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Numerical Analysis of Initial Formation of Thermal Stratification in a Curved Pipe

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Abstract

This paper presents a numerical analysis of transient behaviors of initial formation of thermal stratification in a curved pipe. A finite volume based thermal-hydraulic computer code is developed employing a body-fitted, non-orthogonal curvilinear coordinate and is applied to the analysis of initial formation of thermal stratification in a curved pipe. Using the computer code, detailed analyses of evolutions of temperature and velocity fields are performed. The Richardson number effect on the initial formation of thermal stratification is also investigated. It is shown that the initial formation of thermal stratification in a curved pipe is predicted fairly well by the present calculation method and is strongly dependent on the Richardson number.

1. Introduction

Thermal stratification can occur in a piping system where hot and cold fluids flow in and form two fluid layers due to the difference in fluid density (or temperature), with little mixing. In this situation, the cold (denser) fluid occupies the lower position of the pipe while the hot fluid the upper space. The difference in temperature of the two fluids may lead to considerable circumferential temperature gradients in the pipe wall, which can result in excessive differential expansion at the top and bottom of the pipe threatening the integrity of the piping system.

Some safety-related piping systems connected to reactor coolant systems at operating nuclear power plants are known to be potentially susceptible to unanticipated flow-induced thermal stratification which can lead to thermal fatigue damage to the piping systems. Several nuclear power plants have so far experienced such serious mechanical damages due to thermal fatigue such as pressurizer surge line movements and its support failures, and cracks in feedwater nozzle, high pressure safety injection lines, and residual heat removal lines. The understanding of thermal stratification in a curved piping system is very important for securing the structural integrity of components of nuclear power plant.

The temperature difference in a fluid region due to the thermal stratification produces thermal stress in the pipe in axial and circumferential directions. Several investigators have made efforts to determine the temperature distributions in the pipe wall by means of laboratory testing of particular geometry, field measurement of temperature or fully theoretical predictions. Talja and Hansjosten [1] and Wolf et al. [2] have made experimental efforts to measure the temperature distributions in a horizontal pipe wall and compared the measured and calculated stresses due to the thermal stratification. Basic theoretical predictions either by analytical method [3] or numerical method [4,5] have also been performed to understand the basic features of thermal stratification. Some investigators [6-8] have made practical turbulent calculations for analyzing the thermal stratification in a surge line of pressurized water reactor or in a curved pipe using the $k-e$ turbulence model. The thermal stratification in components of practical nuclear power plant involves temperature gradient, oscillations of the stratification interface and inherent local temperature fluctuations [9]. Time-variations of the temperature due to the turbulent fluctuations can be as critical as the spatial variations. Thus, the simulation of thermal stratification in a component of nuclear power plant by a statistical turbulence model like the $k-e$ turbulence model is not a proper way of predicting the temperature field.

In many practical problems the piping systems have complicated shaped boundaries, which require the use of non-orthogonal curvilinear coordinates for the numerical solution of fluid flow and heat transfer inside the piping system. The present state of the art of computational fluid dynamics can provide numerical solutions for these problems without particular difficulties. To the present author's knowledge there does not exist a detailed measurement that clearly shows the evolution of thermal stratification in a piping system. Ushijima [10] has provided an experimental measurement which shows the transient evolution of temperature field in a curved duct. Ushijima [10] has also carried out numerical calculations for velocity distributions, secondary flow patterns and temperature profiles for the evolution of thermal stratification in a curved duct. However, his measurement and calculation are not detailed enough to fully understand the initial formation of thermal stratification in a curved piping system. The main objective of present study is understanding of the basic physics of initial evolution of thermal stratification in a curved pipe by numerical analysis before going into the practical turbulent simulations using the large eddy simulation method.

The objectives of present study are : i) development and validation of a thermal hydraulic computer code for analysis of thermal stratification in a curved piping system, ii) detailed numerical investigation of initial evolution of thermal stratification in a curved pipe, iii) investigation of Richardson number effect on the initial evolution of thermal stratification. A finite volume based thermal hydraulic computer code, theoretically based on Peric [11] or Majumdar et al. [12], is developed and the computer code is validated through application to the experiment conducted by Ushijima [10]. The evolution of thermal stratification in a curved pipe such as the evolutions of the temperature field, the primary motion as well as the secondary motion is investigated. The Richardson number effect on the evolution of thermal stratification is also investigated by performing calculations changing the Richardson number.

2. Numerical Formulation

Governing Equations

For simplicity, it is assumed the thermally stratified fluid is Newtonian with constant properties and the Boussinesq approximation is valid. Then the governing equations for conservation of mass, momentum and energy in a generalized coordinate system x^j can be written as follows [11]:

$$\frac{\partial}{\partial x^j}(U_j) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}(Jru_i) + \frac{\partial}{\partial x^j}\left[U_j u_i - \frac{m}{J}\left\{\frac{\partial u_i}{\partial x^m} B_m^j + b_k^j w_i^k\right\} + Pb_i^j\right] = rg_i b(T - T_{ref})J \tag{2}$$

$$\frac{\partial}{\partial t}(Jr\,c_p T) + \frac{\partial}{\partial x^j}\left[U_j\,c_p T - \frac{k}{J}\frac{\partial T}{\partial x^m}\,B_m^j\right] = 0 \tag{3}$$

where $$U_i = r\, u_k b_k^i \;, \qquad B_m^j = b_k^j\, b_k^m \;, \qquad w_j^i = \frac{\partial u_i}{\partial x^k}\, b_j^k \qquad (4)$$

and u_i denotes the three Cartesian velocity components in the directions of the transformed coordinates $y^i = y^i(x^j)$, the geometric coefficients b_i^j represent the cofactors of $\partial y^i / \partial x^j$ in the Jacobian matrix of the coordinate transformation, J stands for the determinant of the Jacobian matrix and y^i is the Cartesian coordinate system.

Initial and Boundary Conditions

Consider a general situation of thermally stratified flow in a curved pipe where a fluid of the high temperature is flowing through the pipe at a constant flow rate so that the steady flow condition is maintained, and then the inlet temperature is lowered at a certain point of time. Because the solution domain is symmetric geometrically, only half of the solution domain is solved. Thus, at the symmetry plane, the symmetry boundary conditions are applied for both velocity components and temperature. On the solid wall, no slip and adiabatic boundary conditions are specified. For this situation the boundary conditions are given by

$u_i = u_{i,in}\,,\;\; T = T_{in}$ at the inlet of the pipe, (5a)

$u_i = 0\,,\;\; \left.\frac{\partial T}{\partial n}\right|_{x^2} = \left.\frac{\partial T}{\partial n}\right|_{x^3} = 0$ at the inner surface of the pipe, (5b)

$u_2 = 0\,,\;\; \frac{\partial u_1}{\partial x^2} = \frac{\partial u_3}{\partial x^2} = 0\,,\;\; \frac{\partial T}{\partial x^2} = 0$ at the symmetry plane, (5c)

$\frac{\partial T}{\partial x^1} = 0$ at the outlet of the pipe, (5d)

At the outlet, the velocity components are adjusted to satisfy the overall mass conservation.

3. Numerical Method

The solution domain is divided into a finite number of hexahedral control volume cells. A typical control volume is shown in Fig.1. The discretization of the governing equations is performed following the finite volume approach, and the convection terms are approximated by a higher-order bounded scheme COPLA developed by Choi et al. [13] and the unsteady term is treated implicitly using the three-level second order scheme suggested by Ferziger and Peric [14]. The non-staggered grid arrangement is adopted and the momentum and energy equations are solved at the geometric center of the each control volume cell. The momentum interpolation method originally developed by Rhie and Chow [15] and further modified by Majumdar [16] and Choi [17] is adopted for evaluating the cell-face velocities in the present study. The SIMPLE algorithm [18] is adopted for pressure and velocity coupling.

4. Results and Discussions

The numerical solution method presented in the present study is applied to the analysis of thermal stratification in a curved pipe as shown in Fig.2. Calculations are performed for symmetric half of the solution domain. The 84×22×42 numerical grids are generated algebraically and are shown in Fig.3. The Reynolds number based on the hydraulic diameter of the pipe and the inlet velocity is 500, and the Richardson number is 9.8. Since the Richardson number is relatively high, the buoyancy force affects strongly the flow field. First, the steady state solution is obtained with the temperature maintained at high temperature and then the transient solutions are obtained using the steady state solution as an initial condition. The inlet temperature is assumed to lower instantly by 10 degrees at the moment when the transient begins. Calculations are continued until 150 seconds using the time step size of 0.1 second. The convergence is declared at each time step when the maximum of the absolute sum of the residuals of momentum equations, pressure correction equation and energy equation is less than 10^{-4}.

Before investigating a detailed evolution of thermal stratification in a curved pipe, a primary calculation was performed to validate the computer code for duct flow with the same hydraulic diameter, Reynolds number and Richardson number as the present case except that the inlet temperature is assumed to be lowered linearly by 10 degrees during the initial 30 seconds, which was studied experimentally by Ushijima [10]. Fig.4 shows the comparison of predicted temperature distribution at the symmetry plane with the measured data by Ushijima [10]. We observe that the numerically predicted results fairly well agree with the experimental measurements, especially when the thermal stratification is established (t=120sec). Some discrepancies may be due to the differences in inlet conditions between measurement and calculation, which can be observed in the initial development of temperature in the measured data. However, we can notice that a slightly different imposition of inlet temperature in the initial period of time does not much influence the final formation of thermal stratification. A detailed numerical analysis of this flow is reported in our primary investigation [19], which demonstrated the effectiveness of the computer code.

In this study the computer code developed in the previous study [19] is slightly modified to apply for the simulation of stratified flow in the curved pipe model as depicted in Fig.3.

Fig. 5 shows the transient evolution of temperature field at the symmetry plane. The predicted temperature field is normalized using the hot temperature and cold temperature, and the interval of the isothermal line is 0.1. In the initial stage (t=5~10sec) the inlet flow moves upward pushing the hot fluid in the downstream direction. When the cold fluid reaches the first curved section (t=10~15sec), it begins to flow into the lower side of the pipe with the force balance between inertia of inlet flow and buoyancy of hot water. Even the cold fluid moves further downstream and reaches the outlet (t=20sec), the temperature of fluid in the upper portion of the pipe does not change and a steep temperature gradient is established at the interface between hot and cold fluid regions. The temperature gradient at the interface becomes steeper as time elapses and there exists a strong mixing near the outlet (t=25~50sec). A stable thermal stratification begins to be established in the upper portion of the pipe (t=50~70sec), and then the mixing near the outlet is finished and the stable thermal stratification does not change much with time (t=150sec). Since the Richardson number of the present problem is relatively high, the thermally stratified region covers for most upper portion of the pipe. One thing we note here is that the cold fluid can not mix well with the hot fluid once the thermal stratification is established.

Fig. 6 displays the predicted isothermal lines at the cross section of the center plane of the duct. These figures show well the development of the thermal stratification. At an earlier stage of the mixing process the temperature gradient near the lower wall is small and a little strange evolution of the temperature field in this region is due to the smearing of cold fluid, which can be seen in Fig. 4. The temperature gradient at the interface between cold fluid and hot fluid becomes steeper as the stratification is established (t=25sec). A steep temperature gradient is established around t=40sec. Then the temperature gradient at the upper portion of the interface becomes smaller (t=90-150sec) due to the heat transfer at the interface of hot and cold fluids.

Fig. 7 shows the development of velocity vectors at the symmetry plane. At the initial stage (t= 5sec) the velocity field is not changed much compared with the steady state solution (t=0sec). When the thermal stratification is begun to form balancing the inertia and buoyancy forces, the magnitude of fluid flows near the lower side of pipe becomes larger (t=15sec). There is a strong movement of cold fluid when the cold fluid passes the second curved section (t=25sec) since the incoming cold fluid could not penetrate into the hot fluid region. The strong movement of cold fluid pushes the existing hot fluid in both upward and downward directions and it induces a counterclockwise movement of hot fluid in upper region of the pipe and a strong clockwise vortex near the outlet (t=25sec). There exists a strong mixing near the outlet of the pipe. As the time elapses, the cold fluid flows into the outlet mixing with hot fluid and the counterclockwise vortex in the upper right region of the pipe becomes weak (t=70sec). There still exists a mixing near the outlet and the flow in this region is developing while the flow in upper region of the pipe is nearly quiescent (t=70~90). When the mixing process near the outlet is finished, the velocity field in this region as well as other region is nearly developed (t=120sec). We can see that the development of velocity field is consistent with that of temperature field.

Fig. 8 shows the development of a secondary motion at the cross-section of the center plane of the duct. There exists a relatively strong vortex initially (t=5sec) and the magnitude of vortex becomes weak as the mixing process continues (t=15-17.5sec). Then the secondary motion changes rapidly (t=20-50sec) and the complicated evolution of secondary motion continues until a stable stratification is established (t=70sec). After the stable stratification is established, the secondary motion in the hot fluid region is very weak and there exists a strong clockwise vortex at the right-bottom corner of the duct due to the relatively high strength of the primary motion in this region.

The effect of the Richardson number on the evolution of the thermally stratified flow in the duct is also investigated. For this purpose, a different flow condition that yield different value of the Richardson number for the flow situation is considered in the analysis. The flow condition is the same as that of the above-mentioned flow situation [10] except that the average inlet velocity in the duct is changed to $10\sqrt{2}$ mm/sec so that the Richardson number is decreased to 4.9.

Figs. 9, and 10 show the transient evolutions of temperature fields at the symmetry plane of the pipe and their evolutions at the cross-section of the center plane of the pipe for the different flow condition mentioned above. As shown in the figures, the effect of buoyancy force on the flow is relatively weakened as the Richardson number is lowered and thus the thermally stratified flow in the duct formed in the early stage disappears rapidly. This means that the evolution of temperature field is strongly depends on the magnitude of Richardson number.

It is seen based on the present investigations that the present calculation method can be applied to the prediction of thermal stratification in various piping systems, especially those in the nuclear power plant.

5. Conclusions

An effective numerical method for calculating the thermally stratified flows in a curved piping system has been presented. The method employs a body-fitted, non-orthogonal grid system to accommodate the various shapes of a piping system and an advanced convection model to obtain an accurate solution. The use of the momentum interpolation method for unsteady flow in a non-orthogonal grid system is presented. The computer code developed in the present study has been applied to the prediction of the thermal stratification in a reversed U-pipe of circular cross-section. In addition, the effects of Richardson number on the evolution of thermal stratification have been investigated in detail by analyzing two different situations of stratified flow in the reversed U-pipe.

As the result, it is seen that t the transient behaviors of thermally stratified flow in the reversed U-pipe are also well simulated. Consequently, the favorable results show that the present computer code can be applied to the prediction of practical thermal stratification phenomena in a nuclear power plant.

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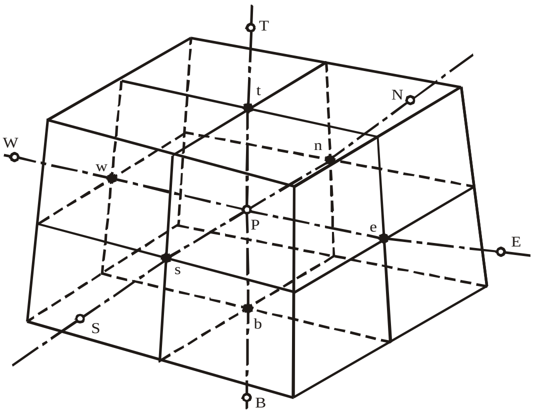

Fig. 1 A typical control volume

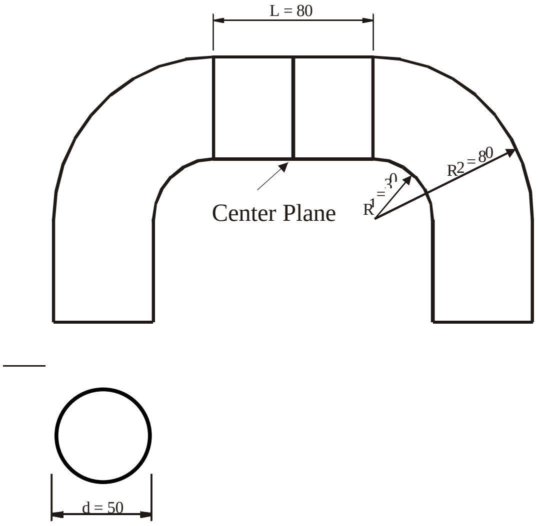

Fig. 2 Geometry of the reversed U-pipe of circular cross-section

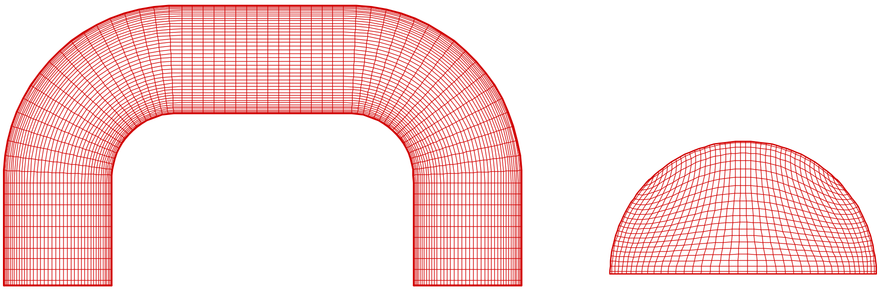

Fig. 3 Numerical grids for the computation model

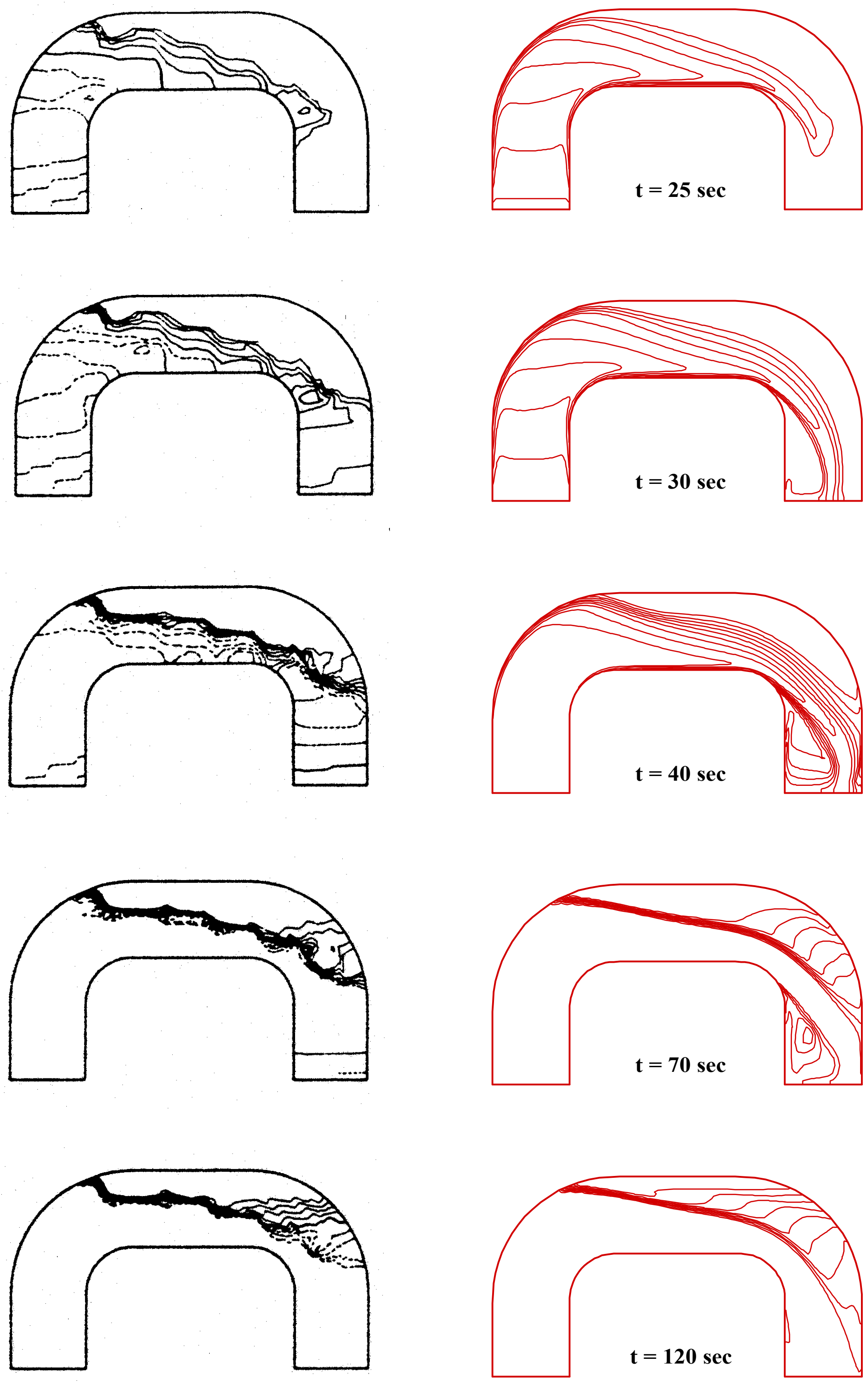

Fig. 4 Comparison between experiments and calculated results for temperature field
(left: experimental results, right: calculated results)

Fig. 5 Development of temperature field at the symmetry plane of the U-pipe for the case where the Richardson number is 9.8

Fig. 6 Development of temperature field at the center cross-sectional plane of the reversed U-pipe for the case where the Richardson number is 9.8

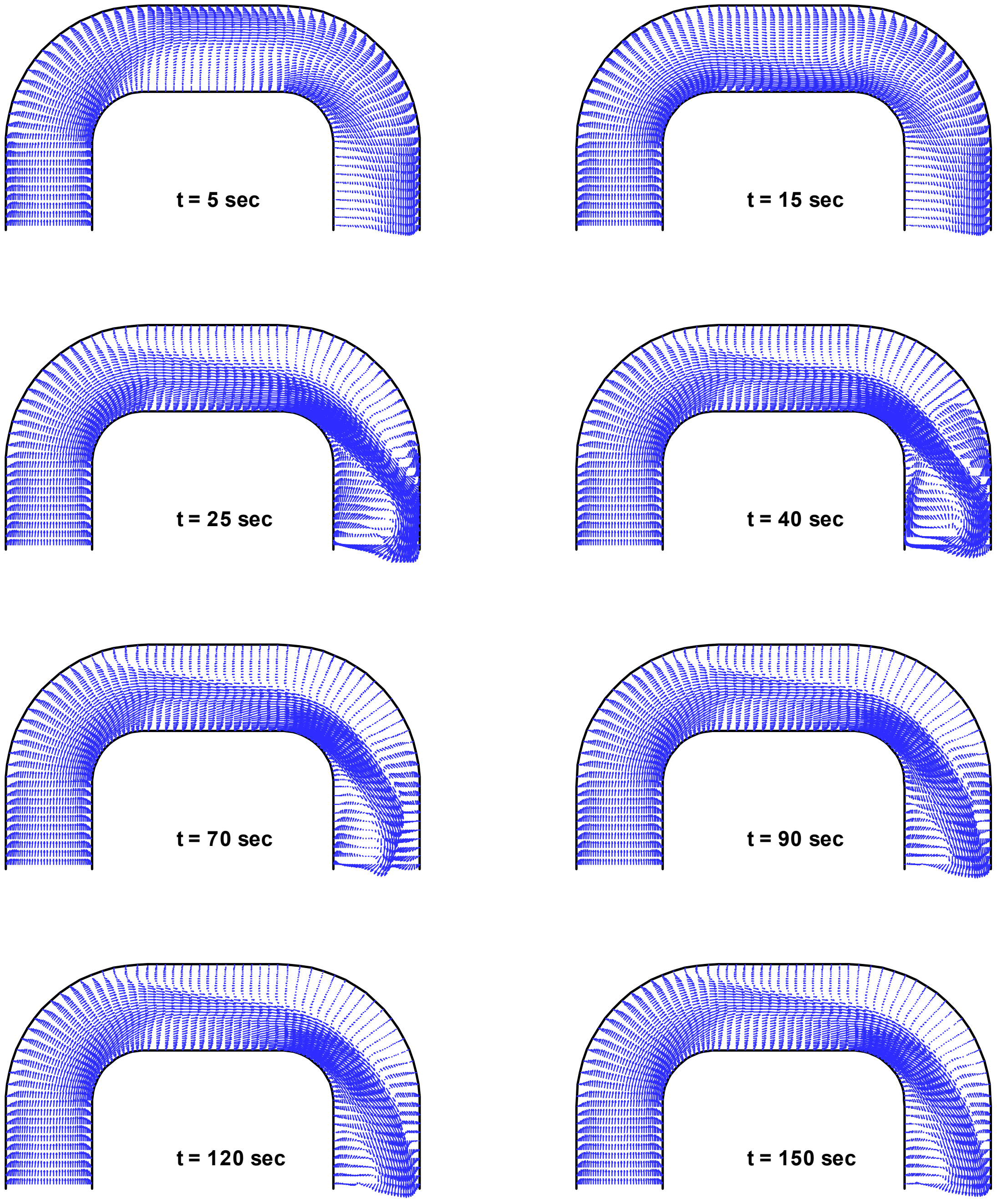

Fig.7 Development of velocity field at the symmetry plane of the U-pipe for the case where the Richardson number is 9.8.

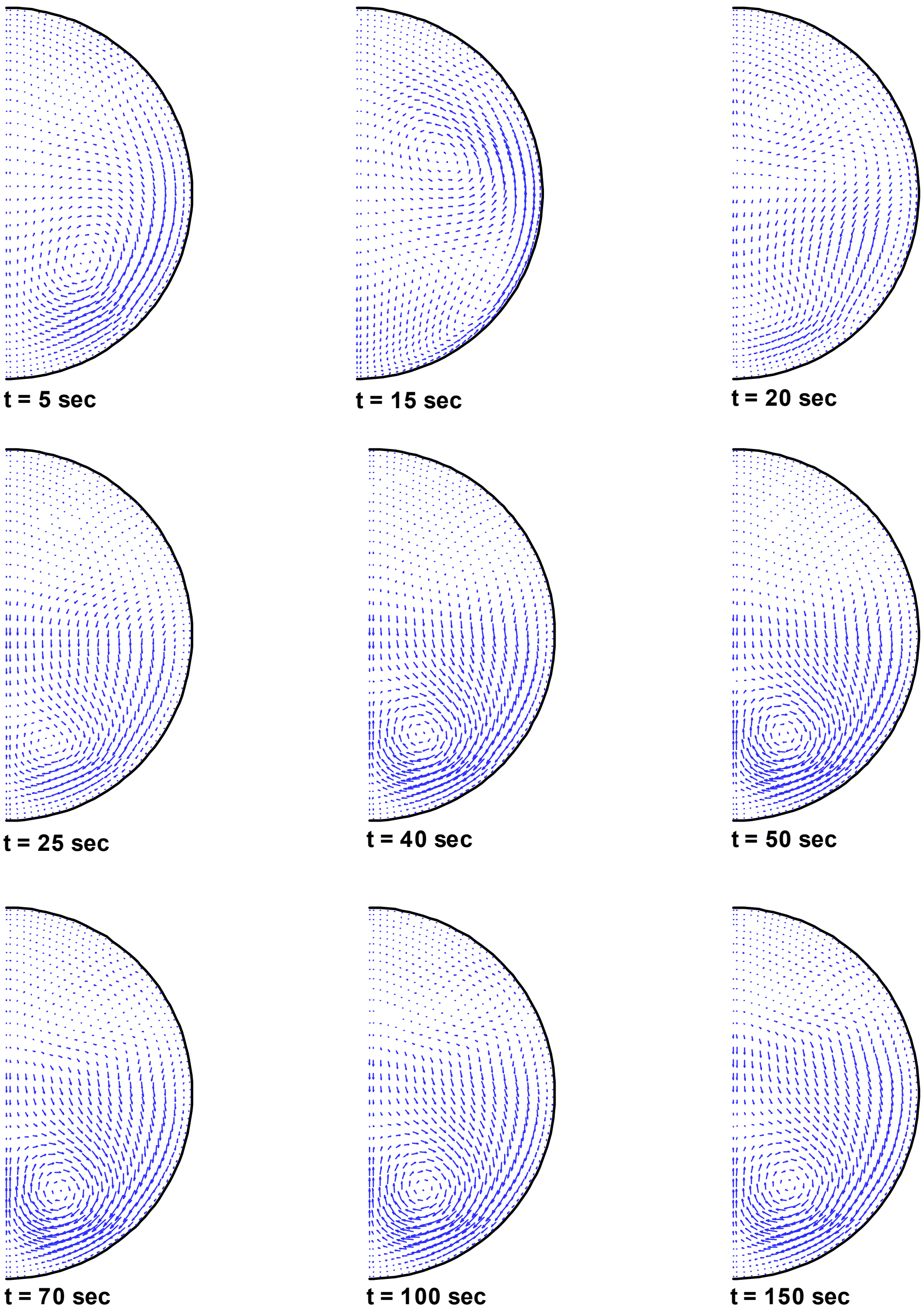

Fig. 8 Development of velocity field at the center plane of the U-pipe for the case where the Richardson number is 9.8

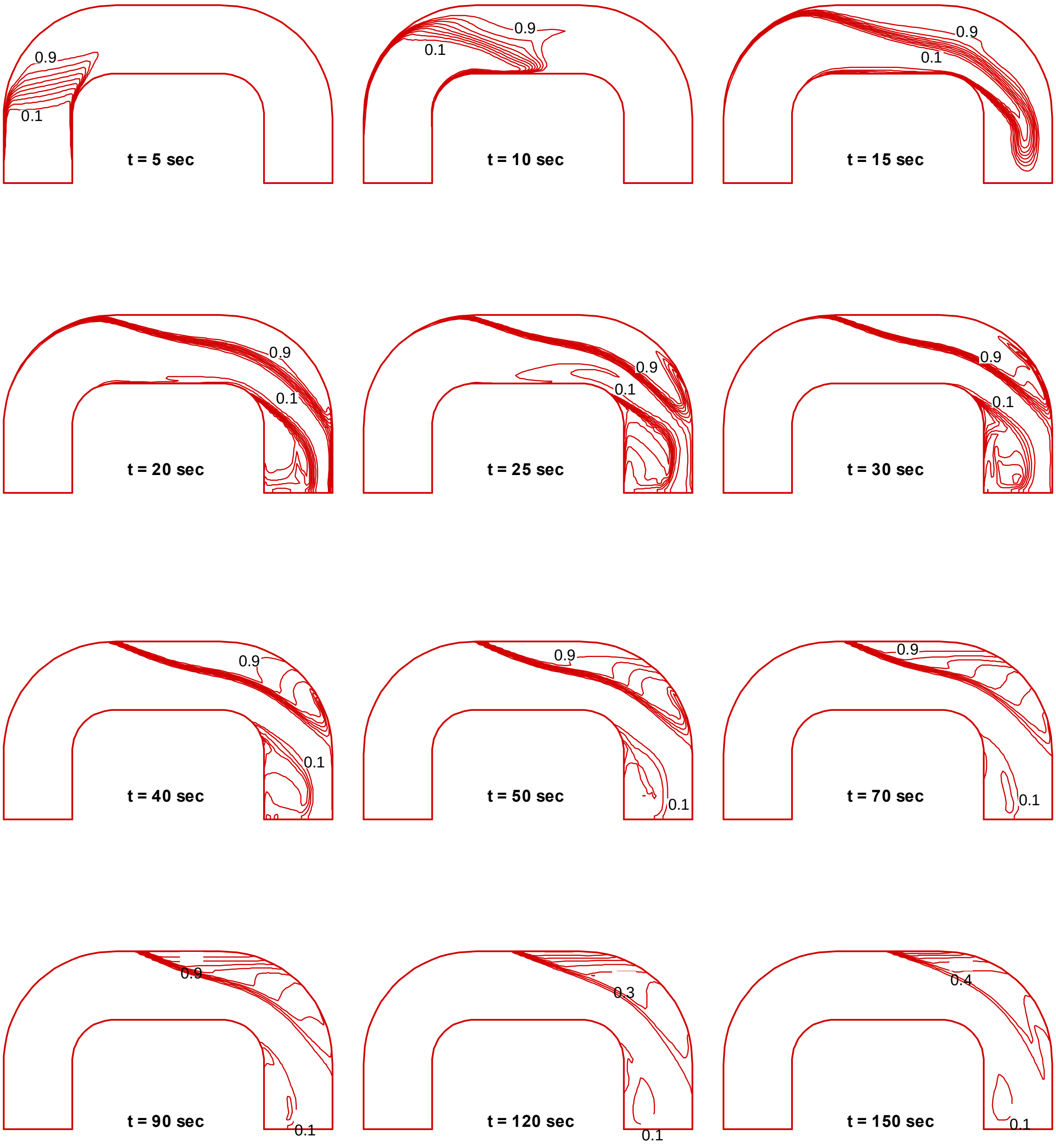

Fig. 9 Development of temperature field at the symmetry plane of the U-pipe for the case where the Richardson number is 4.9

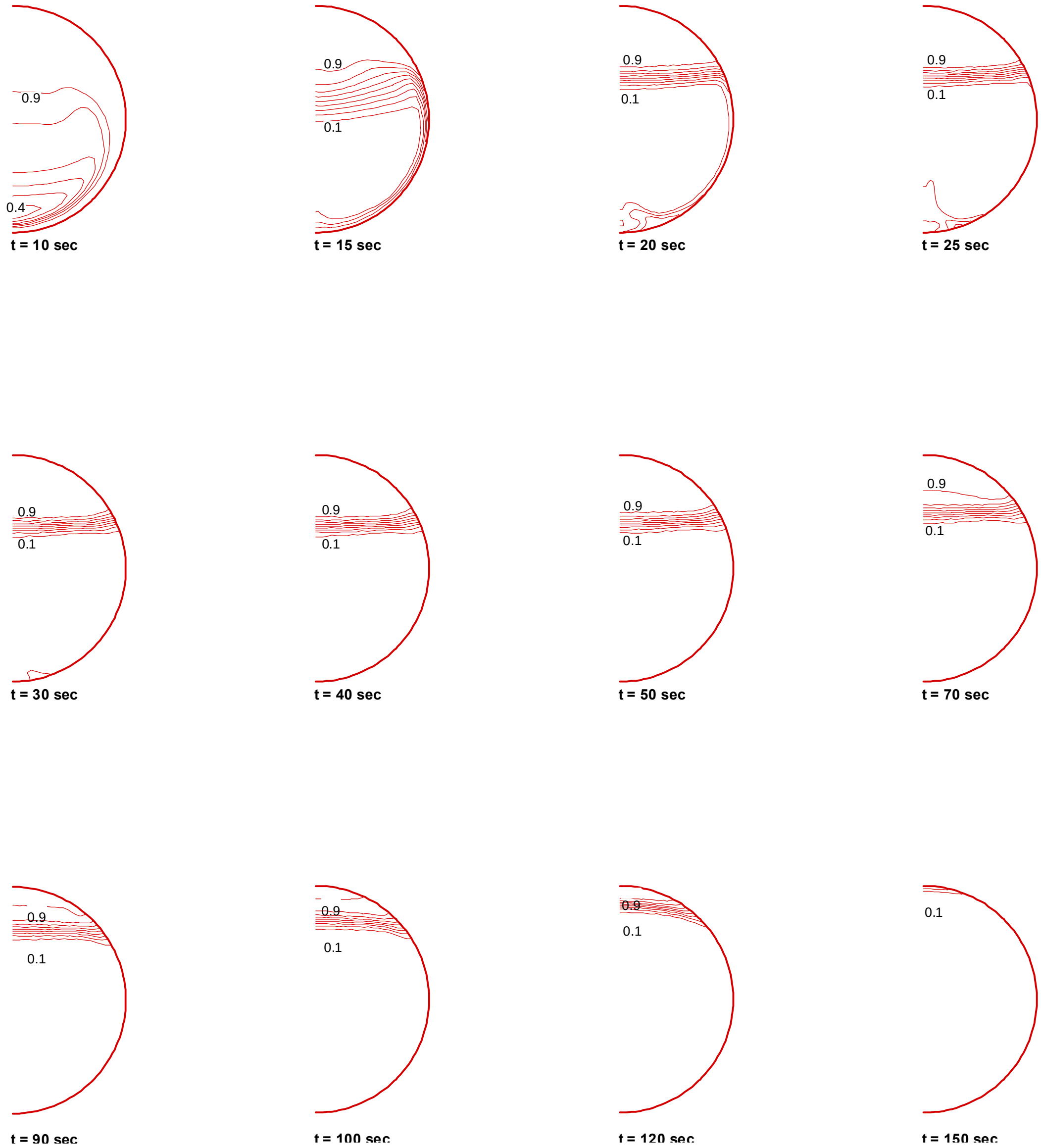

Fig. 10 Development of temperature field at the center cross-sectional plane of the reversed U-pipe for the case where the Richardson number is 4.9