

The Robust Level Control System of the Nuclear Steam Generator

Jong Kwan Hyun, Sin Kim, Yoon Joon Lee

Cheju National University
Department of Nuclear and Energy Engineering,
Ara 1-dong, Cheju City, 690-756

Abstract

The nuclear steam generator level control system is designed by the robust control methods. The design is divided into two steps. First, the feedwater controller is designed by the H_∞ . Then the feedwater controller located on the feedback loop is designed both by a classical PID and by robust techniques. It is found that the controller of simple PID whose coefficients vary with the power is proper for the system performance. The simulations show that the hybrid system of H_∞ and PID has a good performance with proper stability margins.

1. Introduction

The nuclear steam generator has a number of problems in the light of control design. These control problems directly arise from the physical characteristics of the steam generator. The mechanism of the steam generator is based on the thermal-hydraulic phenomena of heat transfer and fluid dynamics. The mathematical modeling of the thermal-hydraulic system is very difficult. It has many intrinsic uncertainties, no matter how exactly it may be modelled. This is mainly due to the theoretical assumptions, linearizations, and experimental correlations. Further, the dynamics of the working fluid gives an additional uncertainty.

The heat transfer mechanism of the steam generator results in the shrink and swell effects. These effects are addressed by the control terminology of non-minimum phase. The control design of the non-minimum phase plant is more difficult than the unstable plant. The effect of the non-minimum phase becomes more salient, resulting in the difficulty in level control as the power becomes lower. This is due to the fact that the plant properties vary with the operation power, which imposes another problem on the control design.

The robust control method could be an alternative to the design of the steam generator level control system. The actual system should work as intended under the

real circumstances even though it is designed with the inexact plant. The ultimate purpose of the control system is to maintain the robustness rather than stability. However, it should be noted that too much stress on the robustness may result in the performance degrades. In the control design, no method can definitely be the best, and compromises between various methods are required in accordance with the system characteristics. In this study, The robust control is applied to the feedwater controller design. For the feedback level controller, the robust method are applied, followed by the variable PID controller to be compared each other. Through the simulation it is found that the hybrid system which is comprised of robust feedwater controller and PID level controller gives the sufficient stability and good performance.

2 The Steam Generator Level Control System

Figure 1 shows the steam generator level control system. The overall system is a kind of regulating system in that the level variation should be kept constant. The steam flow rate change and other feedbacked signals generate a driving signal which controls the feedwater flow rate to keep the level constant. The feedwater station is a servo system in which the feedwater flow rate follows the steam flow rate.

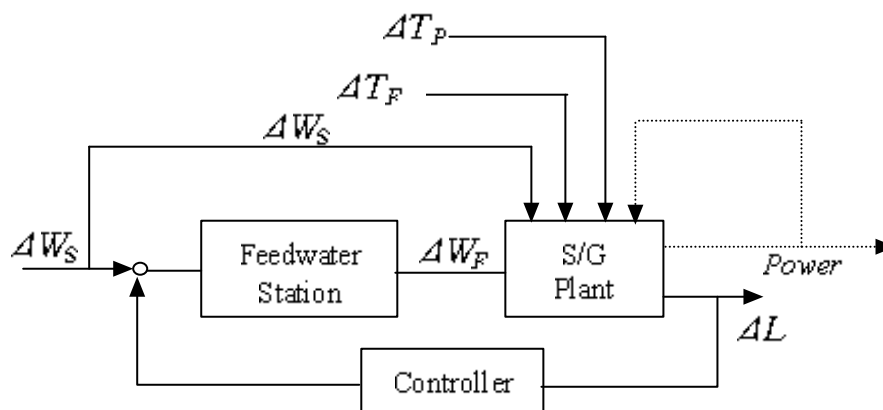


Figure 1. Steam Generator Level Control System

The input and output of the steam generator plant are the feedwater flow rate change (ΔW_F) and level variation (ΔL), respectively. Several noises act on the plant. They are changes of primary coolant temperature (ΔT_P), feedwater temperature (ΔT_F). Also it should be noted that the steam flow rate change (ΔW_S) is not only a command signal to the system but also is a disturbance to the steam generator. Therefore, the relationship between these inputs and the level, should be identified.

Lee[1],[2] developed the MIMO (multi-input, multi-output) transfer functions of the

steam generator from the thermal-hydraulic code which describes the steam generator dynamics in detail. They describe the property changes of the plant. With these open loop transfer functions, the steam generator control system could be put into the block diagram of Fig. 2.

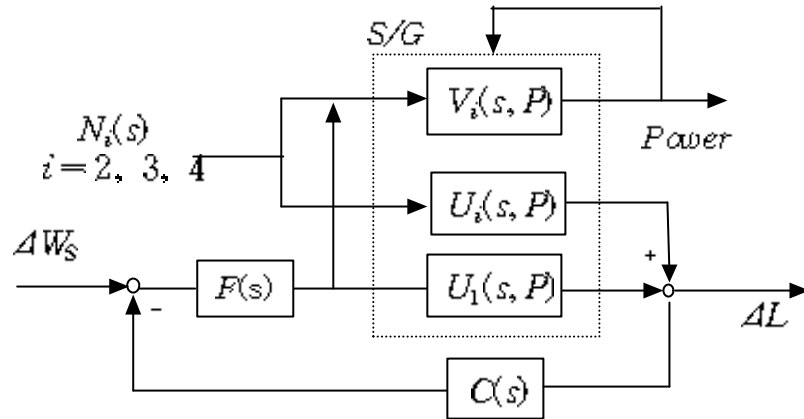


Figure 2. The Block Diagram of the Overall Level Control System

In the figure, $U_i(s, P)$, $i=1, 2, 3, 4$, are open loop transfer functions between the level and each element of input vector of $N_i(s) = [\Delta W_f(s), \Delta W_s(s), \Delta T_p(s), \Delta T_f(s)]$ respectively, and $V_i(s, P)$, $i=1, 2, 3, 4$, are open loop transfer functions between the percent power and the input vector. The feedwater station is represented as a single block $F(s)$. The characteristics of this system can be summarized as

- 1) the plant is dependent on its output, that is, the percent power, P
- 2) the system is comprised of the open loop for power and the closed loop for level
- 3) for the power loop train, all the input vector elements act as system inputs, and for the level train, $N_i(s)$, $i=2, 3, 4$ acts on the system as disturbances
- 4) the system is MIMO. And one of the system output is to be tracked, the other is to be regulated.

The overall control system design can be divided into two steps of the feedwater controller design and the feedback loop controller design.

3. Feedwater Controller Design

3.1 H_∞ controller

Since the feedwater control system is a servo system, at least one integrator is necessary. The valve station is assumed to be a first order lagged of time constant 1 sec. The rationales for this assumption are explained in Ref. [2]. Then the feedwater station of Fig. 1 could be recast into Fig. 3.

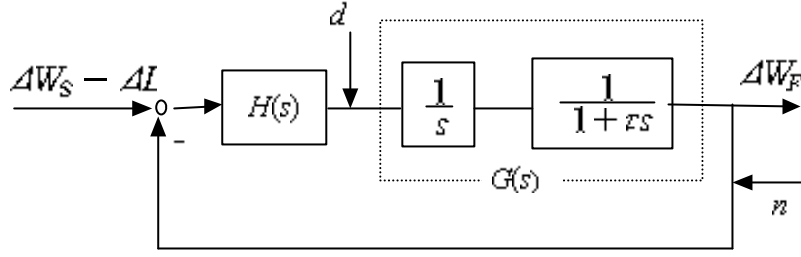


Figure 3. Block Diagram of Feedwater Station

The feedwater system design is to find out the robust controller $H(s)$ in Fig. 3. For the robust design, Fig. 3 is reconstructed as the two-port model of Fig.4.

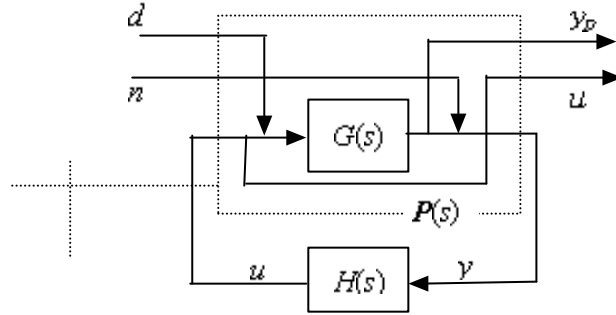


Figure 4. Two-Port Model of Feedwater Control System

The system equations are posed as

$$\begin{aligned}
 \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}_1\mathbf{w} + \mathbf{B}_2u \\
 \mathbf{z} &= \mathbf{C}_1\mathbf{x} + \mathbf{D}_{11}\mathbf{w} + \mathbf{D}_{12}u \\
 y &= \mathbf{C}_2\mathbf{x} + \mathbf{D}_{21}\mathbf{w} + \mathbf{D}_{22}u
 \end{aligned} \tag{1}$$

where \mathbf{A} , \mathbf{B}_2 , \mathbf{C}_2 and \mathbf{D}_{22} are system matrices of $G(s)$, $\mathbf{x} = (x_1, x_2)^T$ is the state variables vector, \mathbf{w} is the input vector of $(d \ n)^T$ and \mathbf{z} is the regulated output vector of $(y_p \ u)^T$.

The packed matrix of Eq. (1) is

$$\mathbf{P}(s) = \begin{pmatrix} \mathbf{A} & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{C}_1 & \mathbf{D}_{11} & \mathbf{D}_{12} \\ \mathbf{C}_2 & \mathbf{D}_{21} & \mathbf{D}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{P}_{11} & \mathbf{P}_{12} \\ \mathbf{P}_{21} & \mathbf{P}_{22} \end{pmatrix} = \begin{pmatrix} -1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & -1 & 0 \end{pmatrix} \tag{2}$$

This packed matrix is obtained with the assumption that the disturbance acts on the state variable x_1 . Eq. (1) satisfies all the conditions for the existence of Riccati solutions. And the design of H_∞ controller is to find out the admissible controller

$H(s)$ which makes the infinity norm of the overall closed loop system have a certain upper bound. That is,

$$\| \mathbf{T}_{\infty} \|_{\infty} = F_L(\mathbf{P}, \mathbf{H}) < \gamma \quad (3)$$

where $F_L(\mathbf{P}, \mathbf{H})$ is the LFT (linear fractional transformation) of the system which is defined as $\mathbf{P}_{11} + \mathbf{P}_{12}\mathbf{H}(\mathbf{I} - \mathbf{P}_{22}\mathbf{H})^{-1}\mathbf{P}_{21}$

There are many reliable algorithms to calculate the controller[3],[4] and the controller is found to be

$$H_{\infty,1}(s) = \frac{5.414 \times 10^4 s + 5.899 \times 10^4}{s^2 + 3.138 \times 10^4 s + 6.758 \times 10^4} \quad (4)$$

with $\gamma = 1.7253$

The controller of Eq. (4) gives the PM (phase margin) of 67.4° and the GM (gain margin) of 91.5dB, which is sufficient to keep the system robustness. The regulated outputs of (y_p, z) converge rapidly to the steady state values, which shows the good robustness. Since the system \mathbf{T}_{∞} is MIMO, it has two singular values, and the infinity norm of the system is 1.

The controller $H_{\infty,1}(s)$ is determined with the assumption that the disturbance acts on x_1 . But it is possible to configure the system in such a way that the disturbance acts on x_2 . In this case, the packed matrix has different elements of $\mathbf{B}_1 = [0 \ 1]^T$ and the controller is

$$H_{\infty,2}(s) = \frac{1.671 \times 10^4 s + 1.671 \times 10^4}{s^2 + 9090 s + 2.166 \times 10^4} \gamma = 1.8392 \quad (5)$$

This controller gives the GM of 81.4 dB and PM of 72.8° .

It is informative to compare H_{∞} controllers with LTR (loop transfer recovery) controller[2]. The H_{∞} problem and LQG (linear quadratic Gaussian) problem have the same paradigm in that both problems are posed as a couple of Riccati equations. The difference between them is the norm used in the performance function. In the LQG, the performance function is the two-norm of output variance which is augmented by the state variable weighting matrix and control effort weighting matrix. The LQG problem is to find the stable controller which minimizes the two-norm of the system, and can be set as

$$\underset{\text{Stable } H(s)}{\text{Min}} \| \mathbf{T}_{\infty} \|_2 \quad (6)$$

The LQG, which incorporates the observer, does not guarantee the margins of the LQR (linear quadratic regulation). However, Doyle[12] showed that the margins of LQR can be recovered with the LTR of

$$\lim_{q \rightarrow \infty} \mathbf{M}(s)_{LQC} = \mathbf{M}(s)_{LQR} \quad (7)$$

where $\mathbf{M}(s)_{LQR} = \mathbf{K}\Phi\mathbf{B}$ $\mathbf{M}(s)_{LQC} = \mathbf{K}\Phi_r(s)\mathbf{L}\mathbf{C}\Phi(s)\mathbf{B}\Phi = (s\mathbf{I} - \mathbf{A})^{-1}$

$$\Phi_r(s) = (s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K} + \mathbf{L}\mathbf{C})^{-1}\mathbf{E}(\mathbf{w}\mathbf{w}^T) = \mathbf{Q}_0 = q^2\mathbf{B}\mathbf{B}^T$$

To obtain the target loop of $\mathbf{M}(s)_{LQR}$, the feedwater servo system is converted to a regulating system by the transformation of

$$\begin{aligned} \dot{\xi} &= \mathbf{A}\xi + \mathbf{B}\mathbf{w}\zeta = \mathbf{C}\xi + \mathbf{D}\mathbf{w} \quad \mathbf{w} = -\mathbf{K}\xi \\ \mathbf{A} &= \begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix} \quad \mathbf{C} = (c, \quad 0) \quad \mathbf{D} = 0 \end{aligned} \quad (8)$$

where (a, b, c) is the system matrix of the first order valve station.

From Eq.(8), the integrator gain and feedback gain are found to be [1.0 0.7321], and the LQR has the target PM of 81° . With this target loop, and by controlling the noise spectral density of $q^2\mathbf{B}\mathbf{B}^T$, where $\mathbf{B} = [b \ 0]^T$, the LTR controller is determined as Eq. (9) and has the PM of 77° and GM of 28 dB.

$$H(s)_{LTR} = \frac{114.1884s + 150}{s^2 + 18.0814s + 162.97} \quad (9)$$

Figure 5 shows the unit step responses of the feedwater system incorporated with each controller designed so far. Comparing $H_{\infty,1}(s)$ with $H_{\infty,2}(s)$, it can be known that the speeds of the both are almost the same, but $H_{\infty,2}(s)$ gives the shorter settling time than $H_{\infty,1}(s)$. Also there is no overshooting for the case of $H_{\infty,2}(s)$. The $H(s)_{LTR}$ seems to be superior to $H_{\infty,1}(s)$ or $H_{\infty,2}(s)$ in the speed and settling time. But the control effort of the $H(s)_{LTR}$ is much larger than those of H_{∞} controllers, which is not desirable with respect to the actuator movement. Accordingly, $H_{\infty,2}(s)$ is adopted as a finally designed controller.

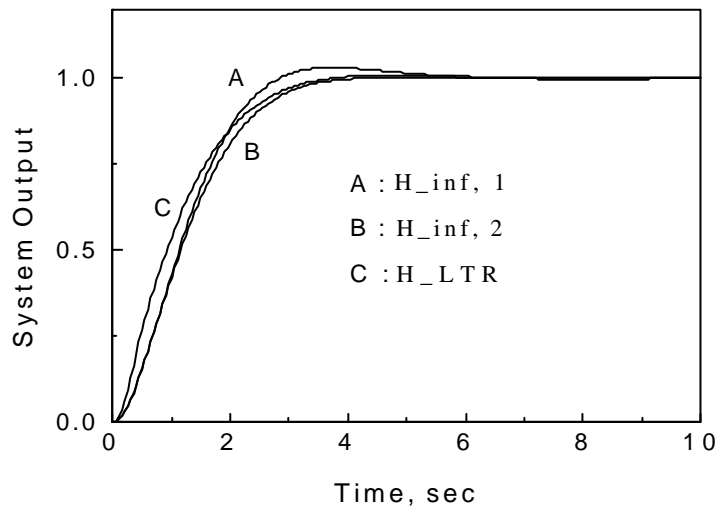


Figure 5. Unit Step Responses of Feedwater Station for Various Controllers

3.2 Robust Controller by MWS

The robust controller can be designed by other offsprings of the H_∞ control algorithms. One of them is the MWS (mixed weight sensitivity) based on the classical loop shaping[5],[6]. The unity feedback system of Fig. 3 can be described by the two-port model of Fig. 6 with the augmented weights.

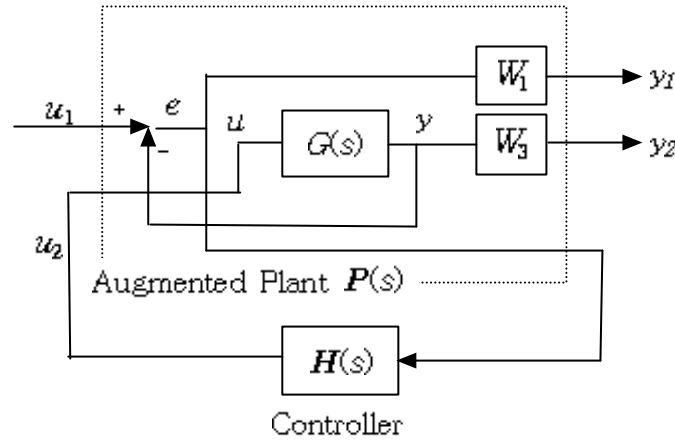


Figure 6. Two-Port Model with Augmented Weights

Considering the external command signal only, the system transfer function is the SIMO(single input, multiple output) of

$$\mathbf{T} = \frac{(y_1 \ y_2)^T}{u_1} = \begin{pmatrix} W_1 S \\ W_3 T \end{pmatrix} \quad (10)$$

where S and T denotes the sensitivity and complementary sensitivity, respectively.

The MWS problem is to determine the stable controller which makes the infinity norm of the closed loop system minimum by selecting proper weighting functions. W_1 and W_3 are functions of frequency, and they are major design factors in MWS. For the desirable loop shapes, they should be determined to satisfy the following conditions.

$$\bar{\sigma}(S(j\omega)) \leq |W_1^{-1}(j\omega)| \quad \bar{\sigma}(T(j\omega)) \leq |W_3^{-1}(j\omega)| \quad \forall \omega \quad (11)$$

where $\bar{\sigma}(\cdot)$ is the singular value.

The MWS requires numerous iterations because the designed system is sensitively related to the selection of weighting functions. And because the H_∞ algorithm are non-convexing problem, it is very difficult to determine the optimal weighting functions. The MGA (modified genetic algorithm)[7],[8], which is an efficient optimizing tool in the non-convexing problem, is applied to determine the weighting functions. The objective function of the GA is

$$C = \sum_{t=0}^{\infty} (\gamma_1 |y(t) - y_d| + \gamma_2 |u(t) - u_d|) \text{ with } \gamma_1 = \gamma_2 = 1 \quad (12)$$

With this objective function, the weighting functions are calculated as

$$W_1(s) = \frac{1}{\gamma_1} \cdot \left(\frac{s+0.4049}{0.5092} \right), \quad W_3(s) = \frac{1}{\gamma_3} \cdot \left(\frac{s^2+2.3833s+1.8338}{s^2+2.3835s} \right)$$

$$W_2(s) = \frac{1}{\gamma_2}, \quad \gamma_1=0.1 \quad \gamma_2=0.2 \text{ and } \gamma_3=5 \quad (13)$$

$W_2(s)$ is applied to the control input to meet the system rank conditions for the solution existence. Then, Eq. (10) is set into the canonical form of Eq. (1), and the controller is calculated in line with the H_∞ algorithms as

$$H(s)_{MUS} = \frac{15.2s^2 + 113.3s + 98.05}{s^3 + 13.12s^2 + 80.79s + 131.6} \quad (14)$$

The simulation shows that the unit step response and the control effort of the $H(s)_{MUS}$ system are almost the same as those of the $H_{2,\infty}(s)$ system. But because of the augmentation of weighting function, the order of the controller increases to the third order (exactly, it is the fourth order but through the model reduction, it becomes of the third order). Also it should be noted that the $H(s)_{MUS}$ could be different depending on the objective function used in the GA. For example, if the larger penalty is given to the output, the system speed increases but at the expense of the larger control effort.

4 Feedback Controller

With the feedwater controller of Eq. (6), the feedwater station, $F(s)$, is represented as

$$F(s) = \frac{1.6714 \times 10^4 (s+1)}{s^4 + 9091s^3 + 3.0746 \times 10^4 s^2 + 3.8371 \times 10^4 s + 1.6714 \times 10^4} \quad (15)$$

By letting $G(s) = F(s)U_i(s, P)$ and by treating the effects due to $U_i(s, P)$ ($i=2, 3,$) as a disturbance, the level control system of Fig. 2 can be simplified as Fig. 7. And this scheme can easily be fashioned into the two port model of Eq. (1) with the coefficients of

$$G(s) \leftrightarrow [A, B, C, D]$$

$$B_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T, \quad B_2 = -B, \quad C_1 = \begin{pmatrix} C & 0 & 0 & 0 & 0 & 0 \end{pmatrix}^T,$$

$$D_{11} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad D_{12} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad C_2 = C, \quad D_{21} = (0 \ 1), \quad D_{22} = 0 \quad (16)$$

For the initial steady state power of 5%, the H_∞ controller is found to be

$$C(s) = \frac{2.174s^5 + 1.976 \times 10^4 s^4 + 4.734 \times 10^4 s^3 + 3.692 \times 10^4 s^2 + 468.7s + 19.26}{s^6 + 9092s^5 + 3.57 \times 10^4 s^4 + 5.034 \times 10^4 s^3 + 2.624 \times 10^4 s^2 + 339.3s + 13.76} \quad (17)$$

and has the PM of 89.2° with the GM of 36dB.

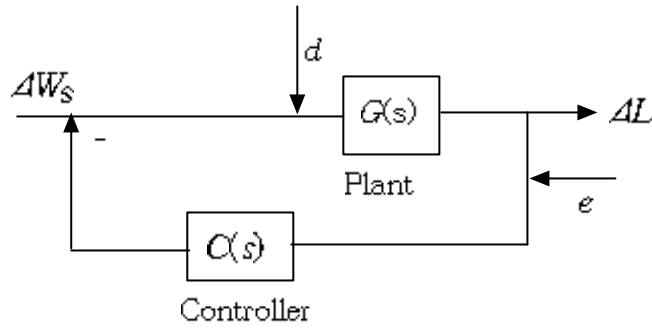


Figure 7. Level Control System with Feedback Controller

This margin is too large, and the system performance degrades to the impracticability. To improve the performance, an additional gain could be introduced. For example, the gain of 30 results in the PM margin of 65° with the plausible speed. But this gain value gives no effects when the power is high. As the power increases, the plant becomes more stable, hence the gain should be increased to maintain the same effects. To keep the same performance as above at 30% power, the gain should be increased to around 120, if the controller designed at 5% power is used.

With an additional gain, the design factors are gain values and the controller coefficients of Eq. (17). The gain value control depending on the power conforms with the concept of gain scheduling. But it is very difficult to provide the system with the flexibility only by gain control. On the other hand, the H_∞ controller becomes different with the power since the plant varies with the power. And the control of controller coefficients with continuously varying power is impractical, since the order of the controller is too high.

Instead of the H_∞ controller, it is found that the PID controller which was proposed in Ref.[2] is more practical since the number of coefficients to be controlled is only two. The proposed controller is

$$C(s, P) = \left(K_p(P) + \frac{K_i(P)}{s} \right) \quad K_p(P) = 34.26 + 3.85P + 0.2P^2$$

$$K_i(P) = \frac{K_p(P)}{641.3 - 60P + 2.1P^2} \quad P = \text{power in percent} \quad (18)$$

With this controller, the system has an almost constant PM of about 30° for the power range of 1% to 30%, and the GM increases slightly from 3dB at 1% power to 5dB at 30%. For the power range of over 30%, the designed controller yields the larger margins and another controller might be defined. However, since the steam generator level control is an issue in low power range, the controller of Eq. (18) is selected as a feedback loop controller.

5. Simulations and Discussions

In Fig. 2, the changes of feedwater flow rate(ΔW_F) and level(ΔL) are calculated by

$$\Delta W_F(s) = \frac{\Delta W_S(s) F(s) \{ (1 - C(s, P) U_2(s, P) \} - F(s) T(s, P)}{1 + F(s) U_1(s, P) C(s)} \quad (19)$$

$$\Delta L(s) = \frac{\Delta W_S(s) (U_2(s, P) + U_1(s, P) F(s)) + T(s, P)}{1 + F(s) U_1(s, P) C(s)} \quad (20)$$

where $T(s, P) = T_P(s, P) U_3(s, P) + T_F(s, P) U_4(s, P)$

And the power included in the transfer functions are obtained from

$$P(t) = L^{-1} \left[\sum_{i=1}^4 V_i(s, P) N_i(s) \right] N_1 = \Delta W_F \quad N_2 = \Delta W_S \quad N_3 = \Delta T_P \quad N_4 = \Delta T_F \quad (21)$$

The level deviation and feedwater flow rate change are calculated by the above equations together with the designed controllers of Eq. (5) and Eq. (18).

The simulations are made in parallel with those of Ref.[2] for the comparison. Two situations are simulated. One is the power increase from 5% to 10% and the other is the power decrease from 10% to 5%. The input conditions are the same as those of Ref. [2]. Also as in Ref.[2], three cases are considered. Case A is such that all the transfer functions and the controller varies with the power, Case B is such that the transfer functions varies continuously with the fixed controller, while in Case C, the transfer functions and controllers determined at the initial power are assumed to be fixed during the transients. In short,

$$\text{Case A : } H_i(s, P) = H_i(s, P(t)) \quad P_i(s, P) = P_i(s, P(t)) \quad C(s, P) = C(s, P(t))$$

$$\text{Case B : } H_i(s, P) = H_i(s, P(t)) \quad P_i(s, P) = P_i(s, P(t)) \quad C(s, P) = C(s, P_0) = \text{Const}$$

$$\text{Case C : } H_i(s, P) = H_i(s, P_0) = \text{Const} \quad P_i(s, P) = P_i(s, P_0) = \text{Const}$$

$$C(s, P) = C(s, P_0) = \text{Const}$$

Figure 8 shows the level variation for each case when the power is increased. Case C shows that if the variation of the plant properties are not considered, the control design is meaningless. Case A and B show the similar level responses. But the transients of Case B is somewhat milder than those of Case A. Further, although not shown in the figure, the feedwater transients of Case A is severer than those of Case B. In summary, Case B shows a little better dynamics than Case A. This is due to the fact that the controller determined at the low power gives a larger margins at high power. Comparing these results with those of Ref.[2], the overall trends are quite similar. But while the Case A in Ref.[2] shows the unstable oscillations, the responses of Case A in this study show a good stability.

The level transients for the power decrease are simulated also. It should be noted ,although the results are not shown, that Case A shows the milder results than Case B, contrary to the power increase. However, the dynamics of the two cases are almost the same as the LTR of Ref.[2], but the peak values of both cases decrease by 5 to 8%

from LTR controller.

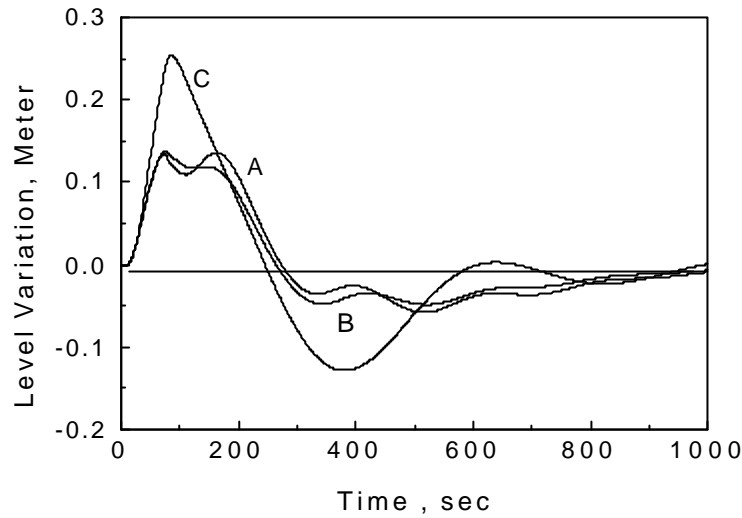


Figure 8. Level Transients, From 5% to 10% Power Increase

6. Conclusion

The control system design starts from the exact description of the plant to be controlled. But all the plant model have uncertainties. The control system based on the uncertain plant might does not work in the actual situation. The robust control takes the uncertainties into account as one of the design factor, and makes the system maintain the sufficient robustness in the real world.

For the steam generator, the H_{∞} method with controlling the state variables is found to be the most appropriate one for the feedwater controller design. The output response of the robust controller is almost the same as that of the LTR controller, but the robust controller decreases the control effort significantly. In contrast with the feedwater station, the robust control method is not proper for the design of the feedback loop controller.

The power dependent PID is preferable to the robust controller in the light of various control specifications. The simulations show a good performance with proper stability margins. However, since the plant varies through the transients, different operational mode is recommended. That is, for the case of power increase, the controller determined at the initial stage of transients is to be fixed, and for the case of power decrease, the controller is to be varied with the power.

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