha}} e^{-(x-x_0)/\nu_\alpha} \vec{\phi}_{\nu_\alpha}(\mu_m), \ x > x_0, \\ \displaystyle -\sum_{\alpha=NG/2+1}^{NG} \frac{[\vec{\phi}^*_{\nu_\alpha}(\mu_p)]_{g_p}}{M_{\nu_\alpha}} e^{-(x-x_0)/\nu_\alpha} \vec{\phi}_{\nu_\alpha}(\mu_m), \ x < x_0, \end{cases} \tag{13}$$

where $[\vec{a}]_p$ represents the p'th component of a vector $\vec{a}$.

II.3. Computational Method

In this section, the computational method using IMGF for obtaining the analytic solutions of the multigroup slab geometry discrete ordinates transport equation is derived. First, consider a homogeneous finite slab problem with given incoming angular fluxes at boundaries:

$$\begin{aligned}\mu_m \frac{d\vec{\psi}_m(x)}{dx} + \Sigma\vec{\psi}_m(x) &= \Sigma_{\mathbf{s0}}\vec{\phi}(x) + 3\mu_m\Sigma_{\mathbf{s1}}\vec{\phi}_1(x) + \vec{q}_m(x),\ x \in [-a,a],\\ \vec{\psi}_m(-a) &= \vec{\psi}^{in}_{L,m},\ \mu_m > 0,\\ \vec{\psi}_m(a) &= \vec{\psi}^{in}_{R,m},\ \mu_m < 0,\end{aligned} \tag{14}$$

where a is the half thickness of the slab. We note that the above finite medium solution $\vec{\psi}_m(x)$ can be represented in terms of an infinite medium solution $[\vec{\psi}^{\infty}_m(x)]$ due to the Placzek's lemma as follows :

$$\begin{aligned}&H_*(x)\vec{\psi}_m(x) = \vec{\psi}^{\infty}_m,\ H_*(x) = \begin{cases} 1,\ x \in [-a,a],\\ 0,\ \text{otherwise},\end{cases}\\ &\mu_m \frac{d\vec{\psi}^{\infty}_m(x)}{dx} + \Sigma\vec{\psi}^{\infty}_m(x) = \Sigma_{\mathbf{s0}}\vec{\phi}^{\infty}(x) + 3\mu_m\Sigma_{\mathbf{s1}}\vec{\phi}^{\infty}_1(x)\\ &\qquad + H_*(x)\vec{q}_m(x) + \mu_m\vec{\psi}_m(x)[\delta(x+a) - \delta(x-a)],\ x \in [-\infty,\infty],\end{aligned} \tag{15}$$

where $H_*(x)$ is a step function. Since IMGF is available, the solution for the finite solution can be given by

$$\begin{aligned}\psi_{p,m}(x) &= \sum_{q=1}^{G}\sum_{n=1}^{N} w_n \int_{-a}^{a} dx_0 G^{q\to p}(x_0,\mu_n;x,\mu_m)q_{q,n}(x_0)\\ &+ \sum_{q=1}^{G}\sum_{n=1}^{N} w_n G^{q\to p}(-a,\mu_n;x,\mu_m)\mu_n\psi_{q,n}(-a) - \sum_{q=1}^{G}\sum_{n=1}^{N} w_n G^{q\to p}(a,\mu_n;x,\mu_m)\mu_n\psi_{q,n}(a),\end{aligned} \tag{16}$$

where the indices p and q were used to represent energy group. In Eq.(15), it must be noted that the finite solution is equal to the infinite medium solution for $x \in [-a,a]$. Inserting $x = a$ into Eq.(16) and separating the boundary angular fluxes into the incoming and outgoing parts give

$$\begin{aligned}\psi_{p,m}(a) &= \sum_{q=1}^{G}\sum_{n=1}^{N} w_n \int_{-a}^{a} dx_0 G^{q\to p}(x_0,\mu_n;a,\mu_m)q_{q,n}(x_0)\\ &+ \sum_{q=1}^{G}\sum_{n=1}^{N/2} w_n G^{q\to p}(-a,\mu_n;a,\mu_m)\mu_n\psi^{in}_{L,q,n} - \sum_{q=1}^{G}\sum_{n=N/2+1}^{N} w_n G^{q\to p}(a,\mu_n;a^-,\mu_m)\mu_n\psi^{in}_{R,q,n}\\ &+ \sum_{q=1}^{G}\sum_{n=N/2+1}^{N} w_n G^{q\to p}(-a,\mu_n;a,\mu_m)\mu_n\psi^{out}_{L,q,n} - \sum_{q=1}^{G}\sum_{n=1}^{N/2} w_n G^{q\to p}(a,\mu_n;a^-,\mu_m)\mu_n\psi^{out}_{R,q,n}.\end{aligned} \tag{17}$$

Multiplying Eq.(17) with $\mu_m[\vec{\phi}^*_{\nu_\alpha}(\mu_m)]_p$ and summing over m and p lead to

$$\begin{aligned}
&\sum_{m=1}^{N} w_m\mu_m < \vec{\phi}^*_{\nu_\alpha}(\mu_m), \vec{\psi}^{out}_{R,m} > + \sum_{m=N/2+1}^{N} w_m\mu_m < \vec{\phi}^*_{\nu_\alpha}(\mu_m), \vec{\psi}^{in}_{R,m} > \\
&= e^{-\frac{2a}{\nu_\alpha}} \sum_{m=1}^{N} w_m\mu_m < \vec{\phi}^*_{\nu_\alpha}(\mu_m), \vec{\psi}^{in}_{L,m} > + e^{-\frac{2a}{\nu_\alpha}} \sum_{m=N/2+1}^{N} w_m\mu_m < \vec{\phi}^*_{\nu_\alpha}(\mu_m), \vec{\psi}^{out}_{L,m} > \\
&+ \int_{-a}^{a} dx_0 e^{-(a-x_0)/\nu_\alpha} \sum_{m=1}^{N} w_m < \vec{\phi}^*_{\nu_\alpha}(\mu_m), \vec{q}_m(x_0) >, \ \alpha = 1, 2, \cdots, NG/2,
\end{aligned} \tag{18}$$

where $< \cdot >$ means the dot product. In derving Eq.(18), the bi-orthogonality of the eigenvectors was used. It must be noted that Eq.(18) can easily treat arbitrarily distributed source. Similarly, the corresponding equation for $x = -a$ can be derived and the two equations can be written in the following vector form :

$$\mathbf{A}\vec{\psi}^{out} = \mathbf{B}\vec{\psi}^{in} + \mathbf{C}\vec{Q}, \tag{19}$$

where $\vec{\psi}^{out}$ and $\vec{\psi}^{in}$ consist of the cell-edge outgoing and incoming angular fluxes for the finite slab, respectively, and $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are $GN \times GN$ matrices. The problems having heterogeneous materials (consisting of multilayered homogeneous slabs) can be solved by an iterative scheme on the interface angular fluxes (e.g., red-black iteration with one-node block inversion) with the continuity of the interface angular fluxes. Finally, if all the cell-edge angular fluxes are calculated, the analytic distribution for each homogeneous slab is easily calculated by Eq.(16). The resulting equation for the case of uniform and isotropic source is explicitly given by

$$\begin{aligned}
\psi_{p,m}(x) = &\sum_{\alpha=1}^{NG/2} \frac{\nu_\alpha}{M_{\nu_\alpha}}(1 - e^{-(x+a)/\nu_\alpha})[\vec{\phi}_{\nu_\alpha}(\mu_m)]_p \sum_{q=1}^{G}[\vec{N}^*_{\nu_\alpha}]_q q_q \\
&+ \sum_{\alpha=1}^{NG/2} \frac{\nu_\alpha}{M_{\nu_\alpha}}(1 - e^{(x-a)/\nu_\alpha})[\vec{\phi}_{-\nu_\alpha}(\mu_m)]_p \sum_{q=1}^{G}[\vec{N}^*_{-\nu_\alpha}]_q q_q \\
&+ \sum_{\alpha=1}^{NG/2} \frac{1}{M_{\nu_\alpha}} e^{-(x+a)/\nu_\alpha}[\vec{\phi}_{\nu_\alpha}(\mu_m)]_p \sum_{q=1}^{G}\sum_{n=1}^{N} w_n\mu_n[\vec{\phi}^*_{\nu_\alpha}(\mu_n)]_q \psi_{L,q,n} \\
&- \sum_{\alpha=1}^{NG/2} \frac{1}{M_{\nu_\alpha}} e^{(x-a)/\nu_\alpha}[\vec{\phi}_{-\nu_\alpha}(\mu_m)]_p \sum_{q=1}^{G}\sum_{n=1}^{N} w_n\mu_n[\vec{\phi}^*_{-\nu_\alpha}(\mu_n)]_q \psi_{R,q,n}.
\end{aligned} \tag{20}$$

Since all derivations except the discrete ordinates approximation are analytic, Eq.(20) is the analytic solution of the multigroup discrete ordinates transport equation in slab geometry. For the case of general source without uniform and isotropic assumptions, the analytic solution can be also written down but omitted here. If only region averaged scalar fluxes are required, the use of the balance equation is more convenient than the use of Eq.(20).

III. Numerical Application and Results

To test our method, two two-energy group benchmark problems are considered. Problem 1 is a homogeneous slab of $100cm$ with isotropic scattering proposed by Barros and Larsen[2].

The left side boundary condition is a predescribed incident angular flux ($\psi^L_{1,m} = 1.0,\ \psi^L_{2,m} = 0.0$) and the right side boundary condition is vacuum. The cross sections of the problem are as follows : $\sigma_1 = \sigma_2 = 1.0cm^{-1},\ \sigma_{0,1\to 1} = 0.99cm^{-1},\ \sigma_{0,1\to 2} = 0.008cm^{-1},\ \sigma_{0,2\to 1} = 0.005cm^{-1}$ and $\sigma_{0,2\to 2} = 0.98cm^{-1}$. The interior source is zero. To solve this problem, the S_4 Gauss-Legendre quadrature set with a pointwise convergence criterion of 10^{-5} is used. The numerical test is performed for several mesh divisions to show that our method gives true analytic solution. In Table I, the numerical results (i.e., scalar fluxes at $x = 0.0cm,\ 50.0cm,\ 100.0cm$) are compared with the diamond difference (DD) method and SGF. The results show that our method gives the analytic solution of multigroup discrete ordinates transport equation. It must be noted that our method used only one mesh to find all information for this problem.

Table 1: Comparison of the scalar fluxes of problem 1

N[a]	x[b]	SGF		DD		present method	
		group 1	group 2	group 1	group 2	group 1	group 2
10	0	0.91268	0.27264E-1	0.91266	0.27271E-1	0.91268	0.27264E-1
	50	0.55129E-3	0.33652E-3	-0.7669E-2	0.32086E-2	0.55129E-3	0.33652E-3
	100	0.62768E-7	0.38442E-7	0.15715E-2	-0.71102E-3	0.62769E-7	0.38443E-7
4	0	0.91268	0.27264E-1	0.91174	0.28700E-1	0.91268	0.27264E-1
	50	0.55129E-3	0.33652E-3	0.11765	-0.78691E-1	0.55129E-3	0.33652E-3
	100	0.62768E-7	0.38442E-7	0.12646E-1	-0.11929E-1	0.62769E-7	0.38443E-7
1	0	-	-	-	-	0.91268	0.27264E-1
	50	-	-	-	-	0.55129E-3	0.33652E-3
	100	-	-	-	-	0.62769E-7	0.38443E-7

[a]Number of meshes, [b]position

Second problem is a heterogeneous slab of $100cm$ thickness with linearly anisotropic scattering. As in Figure 1, this problem consists of three regions. The leftmost region (fuel), $10.0cm$ thick, has $\sigma_1 = 0.3,\ \sigma_2 = 1.0,\ \sigma_{0,1\to 1} = 0.27,\ \sigma_{0,1\to 2} = 0.01,\ \sigma_{0,2\to 1} = 0.001,\ \sigma_{0,2\to 2} = 0.9,\ \sigma_{1,1\to 1} = 0.09,\ \sigma_{1,1\to 2} = 0.002,\ \sigma_{1,2\to 1} = 0.0002,\ \sigma_{1,2\to 2} = 0.08$. This has a uniform isotropic source of $q_1 = 5.0$ and $q_2 = 50.0$. The center region (absorber), $10.0cm$ thick, has $\sigma_1 = 0.2,\ \sigma_2 = 3.53,\ \sigma_{0,1\to 1} = 0.18,\ \sigma_{0,1\to 2} = 0.01,\ \sigma_{0,2\to 1} = 0.001,\ \sigma_{0,2\to 2} = 0.53,\ \sigma_{1,1\to 1} = 0.08,\ \sigma_{1,1\to 2} = 0.003,\ \sigma_{1,2\to 1} = 0.0003,\ \sigma_{1,2\to 2} = 0.06$. The right region (water) is divided into two subregions differing in source strength. The left region ($20cm$ thick) of these two subregions has a uniform and isotropic source of $q_1 = 1.0$ and $q_2 = 10.0$. This water region has $\sigma_1 = 0.401,\ \sigma_2 = 1.30, \sigma_{0,1\to 1} = 0.32,\ \sigma_{0,1\to 2} = 0.08,\ \sigma_{0,2\to 1} = 0.002,\ \sigma_{0,2\to 2} = 1.29,\ \sigma_{1,1\to 1} = 0.07,\ \sigma_{1,1\to 2} = 0.003,\ \sigma_{1,2\to 1} = 0.0004,\ \sigma_{1,2\to 2} = 0.2$. The average scalar fluxes in the absorber region are compared in Table 2. Our results were obtained with the S_4 Gauss-Legendre quadrature set and only three meshes corresponding to three regions. The results show that the values of DD approach the values of the our method as the number of meshes increases. Similar trends are also shown in Table 3 where the scalar fluxes at the boundaries are compared. The analytic solution (scalar flux distribution) by using our method are compared in Figures 2 and 3. These results also show that the results of DD approach those of our method as the number of meshes increases.

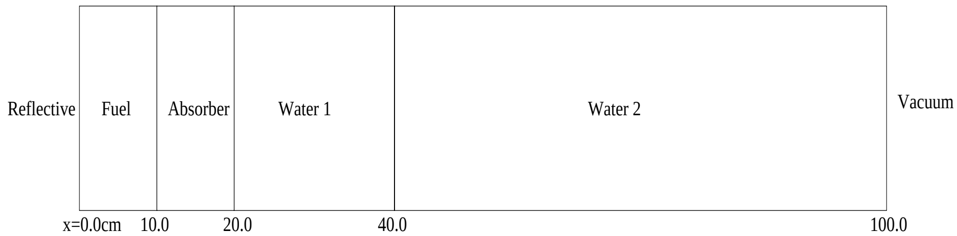

Figure 1: Configuration of problem 2

Table 2: Comparison of the average scalar fluxes in absorber region (problem 2)

Number of meshes	20	50	100	200	3 (present method)
group 1	63.317	59.900	60.04	60.002	59.992
group 2	2.36	4.7438	4.536	4.5421	4.5419

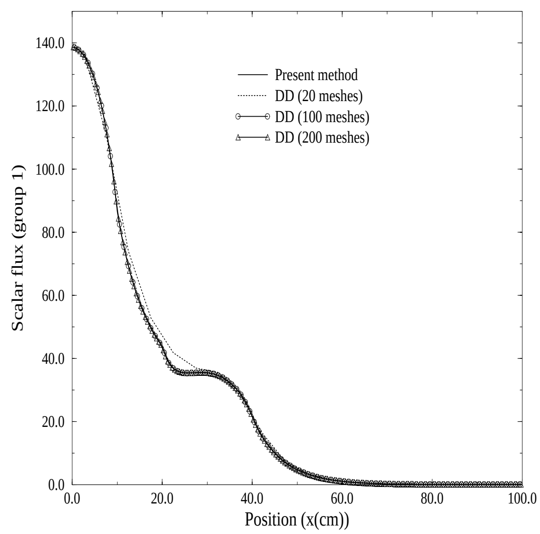

Figure 2: Comparison of the group 1 scalar flux distributions

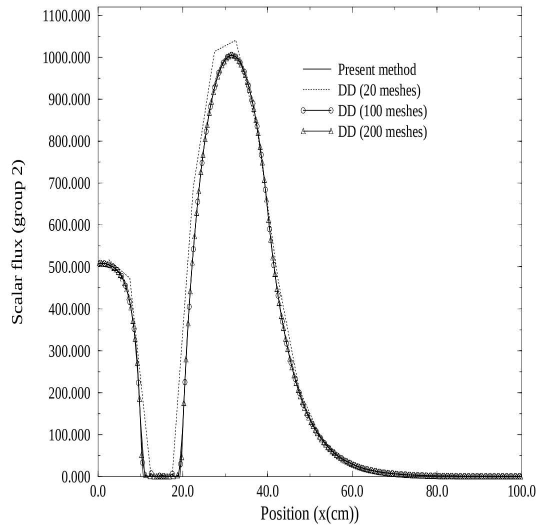

Figure 3: Comparison of the group 2 scalar flux distributions

Table 3: Comparison of the scalar fluxes at $x = 0.0cm,\ 100.0cm$ (problem 2)

N[a]	20	50	100	200	3 (present method)
x=0.0	141.477[b]	138.882	138.732	138.659	138.635
	510.942[c]	511.348	509.692	509.342	509.229
x=100.0	3.72536E-4	7.99657E-4	8.45497E-4	8.56667E-4	8.60428E-4
	3.16601E-2	8.87709E-3	9.38618E-3	9.51024E-3	9.55202E-3

[a]Number of meshes, [b]group 1 scalar flux, [c]group 2 scalar flux

IV. Conclusions

Analytic solutions of the multigroup discrete ordinates transport equation with linearly anisotropic scattering in slab geometry were obtained by using infinite medium Green's function and Placzek's lemma. The infinite medium Green's function is analytically derived by using the spectral analysis of the multigroup discrete oridinates transport equation and its transposed equation. This approach leads to an exact relation in which the outgoing angular fluxes is represented in terms of the incoming angular fluxes. After the interface angular fluxes are calculated, the analytic solution for each slab is calculated straightforwardly with IMGF. Therefore, in our method, the number of meshes that is equal to the number of the homogeneous slabs is sufficient. The numerical tests show that our method gives true analytic solution of the multigroup discrete ordinates transport equation in slab geometry.

References

1. R. C. Barros and E. W. Larsen, "A Numerical Method for One-Group Slab-Geometry Discrete Ordinates Problems with No Spatial Truncation Error," *Nucl. Sci. Eng.*, **104**, 199(1990).

2. R. C. Barros and E. W. Larsen, "A Numerical Method for Multigroup Slab Geometry Discrete Ordinates Problems with No Spatial Truncation Error," *Transp. Th. and Stat. Physics,* **20**, 441(1991).

3. M. T. Vilhena and L. B. Barichello, "An Analytic Solution for the Multigroup Slab Geometry Discrete Ordinates Problems," *Transp. Th. and Stat. Physics,* **24**, 1337(1995).

4. S. G. Hong and N. Z. Cho, "An Analytic Solution Method for Discrete Ordinates Transport Equations in Slab Geometry with No Spatial Truncation Error," Accepted for publication in *Transactions of the American Nuclear Society*, November 1999.

5. K. M. Case, F. De Hoffmann, and G. Placzek, *Introduction to Neutron Diffusion*, LASL Report, Los Alamos Scientific Lab. (1953).

6. K. M. Case and P. F. Zweifel, *Linear Transport Theory*, Addison-Wesley, New York (1967).

7. C. E. Siewart and P. Benoist, "The F_N Method in Neutron-Transport Theory. Part I : Theory and Applications," *Nucl. Sci. Eng.,* **69**, 156(1979).

8. P. Benoist, A. Kharchaf and R. Sanchez, "Multigroup C_N Method-I. Half-Space Albedo Problem," *Ann. Nucl. Energy,* **23**, 1033(1996).

9. B. D. Ganapol and D. K. Parsons, "A Heterogeneous Analytical Benchmark for Particle Transport Methods Development," *Trans. Am. Nucl. Soc.,* **80**, 113(1999).

10. B. D. Ganapol and D. E. Kornreich, "The Green's Function Method for the Monoenergetic Transport Equation with Forward/Backward/Isotropic Scattering," *Ann. Nucl. Energy,* **23**, 301(1996).

11. R. Sanchez and N. J. McCormick, "A Review of Neutron Transport Approximations," *Nucl. Sci. Eng.,* **80**, 481(1982).

12. P. Silvennoinen and P. F. Zweifel, "On Multigroup Transport Theory with a Degenerate Transfer Kernel," *J. Math. Phys.*, **13**, 1114(1972).