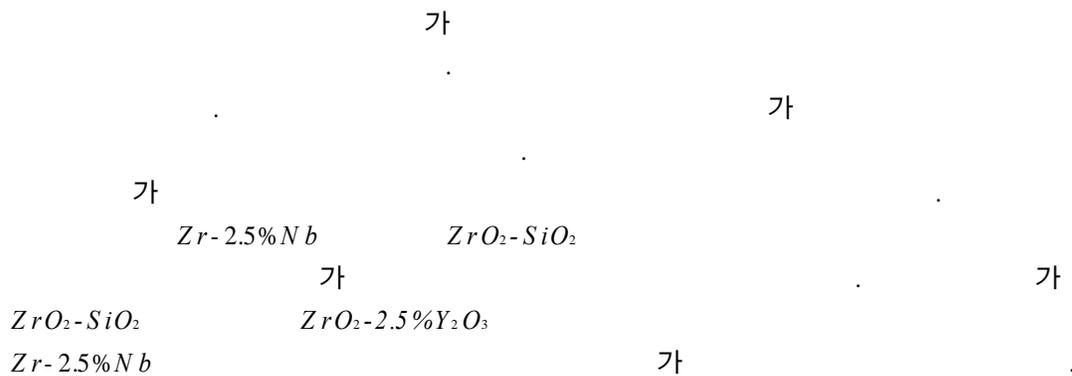


Zr-2.5%Nb

Thermal Stresses of Ceramic Coating on Zr-2.5%Nb Alloy

103-16



Abstract

Based on the principle of complementary energy, an analytical method is developed which focused on the end effects for determining thermal stress distributions in a coated beam. This method gives the stress distributions which completely satisfy the stress-free boundary condition at the edge. A numerical example is given in order to verify the method. The result shows that the shear and peeling stress distributions along the interface between the substrate and coat are significant near the edge and become negligible in the interior region. ZrO_2-SiO_2 layer coated on $Zr-2.5\%Nb$ alloy, which is the material for the pressure tube of PHWR, is investigated to address the coat thickness effects on the thermal stress distributions. The thermal stress distributions of the ZrO_2-SiO_2 coat on the zirconium alloy are compared with those of $ZrO_2-2.5\%Y_2O_3$ coat on that alloy in order to choose the adequate coating material.

1.

가 가

가

[1]. Zr-2.5%Nb

가

가 , 가

가 , 가

Grimado [2] Electronic Packaging,

Suhir [3, 4] Timoshenko [5]

Razaqpur [6] Suhir [7]

Yin [8, 9] Stress Function
(End-Effects)

가

Yin [10]

Zr-2.5%Nb

가

Zr-2.5%Nb

가

가

2.

1(unit)

가 Fig.

1

가

가 ΔT_k

2-

(Isotropic)

$$\sigma_x^k = \frac{E_k}{1 - \nu_k^2} (\epsilon_x^k + \nu_k \epsilon_y^k) - \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (1)$$

$$\sigma_y^k = \frac{E_k}{1 - \nu_k^2} (\epsilon_y^k + \nu_k \epsilon_x^k) - \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (2)$$

$$\tau_{xy}^k = \frac{E_k}{1 + \nu_k} \gamma_{xy}^k \quad (3)$$

, () k k- (k = 1, 2)

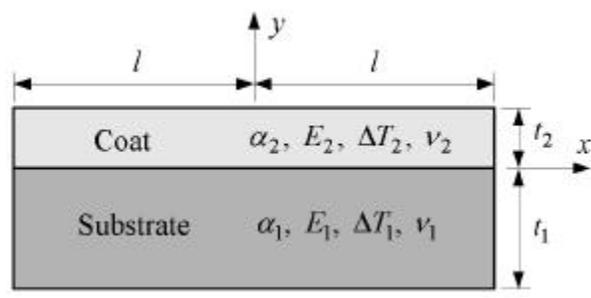


Fig. 1. Material properties of each layer and coordinate system

$k - (k = 1, 2)$

$$\frac{\partial \sigma_x^k}{\partial x} + \frac{\partial \tau_{xy}^k}{\partial y} = 0 \quad (4)$$

$$\frac{\partial \tau_{xy}^k}{\partial x} + \frac{\partial \sigma_y^k}{\partial y} = 0. \quad (5)$$

$$\sigma_x^k = \hat{\sigma}_x^k - \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (6)$$

$$\sigma_y^k = \hat{\sigma}_y^k - \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (7)$$

$$\tau_{xy}^k = \hat{\tau}_{xy}^k \quad (8)$$

가 가 (6), (7), (8)

((4), (5))

$$\frac{\partial \hat{\sigma}_x^k}{\partial x} + \frac{\partial \hat{\tau}_{xy}^k}{\partial y} = 0 \quad (9)$$

$$\frac{\partial \hat{\tau}_{xy}^k}{\partial x} + \frac{\partial \hat{\sigma}_y^k}{\partial y} = 0. \quad (10)$$

“0”

$$\hat{\sigma}_x^k (\pm l, y) = \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (11)$$

$$\hat{\tau}_{xy}^k (\pm l, y) = 0 \quad (12)$$

가

$$\hat{\sigma}_y^2(x, t_2) = \frac{\alpha_2 E_2 \Delta T_2}{1 - \nu_2} \quad (13)$$

$$\hat{\tau}_{xy}^2(x, t_2) = 0 \quad (14)$$

$$\hat{\sigma}_y^1(x, -t_1) = \frac{\alpha_1 E_1 \Delta T_1}{1 - \nu_1} \quad (15)$$

$$\hat{\tau}_{xy}^1(x, -t_1) = 0 \quad (16)$$

$$\hat{\sigma}_y^1(x, 0) = \hat{\sigma}_y^2(x, 0) + \frac{\alpha_1 E_1 \Delta T_1}{1 - \nu_1} - \frac{\alpha_2 E_2 \Delta T_2}{1 - \nu_2} \quad (17)$$

$$\hat{\tau}_{xy}^1(x, 0) = \hat{\tau}_{xy}^2(x, 0) \quad (18)$$

3.

x -

가

$$\hat{\sigma}_x^k(x, y) = \sum_{i=1}^{n_k+1} \sigma_{(i-1)}^k(x) \left(\frac{y}{l_k}\right)^{i-1} \quad (19)$$

$$, \quad n_k (k = 1, 2) \quad \sigma_{(i-1)}^k(x) \quad (n_k + 1) \quad (x = \pm l) \quad (11) \quad (19)$$

$$\sigma_o^k(\pm l) = \frac{\alpha_k E_k \Delta T_k}{1 - \nu_k} \quad (20)$$

$$\sigma_m^k(\pm l) = 0 \quad (m = 1, 2, 3, \dots, n_k) \quad (21)$$

$$\hat{\sigma}_y^k(x, y) \quad \text{가} \quad (9), (10) \quad \sigma_{(i-1)}^k(x) \quad \hat{\tau}_{xy}^k(x, y) \quad \text{가} \quad (12)$$

$$\left. \frac{d\sigma_m^k}{dx} \right|_{x=\pm l} = 0 \quad (m = 1, 2, 3, \dots, n_k) \quad (22)$$

$$(17), (18) \quad (\sigma_y), \quad (\tau_{xy}), \quad (\sigma_o^2, \sigma_1^2, \dots, \sigma_{n_2}^2) \quad 2 \text{ 가} \quad (20), (21), (22) \quad (n_1 + n_2)$$

4.

(Auxiliary Stress Field) 가

$$\frac{\partial^2 \hat{\epsilon}_x^k}{\partial y^2} + \frac{\partial^2 \hat{\epsilon}_y^k}{\partial x^2} = \frac{\partial^2 \hat{\gamma}_{xy}^k}{\partial x \partial y} \quad (23)$$

Complementary

Principle of Complementary Energy

Complementary 0

2 Complementary Energy

$$V^* = \int_{-l}^l \int_t \sum_{k=1}^2 \frac{1}{2E_k} \{ (\hat{\sigma}_x^k)^2 + (\hat{\sigma}_y^k)^2 - 2\nu_k \hat{\sigma}_x^k \hat{\sigma}_y^k + 2(1 + \nu_k) (\hat{\tau}_{xy}^k)^2 \} dy dx \quad (24)$$

(Principle of Stationary Complementary Energy)

$\sigma_{m_1}^1(x)$ ($m_1 = 0, 1, 2, \dots, n_1$), $\sigma_{m_2}^2(x)$ ($m_2 = 0, 1, 2, \dots, (n_2 - 2)$)

$$[L]\{\sigma\} = \{a\} \quad (25)$$

[L] Square Symmetric L_{rs} ($r, s = 1, 2, 3, \dots, (n_1 + n_2)$)

$$L_{rs} = A_{rs} \frac{d^4}{dx^4} + B_{rs} \frac{d^2}{dx^2} + C_{rs} \quad (26)$$

A_{rs}, B_{rs}, C_{rs} (26) y -

(Column Matrix $\{a\}$) Column Matrix $\{\sigma\}$

$$\{\sigma\} = [\sigma_o^1, \sigma_1^1, \dots, \sigma_{n_2-3}^2, \sigma_{n_2-2}^2]^T \quad (27)$$

(22), (23), (24)) (27) [10]

$$(21) \quad n_k (k = 1, 2)$$

$$[11]. \quad (27)$$

$$(1) \quad \text{가}$$

5.

5.1.

Yin [9,

10]

Mathematica [12]

Table 1

2 l

50.8 mm,

2.54 mm

ΔT_k

240 °C가

가

가

가 (19)

가

$(n_k (k = 1, 2) \quad 1 \quad)$

,

$n_1 \quad 3,$

$n_2 \quad 1 \quad 3$

x -

가 Fig.

2 Fig. 3

Yin

가

(가

)

가

, 가

y -

Fig. 4

“0”

Fig. 5

“0”

가

((19) n_k)

가

n_k

“0”

가

(Singular

Point)

가

(, x 가 $\pm (l + |\varepsilon|)$)

“0”

(, x 가 $\pm (l - |\varepsilon|)$)

“0”

. Nissley [13]

(Stress Singularity)

Table 1. Material properties of each layer

| Material | E (GPa) | ν | α (/ °C) |
|----------|-----------|-------|-----------------------|
| Layer 1 | 70.38 | 0.345 | 23.6×10^{-6} |
| Layer 2 | 324.7 | 0.293 | 4.9×10^{-6} |

Table 2. Material properties of Zircaloy and Zircon [12, 13]

| Material | E (GPa) | ν | α (/ °C) |
|------------------------------------|-----------|-------|-----------------------|
| Zr-2.5%Nb | 90 | 0.39 | 5.20×10^{-6} |
| ZrO ₂ -SiO ₂ | 103.5 | 0.30 | 4.50×10^{-6} |

Fig. 6, 7

(, $y \neq 0$)
 $x -$

$x -$ 가
 “0”
 가 가

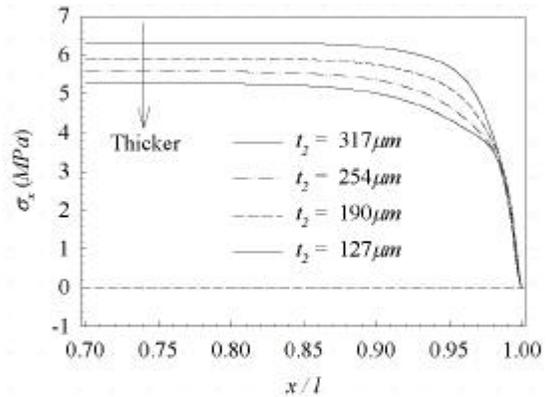
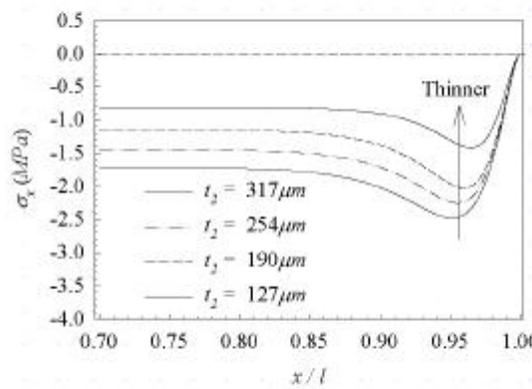
Fig. 8

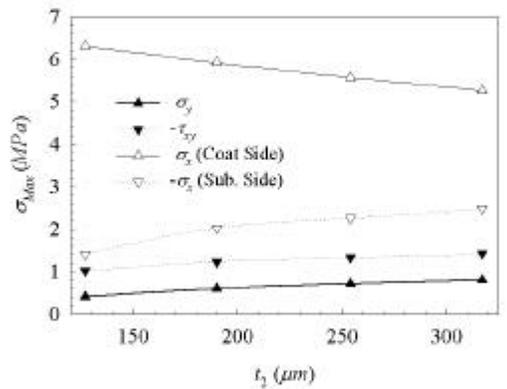
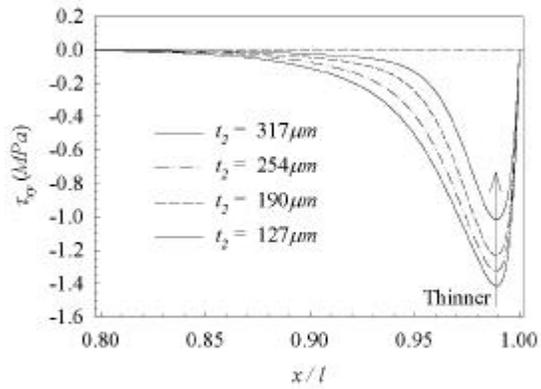
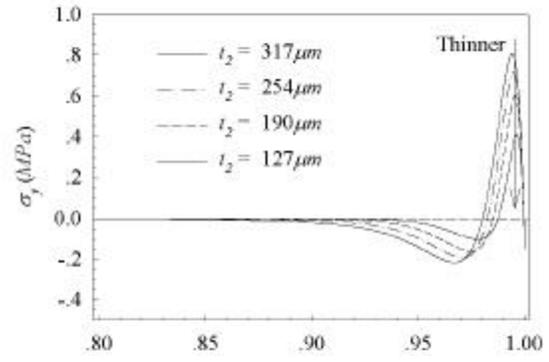
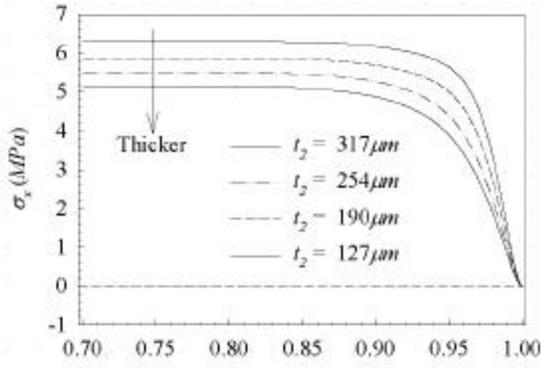
$$(19) \quad n_k \quad \int_0^{t_2} \sigma_x^2(x, y) dy / t_2$$

$x -$ 가
 $y -$ 가

가 Fig. 9

Fig. 10





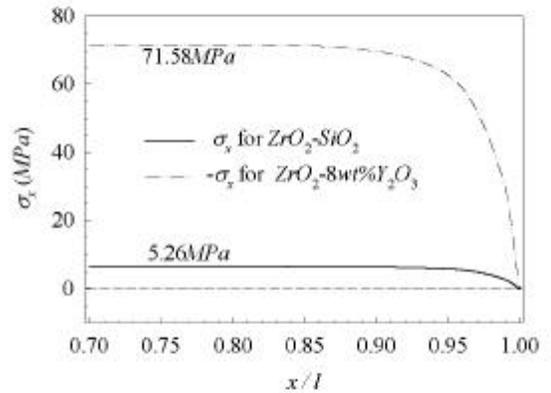
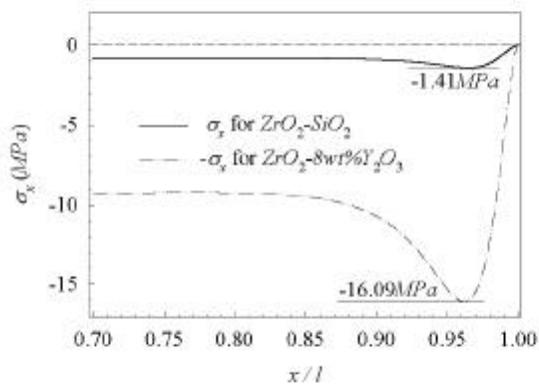
5.3.

$ZrO_2-8wt\%Y_2O_3$, 11.0×10^{-6}
 가 ($ZrO_2-8wt\%Y_2O_3$)
 $9.0 \sim 12.0 \times 10^{-6} / ^\circ C$ (TBC, thermal barrier coating)
 $ZrO_2-8wt\%Y_2O_3$ $ZrO_2-8wt\%Y_2O_3$ 가
 ($Zr-2.5\%Nb$) [1]. $Zr-2.5\%Nb$
 $(5.20 \times 10^{-6} / ^\circ C)$ 가 (Table 3),
 $ZrO_2-8wt\%Y_2O_3$

Table 3. Material properties of Zircaloy, Zircon and Zirconia-Yttria

| Material | E (GPa) | ν | α (/ °C) |
|---|-----------|-------|-----------------------|
| Zr-2.5%Nb | 90 | 0.39 | 5.20×10^{-6} |
| ZrO ₂ -SiO ₂ | 103.5 | 0.30 | 4.50×10^{-6} |
| ZrO ₂ -8wt%Y ₂ O ₃ | 150 | 0.3 | 11.0×10^{-6} |

가
 Zr-2.5%Nb Zr-2.5%Nb
 가 ZrO₂-SiO₂
 ZrO₂-8wt%Y₂O₃
 Table 3 (Zr-2.5%Nb) 4 mm, (ZrO₂-SiO₂)
 127 μm 2l 50 mm (19) n_k
 3, 1
 100 °C
 Zr-2.5%Nb ZrO₂-8wt%Y₂O₃ 가
 ZrO₂-SiO₂ 가 가
 가 “-” Fig. 12 Fig. 15
 ZrO₂-8wt%Y₂O₃ 가 ZrO₂-SiO₂ 가
 Zr-2.5%Nb



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