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A detailed derivation of the Two-Fluid Two-Phase Hydraulic Solver

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Abstract

A computer program, GUARD, semi-implicit multi-dimensional two-fluid three equations hydraulic solver, is formulated and coded. The essential steps of the formulation are presented. Numerical tests are performed to confirm its applicability and/or to identify any potential problems. Some suggestions are made to improve the convergence and stability of the scheme.

1. Introduction

The numerical schemes for solving the two-phase hydraulics have been greatly developed for last three decades. A one-step implicit homogeneous three equation scheme of Porsching[1] is adopted in codes such as RELAP4[2] and FLASH-4[3]. As a semi-implicit two-fluid scheme is formulated by Liles[4], RELAP5[5] and TRAC[6] development groups formulate the similar schemes and use them in their codes. Some theoretical studies on the convergence, stability, and well-posedness are done to identify the potential limitations of the semi-implicit scheme. The outcome of the studies are the formulation of the virtual mass[5] and the two pressure approach[7]. CATHENA[8] development group formulate the two pressure one-step implicit two-fluid scheme. In spatial viewpoint, the three-dimensional analysis capability is implemented in codes TRAC, COBRA-TF[9], and RELAP5.

It is noteworthy that a three-field approach is formulated in COBRA-TF, in which a droplet field is taken as a third field. Even though the droplet field makes the code more complicated, it gives the program developers better chance to follow more realistic phenomena when it plays an important role, such as during the reflood period.

In this study, a numerical scheme to solve the two-phase hydraulics is formulated and tested, in which three-field, three-dimensional and semi-implicit approaches are adopted. This study is the first step of the planned endeavors to develope a system safety analysis code. A formulation of the numerical scheme is presented in section 2, followed by a discussion on the test runs. In section 4, the conclusions are made.

2. Field equations and the Numerical Scheme

The conservation equations that describe a two-phase flow can be derived as a macroscopic time and area average of the local instantaneous conservation equations for a continuum[10]. As are done in COBRA-TF, three field equations are adopted in this study, vapor, continuous liquid, and entrained liquid drops. In addition, a non-condensible gas mixture is allowed to be transported with the vapor field.

2.1 Field equations

Four mass conservation equations are required for the vapor, continuous liquid, entrained liquid and non-condensible gas mixture. They are as follows[1],

$$\frac{\partial}{\partial t}(\alpha_v \rho_v) + \nabla \cdot (\alpha_v \rho_v \vec{U}_v) = \Gamma''' \qquad (1)$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l) + \nabla \cdot (\alpha_l \rho_l \vec{U}_l) = -\Gamma_l''' - S''' \qquad (2)$$

$$\frac{\partial}{\partial t}(\alpha_e \rho_e) + \nabla \cdot (\alpha_e \rho_e \vec{U}_e) = -\Gamma_e''' + S''' \qquad (3)$$

$$\frac{\partial}{\partial t}(\alpha_v \rho_E) + \nabla \cdot (\alpha_v \rho_E \vec{U}_v) = \Gamma_E''' \qquad (4)$$

The continuous liquid and the entrained liquid are assumed to be in thermal equilibrium. Therefore, two energy equations are used for the vapor-gas mixture and the combined liquid fields respectively:

$$\frac{\partial}{\partial t}(\alpha_v \rho_{vE} h_{vE}) + \nabla \cdot (\alpha_v \rho_{vE} h_{vE} \vec{U}_v) = \Gamma''' h_v + q_{iv} + Q_v''' \qquad (5)$$

$$\frac{\partial}{\partial t}((1-\alpha_v)\rho_l h_l) + \nabla \cdot (\alpha_l \rho_l h_l \vec{U}_l) + \nabla \cdot (\alpha_e \rho_l h_l \vec{U}_e)$$
$$= -\Gamma''' h_f + q_{il} + Q_l''' \qquad (6)$$

1 Notations are found in the nomenclature

Having the independent momentum equation, the continuous liquid is allowed to flow with different velocity relative to the entrained liquid phase. Therefore, three momentum equations are adopted as follows:

$$\frac{\partial}{\partial t}(\alpha_v \rho_{vg} \vec{U}_v) + \nabla \cdot (\alpha_v \rho_{vg} \vec{U}_v \vec{U}_v) = -\alpha_v \nabla P + \alpha_v \rho_{vg} \vec{g} - \vec{\tau}_{wv}''' - \vec{\tau}_{lv}''' - \vec{\tau}_{ev}''' + \Gamma''' \vec{U} \quad (7)$$

$$\frac{\partial}{\partial t}(\alpha_l \rho_l \vec{U}_l) + \nabla \cdot (\alpha_l \rho_l \vec{U}_l \vec{U}_l) = -\alpha_l \nabla P + \alpha_l \rho_l \vec{g} - \vec{\tau}_{wl}''' + \vec{\tau}_{lv}''' - \Gamma_l''' \vec{U} - S''' \vec{U} \quad (8)$$

$$\frac{\partial}{\partial t}(\alpha_e \rho_l \vec{U}_e) + \nabla \cdot (\alpha_e \rho_l \vec{U}_e \vec{U}_e) = -\alpha_e \nabla P + \alpha_e \rho_l \vec{g} + \vec{\tau}_{ev}''' - \Gamma_e''' \vec{U} + S''' \vec{U} \quad (9)$$

2.2 Finite Difference Equations and Solution procedure

The solution scheme adopted in this study is the semi-implicit scheme. The dependent variables are junction velocity U_ϕ, gas volume fraction α_g, entrained liquid volume fraction α_e, cell total pressure P, cell gas pressure P_g, vapor enthalpy h_v, and liquid enthalpy h_l. The solution procedure is as follows,

- linearize the conservation equations with respect to the dependent variables
- solve the momentum equations for the new time velocities as a function of adjacent cell pressures.
- insert these new time velocities in the mass and energy equations
- construct the system pressure matrix
- solve the system matrix to get the new time pressures and substitute them in the mass and energy equations and get the rest of the new time variables

2.2.1 Momentum equations

The finite difference forms for the momentum equations can be represented as follows:

$$A_{mj} \frac{\delta x_j}{\delta t} [(\alpha_\phi \rho_\phi)^n U_\phi^{n+1} - (\alpha_\phi \rho_\phi)^n U_\phi^n]_j = \sum_{KB=1}^{NB} [A_{mJ} U_{\phi J}^n (\alpha_\phi \rho_\phi U_\phi)^{n*}]_{KB} - \sum_{KA=1}^{NA} [A_{m(J+1)} U_{\phi(J+1)}^n (\alpha_\phi \rho_\phi U_\phi)^{n*}]_{KA}$$

$$- (\alpha_\phi \rho_\phi)_j g A_{mj} \delta x_j - (P_{J+1}^{n+1} - P_J^{n+1}) \alpha_{\phi_j} A_{mj}$$

$$- A_{mj} \delta x_j \begin{bmatrix} K_\phi U_{\phi j}^{n+1} \\ + K_{\phi\phi'} (U_{\phi j} - U_{\phi' j})^{n+1} \\ + K_{\phi\phi''} (U_{\phi j} - U_{\phi'' j})^{n+1} \end{bmatrix}$$

$$+ A_{mj} \delta x_j (\Gamma_m U_m^{n+1} + S_{m'} U_{m'}^{n+1}) \tag{10}$$

The momentum equations can be rearranged in the following form,

$$U_{\phi j}^{n+1} = C_{\phi\phi} (P_{J+1}^{n+1} - P_J^{n+1}) + C_{\phi\phi'} U_{\phi' j}^{n+1} + C_{\phi\phi''} U_{\phi'' j}^{n+1} + C_{\phi 0} \tag{11}$$

In a matrix form, they can be written,

$$\begin{bmatrix} 1.0 & -C_{fg} & -C_{fe} \\ -C_{gf} & 1.0 & -C_{ge} \\ -C_{ef} & -C_{eg} & 1.0 \end{bmatrix} \begin{bmatrix} U_{fj}^{n+1} \\ U_{gj}^{n+1} \\ U_{ej}^{n+1} \end{bmatrix} = \begin{bmatrix} C_{f\phi} \\ C_{g\phi} \\ C_{e\phi} \end{bmatrix} (P_{J+1}^{n+1} - P_J^{n+1}) + \begin{bmatrix} C_{f0} \\ C_{g0} \\ C_{e0} \end{bmatrix} \tag{12}$$

Equation(12) can be solved for the new time velocities to yield,

$$U_{gj}^{n+1} = \frac{\partial U_{gj}}{\partial P} \Delta P_{j+1,j}^{n+1} + U_{gj}^{exp} \tag{13}$$

$$U_{fj}^{n+1} = \frac{\partial U_{fj}}{\partial P} \Delta P_{j+1,j}^{n+1} + U_{fj}^{exp}$$

$$U_{ej}^{n+1} = \frac{\partial U_{ej}}{\partial P} \Delta P_{j+1,j}^{n+1} + U_{ej}^{exp}$$

2.2.2 Mass conservation equations

The mass conservation equations are written as follows,

$$A_{c_J} \frac{\delta x_J}{\delta t} \left[(\alpha_\phi \rho_\phi)^{n+1} - (\alpha_\phi \rho_\phi)^n \right]$$

$$= \sum_{KB=1}^{NB} [A_{m_{j-1}} U_{\phi_{j-1}}^{n+1} (\alpha_\phi \rho_\phi)^{*n}]_{KB} - \sum_{KA=1}^{NA} [A_{m_j} U_{\phi_j}^{n+1} (\alpha_\phi \rho_\phi)^{*n}]_{KA}$$

$$+ \Gamma_\phi^{n+1} + S_\phi \tag{14}$$

In the above equation, new time variables should be expanded with their old time values and the time increments of them. Interphase mass transfer terms can be expanded as follows,

$$\Gamma_{SCL}^{n+1} \equiv \frac{HA_{SCL}}{C_{pf}(h_v^n - h_{sl}^n)} (h_f^{n+1} - h_{sl}^{n+1}) = \Gamma_1 (h_f^{n+1} - h_{sl}^{n+1}) \quad (h_f^n < h_{sl}^n) \tag{15}$$

$$\Gamma_{SHL}^{n+1} \equiv \frac{HA_{SHL}}{C_{pf}(h_{sv}^n - h_f^n)} (h_f^{n+1} - h_{sl}^{n+1}) = \Gamma_2 (h_f^{n+1} - h_{sl}^{n+1}) \quad (h_f^n > h_{sl}^n) \tag{16}$$

$$\Gamma_{SHV}^{n+1} \equiv \frac{HA_{SHV}}{C_{pv}(h_{sv}^n - h_f^n)} (h_v^{n+1} - h_{sv}^{n+1}) = \Gamma_3 (h_v^{n+1} - h_{sv}^{n+1}) \quad (h_v^n > h_{sv}^n) \tag{17}$$

$$\Gamma_{SCV}^{n+1} \equiv \frac{HA_{SCV}}{C_{pv}(h_v^n - h_{sl}^n)} (h_v^{n+1} - h_{sv}^{n+1}) = \Gamma_4 (h_v^{n+1} - h_{sv}^{n+1}) \quad (h_v^n < h_{sv}^n) \tag{18}$$

$$\Gamma_{12} \equiv \Gamma_1 + \Gamma_2 \qquad \Gamma_{34} \equiv \Gamma_3 + \Gamma_4 \qquad \Gamma_S \equiv \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4$$

$$\Gamma_P \equiv [(\Gamma_3 + \Gamma_4) \frac{\partial h_{sv}}{\partial P_v} + (\Gamma_2 + \Gamma_1) \frac{\partial h_{sf}}{\partial P_v}]$$

$$\Gamma^* \equiv \Gamma^*_{SHV} + \Gamma^*_{SCV} + \Gamma^*_{SHL} + \Gamma^*_{SCL}$$

$$\Gamma^{*+1} = \Gamma^* - \Gamma_P \delta P^{*+1} + \Gamma_P \delta P_g^{*+1} + \Gamma_{12} \delta h_f^{*+1} + \Gamma_{34} \delta h_v^{*+1} \tag{19}$$

The temporal terms are linearized as follows,

$$(\alpha_f \rho_f)^{*+1} = \alpha_f^* \rho_f^* + \delta \alpha_f^{*+1} \rho_f^* + \alpha_f^* \delta \rho_f^{*+1}$$

$$= \alpha_f^* \rho_f^* + \alpha_f^* \frac{\partial \rho_f}{\partial P} \delta P^{*+1} + \alpha_f^* \frac{\partial \rho_f}{\partial h_f} \delta h_f^{*+1} - \rho_f^* \delta \alpha_g^{*+1} - \rho_f^* \delta \alpha_e^{*+1} \tag{20}$$

$$(\alpha_e \rho_f)^{*+1} = \alpha_e^* \rho_f^* + \delta \alpha_e^{*+1} \rho_f^* + \alpha_e^* \delta \rho_f^{*+1}$$

$$= \alpha_e^* \rho_f^* + \alpha_e^* \frac{\partial \rho_f}{\partial P} \delta P^{*+1} + \alpha_e^* \frac{\partial \rho_f}{\partial h_f} \delta h_f^{*+1} + \rho_f^* \delta \alpha_e^{*+1} \tag{21}$$

$$(\alpha_g \rho_v)^{*+1} = \alpha_e^* \rho_f^* + \delta \alpha_g^{*+1} \rho_v^* + \alpha_g^* \delta \rho_v^{*+1}$$

$$= \alpha_g^* \rho_v^* + \alpha_g^* \frac{\partial \rho_v}{\partial P_v} \delta P^{*+1} - \alpha_g^* \frac{\partial \rho_v}{\partial P_v} \delta P_g^{*+1} + \alpha_g^* \frac{\partial \rho_v}{\partial h_v} \delta h_v^{*+1} + \rho_v^* \delta \alpha_g^{*+1} \tag{22}$$

$$(\alpha_g \rho_n)^{*+1} = \alpha_g^* \rho_g^* + \alpha_g^{*+1} \rho_n^* + \alpha_g^* \rho_n^{*+1}$$

$$= \alpha_g^* \rho_g^* + \alpha_g^* \frac{\partial \rho_n}{\partial P_g} \delta P_g^{*+1} + \alpha_g^* \frac{\partial \rho_n}{\partial h_n} \frac{\partial h_n}{\partial h_v} \delta h_v^{*+1} + \rho_n^* \delta \alpha_g^{*+1} \tag{23}$$

The expansion of the temporal and phase exchange terms can be written as follows,

$$A_{c_j} \frac{\delta x_J}{\delta t} [(\alpha_f \rho_f)^{n+1} - (\alpha_f \rho_f)^n] + (1 - \eta) \Gamma^{n+1}$$

$$= A_{c_j} \frac{\delta x_J}{\delta t} [\alpha_f^n \frac{\partial \rho_f}{\partial P} \delta P^{n+1} + \alpha_f^n \frac{\partial \rho_f}{\partial h_f} \delta h_f^{n+1} - \rho_f^n \delta \alpha_g^{n+1} - \rho_f^n \delta \alpha_e^{n+1}]$$

$$+ (1 - \eta) [- \Gamma_P \delta P^{*+1} + \Gamma_P \delta P_g^{*+1} + \Gamma_{12} \delta h_f^{*+1} + \Gamma_{34} \delta h_v^{*+1}] + (1 - \eta) \Gamma^* \tag{24}$$

$$A_{c_j} \frac{\delta x_J}{\delta t} [(\alpha_e \rho_f)^{n+1} - (\alpha_e \rho_f)^n] + \eta \Gamma^{n+1}$$

$$= A_{c_j} \frac{\delta x_J}{\delta t} [\alpha_e^n \frac{\partial \rho_f}{\partial P} \delta P^{n+1} + \alpha_e^n \frac{\partial \rho_f}{\partial h_f} \delta h_f^{n+1} + \rho_f^n \delta \alpha_e^{n+1}]$$

$$+ \eta [- \Gamma_P \delta P^{*+1} + \Gamma_P \delta P_g^{*+1} + \Gamma_{12} \delta h_f^{*+1} + \Gamma_{34} \delta h_v^{*+1}] + \eta \Gamma^* \tag{25}$$

$$A_{c_j}\frac{\delta x_J}{\delta t}\left[(\alpha_g\rho_v)^{n+1}-(\alpha_g\rho_v)^n\right]-\Gamma^{n+1}$$

$$=A_{c_j}\frac{\delta x_J}{\delta t}\left[\alpha_g^n\frac{\partial\rho_v}{\partial P_v}\delta P^{n+1}-\alpha_g^n\frac{\partial\rho_v}{\partial P_v}\delta P_g^{n+1}+\alpha_g^n\frac{\partial\rho_v}{\partial h_v}\delta h_v^{n+1}+\rho_v^n\delta\alpha_g^{n+1}\right]$$

$$-\left[-\Gamma_P\delta P^{n+1}+\Gamma_P\delta P_g^{n+1}+\Gamma_{12}\delta h_f^{n+1}+\Gamma_{34}\delta h_v^{n+1}\right]-\Gamma^n \quad (26)$$

$$A_{c_j}\frac{\delta x_J}{\delta t}\left[(\alpha_g\rho_n)^{n+1}-(\alpha_g\rho_n)^n\right]$$

$$=A_{c_j}\frac{\delta x_J}{\delta t}\left[\alpha_g^n\frac{\partial\rho_n}{\partial P_g}\delta P_g^{n+1}+\alpha_g^n\frac{\partial\rho_n}{\partial h_n}\frac{\partial h_n}{\partial h_v}\delta h_v^{n+1}+\rho_n^n\delta\alpha_g^{n+1}\right] \quad (27)$$

A typical convective terms can be expanded with the new time velocities derived from the momentum equations,

$$\sum_{KA=1}^{NA}\left[A_{m_j}U_{\phi_j}^{n+1}(\alpha_\phi\rho_\phi)^{*n}\right]_{KA}$$

$$=\sum_{KA=1}^{NA}\left[A_{m_j}\left(\frac{\partial U_{\phi j}}{\partial P}(\delta P_{J+1}^{n+1}-\delta P_J^{n+1})+U_{\phi j}^{n+}\right)(\alpha_\phi\rho_\phi)^{*n}\right]_{KA}$$

$$=\sum_{KA=1}^{NA}\left[A_{m_j}(\alpha_\phi\rho_\phi)^{*n}\right]_{KA}\frac{\partial U_{\phi j}}{\partial P}\delta P_{J+1}^{n+1}$$

$$-\sum_{KA=1}^{NA}\left[A_{m_j}(\alpha_\phi\rho_\phi)^{*n}\right]_{KA}\frac{\partial U_{\phi j}}{\partial P}\delta P_J^{n+1}$$

$$+\sum_{KA=1}^{NA}\left[A_{m_j}(\alpha_\phi\rho_\phi)^{*n}\right]_{KA}U_{\phi j}^{n+} \quad (28)$$

2.2.3 Energy conservation equations

The energy conservation equations are written as follows,

$$A_{C_j}\delta x_J\frac{(\alpha_\phi\rho_\phi h_\phi)_J^{n+1}-(\alpha_\phi\rho_\phi h_\phi)_J^n}{\delta t}$$

$$=\sum_{KB=1}^{NB}\left[A_{m_{j-1}}U_{\phi_{j-1}}^{n+1}(\alpha_\phi\rho_\phi h_\phi)^{n*}\right]_{KB}-\sum_{KA=1}^{NA}\left[A_{m_j}U_{\phi_j}^{n+1}(\alpha_\phi\rho_\phi h_\phi)^{n*}\right]_{KA}$$

$$+\Gamma_{E\phi}^{n+1}+q_\phi^{n+1}+Q_\phi \quad (29)$$

The interfacial heat transfer can be written as follows,

$$\begin{aligned} q_v^{n+1} &\equiv HA_{SHV}(T_s^{n+1}-T_v^{n+1})+HA_{SCV}(T_s^{n+1}-T_v^{n+1}) \\ &= \frac{HA_{SHV}(h_{sv}^{n+1}-h_v^{n+1})}{C_{pv}}+\frac{HA_{SCV}(h_{sv}^{n+1}-h_v^{n+1})}{C_{pv}} \\ &= q_{SHV}^{n+1}+q_{SCV}^{n+1} \end{aligned} \quad (30)$$

$$q_{SHV}^{n+1}\equiv\frac{HA_{SHV}(h_{sv}^{n+1}-h_v^{n+1})}{C_{pv}} \qquad q_{SCV}^{n+1}\equiv\frac{HA_{SCV}(h_{sv}^{n+1}-h_v^{n+1})}{C_{pv}}$$

$$q_{SHV}^{n+1} = -\Gamma_{SHV}(h_{sv}^{n} - h_f^{n}) \quad q_{SCV} = -\Gamma_{SCV}(h_v^{n} - h_{sf}^{n})$$

$$\begin{aligned} q_l^{n+1} &\equiv HA_{SHL}(T_s^{n+1} - T_l^{n+1}) + HA_{SCL}(T_s^{n+1} - T_l^{n+1}) \\ &= \frac{HA_{SHL}(h_{sf}^{n+1} - h_f^{n+1})}{C_{pf}} + \frac{HA_{SCL}(h_{sf}^{n+1} - h_f^{n+1})}{C_{pf}} \\ &= q_{SHL}^{n+1} + q_{SCL}^{n+1} \end{aligned} \tag{31}$$

$$q_{SHL}^{n+1} \equiv \frac{HA_{SHL}(h_{sf}^{n+1} - h_f^{n+1})}{C_{pv}} \quad q_{SCL}^{n+1} \equiv \frac{HA_{SCL}(h_{sf}^{n+1} - h_f^{n+1})}{C_{pv}}$$

$$q_{SHL}^{n+1} = -\Gamma_{SHL}^{n+1}(h_{sv}^{n} - h_f^{n}) \quad q_{SCL}^{n+1} = -\Gamma_{SCL}^{n+1}(h_v^{n} - h_{sf}^{n})$$

The energy partitioning can be confirmed as follows,

$$\Gamma_{Ev}^{n+1} \equiv \Gamma_{SHV}^{n+1} h_{sv} + \Gamma_{SCV}^{n+1} h_v + \Gamma_{SHL}^{n+1} h_{sv} + \Gamma_{SCL}^{n+1} h_v \tag{32}$$

$$\Gamma_{El}^{n+1} \equiv \Gamma_{SHV}^{n+1} h_f + \Gamma_{SCV}^{n+1} h_{sf} + \Gamma_{SHL}^{n+1} h_f + \Gamma_{SCL}^{n+1} h_{sf} \tag{33}$$

$$\begin{aligned} & q_v^{n+1} + \Gamma_{Ev}^{n+1} \\ &= -\Gamma_{SHV}^{n+1}(h_{sv}^{n} - h_f^{n}) - \Gamma_{SCV}^{n+1}(h_v^{n} - h_{sf}^{n}) + \Gamma_{SHV}^{n+1} h_{sv} + \Gamma_{SCV}^{n+1} h_v + \Gamma_{SHL}^{n+1} h_{sv} + \Gamma_{SCL}^{n+1} h_v \\ &= \Gamma_{SHV}^{n+1} h_f^{n} + \Gamma_{SCV}^{n+1} h_{sf}^{n} + \Gamma_{SHL}^{n+1} h_{sv} + \Gamma_{SCL}^{n+1} h_v \end{aligned} \tag{34}$$

$$\begin{aligned} & q_l^{n+1} - \Gamma_{El}^{n+1} \\ &= -\Gamma_{SHL}^{n+1}(h_{sv}^{n} - h_f^{n}) - \Gamma_{SCL}^{n+1}(h_v^{n} - h_{sf}^{n}) - \Gamma_{SHV}^{n+1} h_f - \Gamma_{SCV}^{n+1} h_{sf} - \Gamma_{SHL}^{n+1} h_f - \Gamma_{SCL}^{n+1} h_{sf} \\ &= -\Gamma_{SHV}^{n+1} h_f - \Gamma_{SCV}^{n+1} h_{sf} - \Gamma_{SHL}^{n+1} h_{sv}^{n} - \Gamma_{SCL}^{n+1} h_v^{n} \end{aligned} \tag{35}$$

$$q_v^{n+1} + \Gamma_{Ev}^{n+1} + q_l^{n+1} - \Gamma_{El}^{n+1} = 0 \tag{36}$$

Thus, the energy partitioning is confirmed.

$$\Gamma_{Sh} \equiv \Gamma_{SHV}^{n} h_f^{n} + \Gamma_{SCV}^{n} h_{sf}^{n} + \Gamma_{SHL}^{n} h_{sv} + \Gamma_{SCL}^{n} h_v$$

$$\Gamma_{Ph} \equiv [(\Gamma_3 h_f^{n} + \Gamma_4 h_{sf}^{n}) \frac{\partial h_{sv}}{\partial P_v} + (\Gamma_2 h_{sv}^{n} + \Gamma_1 h_v^{n}) \frac{\partial h_{sf}}{\partial P_v}]$$

$$\Gamma_{34h} \equiv (\Gamma_3 h_f^{n} + \Gamma_4 h_{sf}^{n})$$

$$\Gamma_{12h} \equiv (\Gamma_2 h_{sv}^{n} + \Gamma_1 h_v^{n})$$

With the above definitions, the energy transfer rate through the interfacial heat conduction and mas transfer can be expressed as follows,

$$q_v^{n+1} + \Gamma_{Ev}^{n+1} = \Gamma_{Sh} - \Gamma_{Ph} \delta P^{n+1} + \Gamma_{Ph} \delta P_g^{n+1} + \Gamma_{34h} \delta h_v^{n+1} + \Gamma_{12h} \delta h_f^{n+1} \tag{37}$$

The temporal and phase change terms can be expanded using the above informations and similar method used in the mass conservation equations. The results are shown below for each phase.

$$
\begin{aligned}
&A_{C_j}\delta x_J \frac{\partial \alpha_g(\rho_v h_v + \rho_n h_n)_J}{\partial t} - \Gamma_{Ev}^{n+1} - q_v^{n+1} \\
&\quad = A_{C_j}\frac{\delta x_J}{\delta t}\ [(\rho_v h_v + \rho_n h_n)^n \delta\alpha_g^{n+1} + \alpha_g^n h_v^n \frac{\partial \rho_v}{\partial P_v}\delta P^{n+1} \\
&\quad + \alpha_g^n (h_v^n \frac{\partial \rho_v}{\partial h_v} + \rho_v^n + h_n^n \frac{\partial \rho_n}{\partial h_n}\frac{\partial h_n}{\partial h_v} + \rho_n^n \frac{\partial h_n}{\partial h_v})\delta h_v^{n+1} \\
&\quad + \alpha_g^n (h_n^n \frac{\partial \rho_n}{\partial P_g} - h_v^n \frac{\partial \rho_v}{\partial P_v})\delta P_g^{n+1}]_J \\
&\quad + \Gamma_{Ph}\delta P^{\kappa+1} - \Gamma_{Ph}\delta P_g^{\kappa+1} - \Gamma_{34h}\delta h_v^{\kappa+1} - \Gamma_{12h}\delta h_f^{\kappa+1} - \Gamma_{Sh} \qquad (38)
\end{aligned}
$$

$$
\begin{aligned}
&A_{C_j}\delta x_J \frac{\partial(\alpha_f + \alpha_e)\rho_f h_{f_J}}{\partial t} - q_l^{n+1} + \Gamma_{El}^{n+1} \\
&\quad = A_{C_j}\frac{\delta x_J}{\delta t}[-\rho_f^n h_f^n \delta\alpha_g^{n+1} + (1-\alpha_g^n)h_f^n \frac{\partial \rho_f^n}{\partial P}\delta P^{n+1}]_J \\
&\quad + A_{C_j}\frac{\delta x_J}{\delta t}[(1-\alpha_g^n)(h_f^n \frac{\partial \rho_f^n}{\partial h_f} + \rho_f^n)\delta h_f^{n+1}]_J \\
&\quad - \Gamma_{Ph}\delta P^{\kappa+1} + \Gamma_{Ph}\delta P_g^{\kappa+1} + \Gamma_{34h}\delta h_v^{\kappa+1} + \Gamma_{12h}\delta h_f^{\kappa+1} + \Gamma_{Sh} \qquad (39)
\end{aligned}
$$

A typical energy convective terms can be expanded by substituting the new time velocities derived from the momentum equations,

$$
\begin{aligned}
&\sum_{KA=1}^{NA}[A_{m_j} U_{g_j}^{n+1}(\alpha_g(\rho_v h_v + \rho_n h_n))^{*n}]_{KA} \\
&\quad = \sum_{KA=1}^{NA}[A_{m_j}(\alpha_g(\rho_v h_v + \rho_n h_n))^{*n}]_{KA}\frac{\partial U_{gi}}{\partial P}\delta P_{J+1}^{n+1} \\
&\quad - \sum_{KA=1}^{NA}[A_{m_j}(\alpha_g(\rho_v h_v + \rho_n h_n))^{*n}]_{KA}\frac{\partial U_{gi}}{\partial P}\delta P_J^{n+1} \\
&\quad + \sum_{KA=1}^{NA}[A_{m_j}(\alpha_g(\rho_v h_v + \rho_n h_n))^{*n}]_{KA} U_{gi}^{n+} \qquad (40)
\end{aligned}
$$

2.2.4 Cell and System matrix treatment

Collecting the incremental terms of dependent variables to the left side of the equation, the following cell matrix equation is obtained,

$$
\begin{bmatrix}
x & x & 0 & x & 0 & 0 \dots x & x \\
x & x & 0 & x & 0 & x \dots x & x \\
0 & 0 & x & 0 & x & x \dots x & x \\
x & x & 0 & x & 0 & x \dots x & x \\
x & x & 0 & x & x & x \dots x & x \\
0 & x & x & 0 & x & x \dots x & x
\end{bmatrix}
\begin{bmatrix}
\delta P_g \\ \delta\alpha_v \\ \delta\alpha_e \\ \delta h_v \\ \delta h_l \\ \delta P_J \\ \cdot \\ \cdot \\ \delta P_{J+1}
\end{bmatrix}
=
\begin{bmatrix}
RC_g \\ RC_v \\ RC_e \\ RE_{vg} \\ RE_l \\ RC_l
\end{bmatrix}
\qquad (41)
$$

This matrix can be solved for $\delta P_J, \delta P_{J+1}$. This procedure is repeated for all cells and the system matrix is constructed by collecting the pressure equations from the individual cells. Solving the system matrix with a sparse matrix solver, pressure increments of cells can be obtained. The new time dependent variables are obtained by back-substituting the pressure increments to the corresponding equations.

3. Numerical Tests

3.1 Connection tests

The code structure of GUARD is similar to that of COBRA-TF. It has the concepts such as channels, gaps and sections. Howeve, there are several concepts that are not found in COBRA-TF. Two types of vertical connections are intoduced in GUARD. The first type is the internal junction that connects between internal cell faces. The second one is the external junction that connects the cell faces of the different channels. Channels in different sections should be connected by the external junctions. Boundary nodes and junctions are newly defined in GUARD, that are very similar to the time dependent volumes and junctions of RELAP5. As pointed out by Lee[11], the application of COBRA-TF is limited to the analysis of the vertical hydraulic structures. In other words, it is very difficult to model the horizontal pipe with COBRA-TF. Such limitations can be avoided by introducing the horizontal channel with some provisions to connect it to the vertical or another horizontal channel. 'CONNECTION' is provided for this purpose. Several simple geometrical test cases are constructed and utilized for validation of those concepts of connections. One of them is shown in Figure-1. Newly introduced concepts, external junction, connection and boundary channel are proved to be properly working.

3.2 Convergence and stability tests

A simple pipe, as shown in Figure-2, is used to check the convergence and stability of the employed numerical scheme. A wide variety of initial and boundary conditions are tested with this simple problem. The ranges of tested parameters are found in Table-1.

While GUARD shows a stable behavior for almost all the ranges of parameters, it fails to converge when the initial value of the channel entrained liquid fraction is nealy zero and the gas velocity is relatively high in a vertical pipe. Compared with COBRA-TF, GUARD, in general, shows a slower convergence and a weaker stability.

This convergence problem may be relaxed if the void fraction weighted dependent variables are used. In COBRA-TF, $A_{mj}\alpha_\phi\rho_\phi U_\phi$, $\alpha_g P_g$, $\alpha_g h_v$ and $\alpha_{le} h_l$ are adopted as dependent variables instead of U_ϕ, P_g, h_v and h_l in GUARD. Adopting the volume fraction weighted dependent variables makes the code work better when phases appear or disappear. However, the formulation of the numerical scheme gets more difficult because more involved derivatives are to be handled.

4. Conclusions

A formulation of the semi-implicit multi-dimensional three-fluid hydraulic solver is presented. Although this scheme works successfully in the wide range of two-phase regime, it shows some numerical difficulties when the droplet volume fraction starts growing from zero in the upward co-current flow in a vertical pipe. This convergence problem may be relaxed if the void fraction weighted dependent variables are used.

Nomenclature

α; volume fraction — ρ; density — U; velocity

Γ ; vaporization — S; entrainment source — h; enthalpy

g; gravitaional constant — P; pressure — τ ; shear force

A; area — H; heat transfer area — x,t; distance and time

T ; temperature — C_p ; specific heat — Q ; wall heat

ch; channel — bch;boundary channel — c ; cell

C_{xv} ; coefficients in velocity equations

q ; interfacial heat transfer rate

RE ; residual of energy equations

RC ; residual of continuity equations

K ; friction and form loss factor

superscripts

''' ; volumetric source — NA ; total number of connections

* ; donner property

subscripts

g ; gas — l ; liquid — e; entrained liquid

v; vapor — f; liquid — i; interface

ϕ ; phase parameters(g=gas, l=liquid, le=liquid and drop, v=vapor...)

KA or KB; connection index — J ; node J — SHV ; super heated vapor

SHL ; super heated liquid — SCV ; subcooled vapor — SCL ; sucooled liquid

sv ; saturated vapor — sf ; saturated liquid — m ; momentum

C ; continuity — E ; energy

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Table-1. test parameter ranges

parameters	ranges
reference pressures	50-2500 psia
pressure difference	0-100 psid
inlet void fraction	0-1.0
channel void fraction	0-1.0
flow direction	vertical, horizontal
number of nodes	1-100

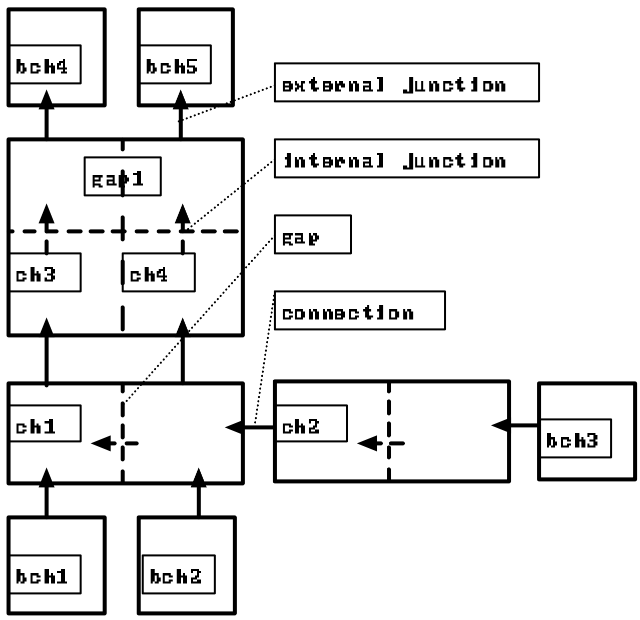

Figure-1. Connection test problem

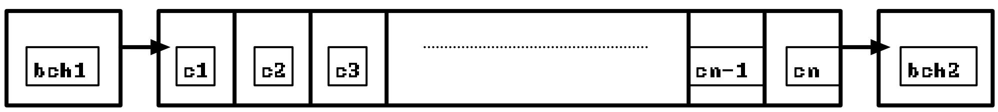

Figure-2. Convergence and stability test ptoblem